Postprint: Probabilistic Power Flow Algorithm Based on Edgeworth Series and Multilinearization

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Abstract

For large-scale probabilistic power flow problems involving random variables, the Gram-Charlier series expansion approach for approximating probability distributions entails significant computational burden. A probabilistic power flow model is established that considers the stochastic nature of wind power generation and load variability, and the recursive property of Hermite orthogonal polynomials in the Edgeworth expansion is utilized to reduce the computational complexity of high-order series expansions. To address the degradation of approximation accuracy caused by truncating high-order terms in the series transformation method, a multiple linearization approach is proposed, which reduces truncation error by uniformly partitioning the total active power of the system, thereby mitigating errors arising from the large variation range of input random variables during power flow equation linearization. Simulation results on the IEEE 39-bus system validate the effectiveness of the proposed method.

Full Text

Preamble

A Probabilistic Power Flow Algorithm Based on Edgeworth Series and Multiple Linearization

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Abstract: For large-scale probabilistic power flow problems with random variables, using Gram-Charlier series expansion to approximate the probability distribution of random variables entails significant computational burden. This paper establishes a probabilistic power flow model that accounts for the randomness of wind power output and load fluctuations. To reduce the computational complexity of high-order series expansion, the method leverages the recursive property of Hermite orthogonal polynomials in Edgeworth expansion. To address the accuracy loss caused by truncating high-order terms in series transformation methods, a multiple linearization approach is introduced. By uniformly partitioning the system' s total active power, this approach reduces truncation errors arising from the large variation range of input random variables during power flow linearization. Simulation results from the IEEE 39-bus system validate the effectiveness of the proposed method.

Keywords: Probabilistic power flow, multiple linearization, recursive property, Edgeworth series

1 Introduction

Probabilistic power flow analysis enables dispatch and maintenance personnel to evaluate the impact of random factor variations on secure and stable power flow operation, thereby determining safe dispatch strategies. In probabilistic power flow analysis, beyond predictable factors such as conventional thermal generator outputs and network impedance, it is essential to consider the influence of random disturbances like the unstable output of renewable energy sources such as wind power and load fluctuations on system security [1].

Reference [2] calculates the semi-invariants of load and wind turbine output based on the expected values of bus voltages and branch power flows and the sensitivity matrix, then obtains the probability distribution function using Gram-Charlier series expansion. However, Gram-Charlier expansion requires calculating each order moment of the expansion term through various simulated sampling sequences, resulting in substantial computational effort for high-order expansion. Reference [3] employs Monte Carlo Simulation (MCS) to obtain input variable samples through simple random sampling, performs deterministic power flow calculations for each sample point, and finally obtains the distribution of state variables through statistical analysis. This method is conceptually simple and can achieve high accuracy with sufficiently large sample sizes, but its drawback lies in the large number of simulations required, making it time-consuming for large-scale probabilistic power flow problems.

This paper first establishes an AC probabilistic power flow model considering wind power output instability and load fluctuations, and uses a multiple linearization method to linearize the power flow equations by uniformly partitioning the system' s total active power. The probability distribution of random state variables in the power flow equations is then calculated through Edgeworth series expansion. Finally, the impact of wind power output uncertainty and load fluctuations on the approximation accuracy of series-expansion-based probabilistic power flow is analyzed through case simulations on the IEEE 39-

bus system, with a comparison of approximation accuracy and computational speed across different series expansion orders.

2.1 AC Power Flow Model and Its Cumulants

The AC power flow model is given by:

$$\sum_{j\in i} Y_{ij}V_j\cos\theta_{ij} + P_{Li} - P_{Gi} - P_{Wi} = 0 \quad i \in S_{PV} \cup S_{PQ} \tag{1}$$

$$\sum_{j\in i} Y_{ij}V_j\sin\theta_{ij} + Q_{Li} - Q_{Gi} - Q_{Wi} = 0 \quad i \in S_{PQ} \tag{2}$$

where S_{PV} and S_{PQ} represent the sets of PV and PQ buses in the system; P_{Wi} and Q_{Wi} are the active and reactive power outputs of wind turbines at bus i; P_{Li} and Q_{Li} are the active and reactive loads at bus i; V_i and δ_i are the voltage magnitude and angle at bus i; Y_{ij} is an element of the bus admittance matrix; and α_{ij} is the phase angle of Y_{ij}, with $\delta_{ij} = \delta_i - \delta_j - \alpha_{ij}$. This probabilistic power flow considers load fluctuations and wind turbine output uncertainties, making P_{Wi}, Q_{Wi}, P_{Li}, and Q_{Li} random variables.

Linearizing the power flow equations in (1) and assuming independent random variations in power injection at each bus, the system state variables become a linear sum of the random variables of power injection, with weighting coefficients equal to sensitivity coefficients [4]. Based on this, Taylor expansion is applied at the base operating point while ignoring higher-order terms above the second order, yielding the single-point expansion model:

$$X = X_0 + S_0(W - W_0) \tag{3}$$

$$Z = Z_0 + T_0(W - W_0) \tag{4}$$

where X_0, Z_0, and W_0 are the bus voltages, branch power flows, and bus power injections at the base operating point; W is the random variable of bus power injection; and S_0 and T_0 are sensitivity matrices of bus voltages and branch power flows with respect to changes in power injection [5], with $S_0 = J_0^{-1}$ and $T_0 = G_{0J}0^{-1}$, where J_0 is the Jacobian matrix and $G_0 = (\partial Z_0/\partial X_0)|_{X=X_0}$.

Using the properties of cumulants, the k-th order cumulants of state variables ΔX and branch power flows ΔZ can be obtained as [6]:

$$\Delta X^{(k)} = S_0^{(k)} \cdot \Delta W^{(k)} \tag{5}$$

$$\Delta Z^{(k)} = T_0^{(k)} \cdot \Delta W^{(k)} \tag{6}$$

where $S_0^{(k)}$ and $T_0^{(k)}$ are matrices composed of the k-th powers of elements in S_0 and T_0, and $\Delta W^{(k)}$ is the k-th power of the bus injection power variation. This allows calculation of the cumulants of generators and loads based on the distribution of generator outputs, wind turbine outputs, and loads [7].

2.2 Edgeworth Series Expansion for Probabilistic Power Flow Equations

Traditional Gram-Charlier series expansion for approximating the Cumulative Distribution Function (CDF) of state variables and branch power flows requires 逐项计算各阶 Hermite 正交矩阵, resulting in large computational effort. To address this, the Edgeworth expansion method is adopted, which can approximate the probability distribution function of random variables following any distribution through the standard normal distribution function, with relatively simple asymptotic expansion calculations [8].

First, define the standardized form y of random variable x as:

$$y = \frac{x-\mu}{\sigma}$$

where μ is the mean of x and σ is the standard deviation. Let $F(x)$ and $f(x)$ be the CDF and Probability Density Function (PDF) of y, respectively, with $f(x) = F'(x)$. The Edgeworth expansion of $f(x)$ is expressed as:

$$f(t) = \phi(y)\left[1 + \frac{k_3}{3!}H_3(y) + \frac{k_4}{4!}H_4(y) + \frac{10k_3^2}{6!}H_6(y) + \frac{k_5}{5!}H_5(y) + \frac{35k_{3k}4}{7!}H_7(y) + \cdots\right]$$

where $\phi(y)$ is the PDF of the standard normal distribution, $H_i(y)$ is the i-th order Hermite polynomial, and k_i is the i-th order cumulant of y, with $k_i = E[(y - E(y))^i]$. The Hermite polynomials have the following recursive relationship:

$$H_{j+1}(y) = yH_j(y) - jH_{j-1}(y) \tag{7}$$

$$H_0(y) = 1, \quad H_1(y) = y \tag{8}$$

Due to the orthogonality of Edgeworth' s Hermite polynomials in Hilbert space, subsequent terms of k_i can be determined from previous terms through recursive relationships [9]. After obtaining the initial terms, higher-order terms can be quickly derived using polynomial techniques from the previous terms, thereby reducing computational effort.

2.3 Multiple Linearization of Power Flow Equations

When the variation range of input random variables is large, truncating high-order terms after series expansion introduces significant truncation errors in the single-point linearization model of (2). To improve the computational speed of series transformation methods without sacrificing excessive accuracy, a multiple linearization method is introduced.

Let P_{tot} be the system' s total active power:

$$P_{tot} = \sum_{i=1}^{N_N} P_{Li} + \sum_{j=1}^{N_G} P_{Gj}$$

where N_N is the number of PQ buses, P_{Li} is the active load at bus i, N_G is the number of PV buses, and P_{Gj} is the active power at generator bus j. Since both load power and renewable energy output are random variables, P_{tot} is also a random variable.

The multiple linearization method first partitions P_{tot} into R_T regions with equal spacing. Corresponding base operating points are selected within regions R_1 to R_T, and the power flow equations are linearized at each point:

$$X_{R1} = X_{R10} + S_{R1}(W_{R1} - W_{R10}) \quad (9)$$

$$X_{R2} = X_{R20} + S_{R2}(W_{R2} - W_{R20}) \quad (10)$$

$$\vdots \quad (11)$$

$$X_{RT} = X_{RT0} + S_{RT}(W_{RT} - W_{RT0}) \quad (12)$$

where S_{Ri} is the sensitivity matrix for region R_i, X_{Ri0} is the state vector for region R_i, and W_{Ri} is the base power vector for region R_i.

The algorithm flowchart for the proposed probabilistic power flow method based on multiple linearization and Edgeworth series expansion is shown in [Figure 1: see original paper].

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3 Case Study Analysis

To verify the feasibility of the proposed method, the IEEE 39-bus system is used as a test case. This system contains 39 buses and 46 branches. Wind turbine output is assumed to follow a Weibull distribution, while loads follow a normal distribution. All random variables are independent, with models described in reference [10], and the original system load forecast data serves as the expected value of the distribution.

To validate the accuracy of the proposed method, results from both the proposed method and the Gram-Charlier series method are compared against those obtained using MCS. The MCS sampling count N_d is set to 1,000, and variance and Average Root Mean Square (ARMS) [11] are used as accuracy evaluation metrics:

$$\text{ARMS} = \sqrt{\frac{\sum_{i=1}^{N}(C_{CEi} - C_{MCi})^2}{N}}$$

where C_{CEi} and C_{MCi} are the values at the i-th point on the cumulative distribution curves of output random variables obtained by the cumulant method and MCS method, respectively, and N is the number of buses.

Two scenarios are analyzed: Scenario 1 considers only load fluctuations, while Scenario 2 considers both load fluctuations and wind turbine output variations.

[Figure 2: see original paper] shows the average and maximum values of the bus voltage magnitude accuracy metrics for Scenario 1 as a function of load fluctuation standard deviation. When only load fluctuations are considered, both series expansion methods exhibit high approximation accuracy. This is because load fluctuations are assumed to follow a normal distribution, and both series represent the input random variable distribution functions as series composed of derivatives of normal random variables [12]. As load fluctuation increases, the approximation accuracy of the cumulant method decreases. This occurs because larger load fluctuation standard deviations increase the probability that injection power deviates far from the expected load power value, thereby increasing errors introduced by power flow linearization.

In Scenario 2, a Vestas V112-2.5MW wind turbine is connected to bus 11 on the basis of Scenario 1, with cut-in, cut-out, and rated wind speeds of 4 m/s, 25 m/s, and 16 m/s, respectively. A constant power factor control strategy is adopted, and the two-parameter Weibull distribution parameters for wind speed are set to $k = 3.97$ and $c = 10.7$.

[Figure 3: see original paper] shows the voltage magnitude approximation accuracy metrics for both methods under different wind turbine capacities in Scenario 2. The results indicate: (1) As wind turbine capacity increases, the ARMS metric values for both methods increase, showing a decreasing trend in approximation accuracy; (2) Since wind turbine capacity corresponds to the maximum range of random variable values, the maximum ARMS value shows a more pronounced increasing trend with wind turbine capacity, while the mean value changes more gradually due to averaging over a large sampling base; (3) The fitting accuracy of the proposed Edgeworth series expansion method based on multiple linearization is slightly better than that of the Gram-Charlier series, with the difference becoming more significant as wind turbine capacity increases. This improvement stems from the multiple linearization method, which partitions the system' s total active power into several regions and linearizes the power flow equations at base operating points in each region, thereby reducing truncation errors caused by deviations between the base point and actual operating points in single-point linearization models.

To compare the computational time of the proposed method, Gram-Charlier expansion, and MCS, the probabilistic distribution of active power flow on line 13-14 is calculated for Scenario 2 with a wind turbine capacity of 25 MW. The results are shown in (MCS is not compared with itself). The proposed method' s approximation accuracy is slightly lower than that of Gram-Charlier expansion because the Edgeworth expansion approximates high-order terms using only Hermite polynomial information, introducing fitting errors. With 1,000 sampling points, the proposed method is approximately 22% faster than MCS, as MCS requires generating random samples of corresponding distributions and repeatedly performing deterministic power flow calculations for each simulation scenario. The proposed method is also slightly faster than Gram-Charlier expansion because Gram-Charlier requires recalculating each order moment for every expansion order, whereas Edgeworth polynomials have orthogonality in Hilbert space, allowing high-order Hermite polynomials to be determined recursively from low-order terms. After obtaining low-order moments, high-order moments can be quickly derived recursively, thus accelerating computation.

Comparison of calculation time of different methods

Method	ARMS (%)	Calculation Time (s)
Proposed method	[value]	[value]
Gram-Charlier series	[value]	[value]

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4 Conclusion

This paper proposes a probabilistic power flow algorithm based on Edgeworth series expansion, which reduces the computational complexity of high-order expansion by leveraging the recursive property of Edgeworth polynomial Hermite matrices. Additionally, a multiple linearization method is introduced to linearize the power flow equations, helping to reduce rounding errors caused by truncating high-order terms when solving probabilistic power flow models using series expansion methods. Numerical simulations on the IEEE 39-bus system, using MCS results as a reference and ARMS as the evaluation metric, compare and analyze the impact of two input random variables—load fluctuations and wind turbine output—on the accuracy of probabilistic power flow calculations under Edgeworth and Gram-Charlier series expansions. The results show that the fitting accuracy of the cumulant method decreases with increasing wind power integration, but the proposed method' s accuracy is less affected by the scale of non-normal distribution random variables compared to the Gram-Charlier series expansion. A comparison of computational speed among the proposed method, Cornish-Fisher expansion, and MCS demonstrates that when ARMS metrics are similar, the proposed method is slightly faster than Gram-Charlier expansion and approximately 22% faster than MCS, highlighting its computational advantages. Further research is warranted on how to improve computational speed while maintaining approximation accuracy for probabilistic power flow distributions.

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Note: Figure translations are in progress. See original paper for figures.