Fast Convergence of Flocking Motion for Multi-Leader Systems with Disturbances: Postprint

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Abstract

To address the discrete-time flocking motion problem for multi-leader networked systems, containment control algorithms for first-order/second-order networked systems are proposed. Employing analytical tools such as modern control theory, algebraic graph theory, and linear matrix inequalities, a theoretical analysis of the proposed control algorithms is conducted, yielding convergence conditions for multi-leader networked systems with disturbances to achieve flocking motion within finite time under discrete-time settings. Finally, numerical simulations utilizing the LMI toolbox are performed to obtain the range of positive definite matrices, thereby determining the stability of the linear system. System simulations validate the correctness of the derived conclusions.

Full Text

Preamble

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Fast Convergence for Flocking Motion of Multi-Leader Networked Systems with Disturbance

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Abstract: This paper proposes containment control algorithms for first-order/second-order networked systems to address discrete-time flocking motion problems in multi-leader networked systems. Using modern control theory, algebraic graph theory, and linear matrix inequalities as analytical tools, we derive convergence conditions for achieving flocking motion within finite time for multi-leader networked systems with disturbances under discrete-time conditions. Finally, numerical simulations utilize the LMI toolbox to determine the range of positive definite matrices, thereby establishing the stability of the linear system. The simulation results verify the correctness of the obtained conclusions.

Keywords: multi-agent systems; flocking motion; finite time; disturbances

0 Introduction

Multi-agent systems, as a major branch of distributed systems, have attracted widespread attention from researchers for their applications in unmanned aerial vehicle formations, robot control, distributed sensor networks, and other fields. The flocking motion of multi-agent systems has become a crucial research topic in distributed systems. Researchers have primarily focused on coordinated control problems such as consensus, flocking motion, and dynamic formation in multi-agent systems.

Numerous studies have addressed consensus problems in continuous-time multi-agent systems [1-4, and references therein]. Reference [1] investigated consensus in multi-agent systems with a dynamic leader and heterogeneous communication delays. Reference [2] studied distributed containment control for linear multi-agent systems and proposed an effective algorithm for designing H∞-type Riccati-based control gain matrices. Reference [3] examined leader-following consensus control for second-order multi-agent systems based on uniform sampling control. Reference [4] investigated distributed containment control for mobile agents with multiple stationary or dynamic leaders under fixed and switching directed networks. However, these results primarily focused on asymptotic convergence over infinite time, whereas practical engineering applications require systems to reach desired objectives within finite time—that is, systems must achieve stability within a finite time interval.

Regarding finite-time convergence for multi-agent systems, reference [5] studied finite-time control problems for multi-agent systems with general linear dynamics. Reference [6] designed control protocols for multi-agent systems and analyzed finite-time consensus for leader-following systems in detail. Reference [7] proposed a continuous-time nonlinear distributed cooperative control protocol to investigate finite-time consensus for heterogeneous multi-agent systems. Reference [8] discussed finite-time consensus problems for multi-agent systems both with and without leaders. Reference [9] designed active disturbance observers to estimate disturbances for each agent and established output feedback control based on disturbance feedforward compensation, proposing an output consensus control algorithm for multi-agent systems with multiple disturbance sources. For finite-time containment control of second-order systems with mismatched disturbances, reference [10] designed nonlinear observers to estimate unknown system states and disturbances, and based on these state estimates, constructed cooperative control algorithms for multi-agent systems using disturbance observers. The aforementioned literature primarily focused on motion consensus within finite time for multi-agent systems but neglected the influence of external disturbances.

In practical engineering applications, physical systems employ discrete-time systems for data transmission. The presence of uncertain factors such as external disturbances significantly affects the motion trajectories of discrete-time multi-agent systems. Research on consensus problems for discrete-time systems with disturbances includes reference [11], which utilized network communication protocols and performance Laplacian matrices to equivalently transform the motion control problem of continuous-time linear multi-agent systems into a stability problem for discrete-time linear systems. Reference [12] proposed dilated LMI characteristics for robust finite-time control of discrete-time uncertain linear systems. The containment control problem for multi-agent systems essentially represents a class of flocking motion in networked systems. In recent years, few studies have investigated flocking motion in discrete-time multi-agent systems, particularly regarding finite-time containment control for multi-leader networked systems with external disturbances.

This paper employs modern control theory and linear matrix inequalities to study flocking motion problems for multi-agent systems with disturbances under discrete-time conditions. The innovations of this paper are: proposing finite-time containment control based on discrete-time multi-agent systems, and investigating fast target tracking control for dynamic networked systems with multiple leaders. We design state feedback control protocols for first-order and second-order networked systems separately, and analyze the finite-time convergence conditions under which the system can achieve group tracking in bidirectional communication network structures. Finally, numerical simulations verify that the entire system achieves stability within finite time.

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1 Algebraic Graph Theory and Related Lemmas

This paper investigates flocking motion in dynamic multi-agent systems where agents communicate via sensors. Assuming each agent is a point and mutual sensing between agents constitutes connections, the dynamic multi-agent system forms a bidirectional communication network topology graph.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted undirected graph with n nodes, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the vertex (or node) set, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. The adjacency matrix is $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$, where $a_{ij} \geq 0$ for $i \neq j$, and $a_{ii} = 0$ (assuming no self-loops). The neighbor set of node i is defined as $N_i = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}$. The degree matrix is $D = \operatorname{diag}(d_1, d_2, \ldots, d_n) \in \mathbb{R}^{n\times n}$, where

$d_i = \sum_{j \in N_i} a_{ij}$ for $i = 1, 2, \ldots, n$. The Laplacian matrix of graph $\mathcal{G}$ is $L = D - \mathcal{A} \in \mathbb{R}^{n \times n}$.

Remark 1: If the network formed by n followers is an undirected connected graph and at least one follower is connected to a leader, then $L_{\mathcal{F}\mathcal{F}}$ is a positive definite matrix.

Definition 1: Let $X = \{x_1, x_2, \ldots, x_n\}$ be a subset of real vector space $\mathbb{R}^n$. The convex hull of set X is defined as $\mathrm{co}(X) = \{\sum_{i=1}^n \alpha_i x_i \mid x_i \in X, \alpha_i \geq 0, \sum_{i=1}^n \alpha_i = 1\}$.

Definition 2: For the networked system (2), if there exists $x_i(k) \in \mathrm{co}\{x_j(k) \mid j \in \Gamma\}$ for all $i \in \mathcal{F}$, then the networked system achieves containment control.

Lemma 1 (Finite-Time Boundedness): The discrete-time linear system $x(k+1) = Ax(k) + Gw(k), w(k+1) = Fw(k)$ is finite-time bounded if there exist positive definite matrices P_1 and P_2 satisfying:

$$\begin{bmatrix} A^T P_1 A - \gamma P_1 & A^T P_1 G \\ G^T P_1 A & G^T P_1 G - \gamma P_2 \end{bmatrix} < 0$$

where R, δ_x, δ_w, and ε are positive scalars with $0 < \delta_x < \varepsilon$, $\delta_w \geq 0$, and $\gamma > 1$.

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2 First-Order Discrete-Time System Flocking Motion Control

2.1 First-Order Discrete-Time Networked System Containment Control Algorithm Model

Consider a networked system consisting of n followers and m leaders, with follower set $\mathcal{F} = \{1, 2, \ldots, n\}$ and leader set $\Gamma = \{n+1, n+2, \ldots, n+m\}$. The system dynamics are modeled as:

$$\dot{x}_i(t) = u_i(t) + w_i(t), \quad i = 1, 2, \ldots, n+m$$

where $x_i(t) \in \mathbb{R}$ represents the position state of agent i, $u_i(t) \in \mathbb{R}$ is the control input of agent i, and $w_i(t) \in \mathbb{R}^n$ denotes the disturbance affecting agent i. This paper assumes system states are in real space $\mathbb{R}^n$, but the conclusions can be extended to $\mathbb{R}^{nm}$ using the Kronecker operator. Assuming static leaders with zero velocity, the discrete-time multi-agent system can be described as:

$$x(k+1) = x(k) + Tu(k) + Gw(k), \quad w(k+1) = Fw(k)$$

where T is the sampling period and G is the disturbance system matrix. The disturbance system satisfies $\|F\| \leq 1$.

Let the control protocol for the multi-leader discrete-time system be:

$$u_i(k) = \sum_{j \in N_i} a_{ij}(x_j(k) - x_i(k)), \quad i \in \mathcal{F}$$

which can be written in compact form as $u(k) = -Lx(k)$. Define $x_{\mathcal{F}}(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T$ and $x_\Gamma(k) = [x_{n+1}(k), x_{n+2}(k), \dots, x_{n+m}(k)]^T$, then the system dynamics become:

$$x_{\mathcal{F}}(k+1) = x_{\mathcal{F}}(k) - TL_{\mathcal{F}\mathcal{F}}x_{\mathcal{F}}(k) - TL_{\mathcal{F}\Gamma}x_\Gamma(k) + Gw(k)$$

Let $y(k) = x_{\mathcal{F}}(k) + L_{\mathcal{F}\mathcal{F}}^{-1}L_{\mathcal{F}\Gamma}x_\Gamma(k)$. Then the system can be transformed into:

$$y(k+1) = (I - TL_{\mathcal{F}\mathcal{F}})y(k) + Gw(k), \quad w(k+1) = Fw(k)$$

2.2 First-Order Discrete-Time Networked System Containment Control Algorithm Analysis

Theorem 1: Consider a multi-leader networked system with n followers and m leaders. Assume the network topology formed by the n followers is undirected and connected, with at least one follower receiving information from the leaders. The discrete-time multi-leader networked system (2) with disturbances can achieve finite-time boundedness under control protocol (3) if there exist positive definite matrices P_1 and P_2 satisfying:

$$\begin{bmatrix} (I - TL_{\mathcal{F}\mathcal{F}})^T P_1 (I - TL_{\mathcal{F}\mathcal{F}}) - \gamma P_1 & (I - TL_{\mathcal{F}\mathcal{F}})^T P_1 G \\ G^T P_1 (I - TL_{\mathcal{F}\mathcal{F}}) & G^T P_1 G - \gamma P_2 \end{bmatrix} < 0$$

where $\gamma > 1$.

Proof: Assuming leaders are unaffected by disturbances, the discrete-time linear system becomes $y(k+1) = (I - TL_{\mathcal{F}\mathcal{F}})y(k) + Gw(k)$. Construct the Lyapunov function $V(k) = y^T(k)P_1y(k) + w^T(k)P_2w(k)$, where P_1 and P_2 are positive definite matrices. Then:

$$\Delta V(k) = V(k+1) - V(k) = \begin{bmatrix} y(k) \\ w(k) \end{bmatrix}^T \Omega \begin{bmatrix} y(k) \\ w(k) \end{bmatrix}$$

where:

$$\Omega = \begin{bmatrix} (I - TL_{\mathcal{F}\mathcal{F}})^T P_1 (I - TL_{\mathcal{F}\mathcal{F}}) - P_1 & (I - TL_{\mathcal{F}\mathcal{F}})^T P_1 G \\ G^T P_1 (I - TL_{\mathcal{F}\mathcal{F}}) & G^T P_1 G - P_2 \end{bmatrix}$$

According to Theorem 1’ s condition, we have $\Delta V(k) < -\gamma V(k)$ for some $\gamma > 0$. Through iterative application, we obtain $V(k) < \gamma^k V(0)$. By Lemma 1, the discrete-time multi-agent system achieves finite-time boundedness with respect to $(\delta_y, \delta_w, R, N, \varepsilon)$. Furthermore, using the relationship $y(k) = x_{\mathcal{F}}(k) + L_{\mathcal{F}\mathcal{F}}^{-1}L_{\mathcal{F}\Gamma}x_\Gamma(k)$ and the condition that $L_{\mathcal{F}\mathcal{F}}$ is positive definite, we can conclude that the system achieves finite-time asymptotic stability, i.e., finite-time containment control.

Corollary 1: For the multi-leader networked system described in Theorem 1, if there exist positive definite matrices P_1 and P_2 satisfying the LMI condition (7), then the discrete-time multi-agent system achieves containment control in finite time under control protocol (3).

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3 Second-Order Discrete-Time System Flocking Motion Control

3.1 Second-Order Discrete-Time Networked System Model

This section considers containment control for second-order discrete multi-agent systems. Let the follower set be $\mathcal{F} = \{1, 2, \dots, n\}$ and the leader set be $\Gamma = \{n+1, n+2, \dots, n+m\}$. The system dynamics are modeled as:

$$\begin{cases} x_i(k+1) = x_i(k) + Tv_i(k) \\ v_i(k+1) = v_i(k) + Tu_i(k) + w_i(k) \end{cases}$$

where $x_i(k) \in \mathbb{R}$ represents the position state, $v_i(k) \in \mathbb{R}$ represents the velocity state, $u_i(k) \in \mathbb{R}$ is the control input, and $w_i(k) \in \mathbb{R}^n$ is the disturbance. We assume static leaders with zero velocity. The control protocol for the multi-leader discrete-time system is:

$$u_i(k) = \sum_{j \in N_i} a_{ij}[(x_j(k) - x_i(k)) + (v_j(k) - v_i(k))], \quad i \in \mathcal{F}$$

The system can be written in compact form as:

$$\begin{cases} x_{\mathcal{F}}(k+1) = x_{\mathcal{F}}(k) + Tv_{\mathcal{F}}(k) \\ v_{\mathcal{F}}(k+1) = v_{\mathcal{F}}(k) - TL_{\mathcal{F}\mathcal{F}}x_{\mathcal{F}}(k) - TL_{\mathcal{F}\Gamma}x_{\Gamma}(k) - TL_{\mathcal{F}\mathcal{F}}v_{\mathcal{F}}(k) - TL_{\mathcal{F}\Gamma}v_{\Gamma}(k) + Gw(k) \end{cases}$$

Define $y_1(k) = x_{\mathcal{F}}(k) + L_{\mathcal{F}\mathcal{F}}^{-1}L_{\mathcal{F}\Gamma}x_{\Gamma}(k)$ and $y_2(k) = v_{\mathcal{F}}(k) + L_{\mathcal{F}\mathcal{F}}^{-1}L_{\mathcal{F}\Gamma}v_{\Gamma}(k)$. The system becomes:

$$\begin{cases} y_1(k+1) = y_1(k) + Ty_2(k) \\ y_2(k+1) = (I - TL_{\mathcal{F}\mathcal{F}})y_2(k) + Gw(k) \end{cases}$$

Definition 3: For the second-order networked system, if there exists $x_i(k) \in \mathrm{co}\{x_j(k) \mid j \in \Gamma\}$ for all $i \in \mathcal{F}$, then the second-order networked system achieves containment control.

Definition 4: A second-order multi-agent system is said to achieve finite-time consensus-based containment control if there exists a time $T_0 \in [0, +\infty)$ such that the final states of all agents satisfy $\lim_{k\to\infty} y_1(k) = 0$ and $\lim_{k\to\infty} y_2(k) = 0$ for $k \geq T_0$.

3.2 Second-Order Discrete-Time Networked System Containment Control Algorithm Analysis

Theorem 2: Consider a multi-leader networked system with n followers and m leaders. Assume the network topology formed by the n followers is undirected and connected, with at least one follower receiving information from the leaders. The discrete-time multi-agent system (20) under control protocol (21) can achieve finite-time boundedness if there exist positive definite matrices P_1 and P_2 satisfying:

$$\begin{bmatrix} A^T P_1 A - \gamma P_1 & A^T P_1 G \\ G^T P_1 A & G^T P_1 G - \gamma P_2 \end{bmatrix} < 0$$

where $A = \begin{bmatrix} I & TI \\ 0 & I - TL_{\mathcal{F}\mathcal{F}} \end{bmatrix}$ and $\gamma > 1$.

Proof: Assuming leaders are unaffected by disturbances, the discrete-time linear system can be written as $y(k+1) = Ay(k) + Gw(k)$. Let the Lyapunov function be $V(k) = y^T(k)P_1 y(k) + w^T(k)P_2 w(k)$. Following a similar proof procedure as Theorem 1 and applying Lemma 1, we can conclude that the discrete-time second-order multi-agent system achieves finite-time boundedness with respect to $(\delta_y, \delta_w, R, N, \varepsilon)$. Using the transformation relationships and the positive definiteness of $L_{\mathcal{F}\mathcal{F}}$, the system achieves finite-time asymptotic stability, i.e., finite-time containment control.

Corollary 2: For the second-order multi-leader networked system described in Theorem 2, if there exist positive definite matrices P_1 and P_2 satisfying the LMI condition (23) with $\gamma = 1$, then the discrete-time multi-agent system achieves containment control in finite time under control protocol (21).

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4 Numerical Simulations

4.1 First-Order Discrete-Time Networked System Simulation

Consider a second-order multi-agent network with three leaders and five followers [Figure 1: see original paper], where agents 6, 7, and 8 are leaders and the remaining agents are followers. Assume all edge weights in the topology graph are 1. The Laplacian matrix is:

$$L = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 3 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Assume the initial states of the five followers are $x_1(0) = (0, 4)$, $x_2(0) = (4, 0)$, $x_3(0) = (0, 2)$, $x_4(0) = (2, 0)$, and $x_5(0) = (0, 0)$. The leaders' initial states are $x_6(0) = (6, 8)$, $x_7(0) = (8, 8)$, and $x_8(0) = (8, 6)$. The disturbance is $w(k) = 0.1\sin(kT)$. Using the LMI toolbox, we obtain $P_1 = 10.14$ and $P_2 = 21.66$ with parameters $G = 0.6$, $F = 1$, $\gamma = 1$, $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\delta_y = 1.3$, $\delta_w = 0.01$, and $\varepsilon = 1.6$.

The system trajectory is shown in [Figure 2: see original paper]. It can be observed that despite disturbances, all followers asymptotically converge to the triangular region formed by the three leaders' positions, demonstrating that the multi-leader system achieves containment control. [Figure 3: see original paper] shows the state errors of the five followers in the first-order multi-agent system. Starting from their initial states, the followers' state errors eventually converge to zero, indicating that the relative state distance between followers and leaders becomes zero. This simulation verifies that the proposed algorithm achieves containment control within finite time, with a convergence time of 25 seconds.

4.2 Second-Order Discrete-Time Networked System Simulation

Consider the same network with three leaders and five followers [Figure 1: see original paper]. Assume the initial states of the five followers are $x_1(0) = (0, 4)$, $x_2(0) = (4, 0)$, $x_3(0) = (0, 2)$, $x_4(0) = (2, 0)$, and $x_5(0) = (3, 0)$, with initial velocities $v_i(0) = (0, 0)$ for all followers. The leaders' initial states are $x_6(0) = (6, 8)$, $x_7(0) = (8, 8)$, and $x_8(0) = (8, 6)$. The disturbance is $w(k) = \sin(kT)$. Using the LMI toolbox, we obtain the positive definite matrices P_1 and P_2 satisfying condition (23).

The system trajectory is shown in [Figure 4: see original paper]. Despite disturbances, all followers asymptotically converge to the triangular region formed by the three leaders' positions, demonstrating that the multi-leader system achieves containment control. [Figure 5: see original paper] shows the state errors of the five followers in the second-order multi-agent system. Starting from their initial states, the followers' state errors eventually converge to zero, indicating that the relative state distance between followers and leaders becomes zero. This simulation verifies that the proposed algorithm achieves containment control within finite time, with a convergence time of 37 seconds.

5 Conclusion

This paper studied discrete-time containment control motion problems for multi-leader systems, considering finite-time control algorithms for multi-agent systems with external disturbances. We designed coordinated control protocols for first-order and second-order discrete-time networked systems and investigated flocking motion for discrete-time multi-agent systems with disturbances using theoretical tools such as Lyapunov functions and algebraic graph theory. Finally, computer simulations demonstrated that the multi-agent systems can rapidly converge to the region surrounded by multiple leaders.

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