Postprint of Adaptive Zero-Watermarking Algorithm Based on NMF and Enhanced Singular Value Decomposition

Authors: Xiao Zhenjiu, Ning Qiuying, Zhang Han, Tang Xiaoliang, Chen Hong

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Abstract

To address the issues of high false positive rate and insufficient robustness in singular value decomposition (SVD)-based watermarking algorithms, this paper proposes an adaptive zero-watermarking algorithm that combines block Non-negative Matrix Factorization (NMF) and Boosted Non-negative Singular Value Decomposition (BN-SVD). Firstly, a two-level Discrete Wavelet Transform (DWT) is performed on the original grayscale image, and the resulting second-level low-frequency subband (LL2) is divided into non-overlapping blocks, each of which undergoes rank-r NMF decomposition; subsequently, BN-SVD is applied to the feature matrices obtained from NMF decomposition, and a feature vector is constructed based on the relationship between the maximum singular value of each block matrix and the mean of all maximum singular values; the generated feature vector is then XORed with a watermark image that has been doubly scrambled and encrypted via Arnold transform and chaotic mapping to produce the zero-watermark, while the Beetle Antennae Search (BAS) algorithm is employed to adaptively determine the scaling parameter in BN-SVD that offers the strongest attack resistance. Experimental results indicate that the NC value falls below 0.4 for false positive problems, and under JPEG compression, noise, filtering, rotation, cropping, and hybrid attacks, the Normalized Correlation (NC) values between the extracted watermark image and the original watermark image can all exceed 99%, thereby effectively resolving the false positive issue with strong robustness and effective resistance against various attacks.

Full Text

Adaptive Zero-Watermarking Algorithm Based on Block NMF and Boost Normed Singular Value Decomposition

Xiao Zhenjiu, Ning Qiuying, Zhang Han, Tang Xiaoliang, Chen Hong
(College of Software, Liaoning Technical University, Huludao, Liaoning 125105, China)

Abstract: To address the high false alarm rate and weak robustness issues inherent in singular value decomposition (SVD) watermarking algorithms, this paper proposes an adaptive zero-watermarking algorithm that combines block Non-negative Matrix Factorization (NMF) with Boost Normed Singular Value Decomposition (BN-SVD). The algorithm first performs a two-level Discrete Wavelet Transform (DWT) on the original grayscale image, then divides the resulting second-level low-frequency subband (LL2) into non-overlapping blocks, and decomposes each sub-block using NMF with rank r. Next, boost normed singular value decomposition is applied to the feature matrices obtained from NMF decomposition. A feature vector is constructed based on the relationship between each block matrix' s maximum singular value and the mean of all global maximum singular values. The generated feature vector is then XORed with a watermark image that has been scrambled through Arnold transformation and chaotic mapping to generate the zero watermark. The Beetle Antennae Search (BAS) algorithm is employed to adaptively determine the scaling parameter in the BN-SVD that provides the strongest attack resistance. Experimental results demonstrate that the proposed scheme effectively solves the false alarm problem with NC values below 0.4, while achieving normalized correlation (NC) values above 99% between extracted and original watermark images under JPEG compression, noise, filtering, rotation, cropping, and hybrid attacks. The method efficiently resolves false positive errors and exhibits strong robustness against various attacks.

Keywords: Non-negative Matrix Factorization; Boost Normed Singular Value Decomposition; Arnold Transform; Chaotic Mapping; Beetle Antennae Search Algorithm

0 Introduction

Over the past two decades, improvements in data compression capabilities and increasing internet bandwidth have made digital images, video, and audio easier to playback, copy, and tamper with. As more individuals seek to share their creative digital works, copyright protection and ownership verification for all digital content have become critically important. To protect digital works from piracy, digital watermarking offers a potential solution, driving continuous advancements in digital watermarking research due to its significant applications in digital rights management and protection.

Digital watermarking [1,2] refers to the process of imperceptibly embedding information (the watermark) into digital documents to provide content protection or authentication. Based on embedding algorithms, watermarking can be classified into two main categories: spatial domain algorithms and transform domain algorithms. Spatial domain watermarking typically modifies pixel intensity values directly, while transform domain watermarking achieves embedding by modifying frequency domain coefficients of the host image. Transform domain methods are generally more robust than spatial domain approaches. Most wavelet-domain watermarking schemes embed watermarks in the mid-frequency region. Reference [3] proposed a digital image watermarking algorithm based on wavelet transform and NMF, which performs a two-level DWT on the original image, selects the mid-frequency region for embedding, applies NMF decomposition to LH2, and embeds an Arnold-transformed watermark into the decomposed feature matrix with embedding strength . Reference [4] improved upon this approach by performing three-level wavelet decomposition and selecting the low-frequency approximation component for embedding, ultimately obtaining a feature vector representing the original image through quantization of the coefficient matrix and embedding copyright information by computing the watermark and feature vector. Algorithms incorporating DWT improve watermark imperceptibility and enhance robustness against certain image degradation processes and JPEG compression.

However, single wavelet transform cannot achieve strong robustness. Reference [5] proposed a watermarking algorithm combining SVD with wavelet transform, which performs wavelet decomposition first and then embeds watermark content after applying SVD to the low-frequency component. Reference [6] utilized DCT and DWT to embed robust and fragile watermarks, achieving a dual-function watermarking algorithm that effectively resists compression, noise, and local cropping attacks. Although both algorithms in [5,6] satisfy the requirements of imperceptibility and robustness, they fail to adequately address the diagonal distortion and false alarm rate issues caused by singular value decomposition. Reference [7] presented a robust watermarking algorithm based on SVD and bee colony optimization, which uses swarm intelligence to select embedding strength and adaptively balance robustness and transparency, but suffers from slow convergence and long optimization times.

To fundamentally address the imbalance between watermark transparency and robustness, Wen Quan et al. [8] proposed the zero-watermarking concept, which constructs zero watermarks using intrinsic features of the original host image. This approach preserves the integrity of the original carrier image while effectively resolving the conflict between robustness and transparency. Qu Changbo et al. [9] proposed a strongly robust zero-watermarking scheme based on Curvelet-DSVD and visual cryptography, which generates zero watermarks by XORing shares produced by visual cryptography with feature matrices obtained through double singular value decomposition. While demonstrating good robustness against JPEG compression, noise, filtering, and cropping, this algorithm is complex and offers considerable room for improvement in resisting geometric attacks, with experimental results remaining unsatisfactory regarding false alarm rates. Drawing inspiration from [10], this paper proposes an adaptive zero-watermarking algorithm based on block NMF and BN-SVD. The algorithm divides the low-frequency region after two-level wavelet transform into non-overlapping blocks, leverages the non-negative property of images to decompose each block using NMF to obtain basis matrix W and feature matrix H, applies boost normed singular value decomposition to the feature matrix to generate a feature vector, and XORs this vector with a watermark image scrambled through Arnold transformation and chaotic mapping to create the zero watermark. The Beetle Antennae Search (BAS) algorithm adaptively determines the parameter in BN-SVD that provides optimal attack resistance. The proposed algorithm' s advantage lies in solving the high false alarm errors and weak robustness issues in SVD-based watermarking extraction while utilizing the beetle antennae search algorithm to achieve an adaptive process that enhances robustness.

1.1 Non-negative Matrix Factorization

Non-negative Matrix Factorization (NMF) [11,12] is one of the most popular multidimensional data processing tools in signal processing, computer vision, and image engineering. It is a matrix decomposition method based on the constraint that all elements in the matrix are non-negative. The algorithm is described as follows:

Given a non-negative matrix $V \in \mathbb{R}_+^{n\times m}$, NMF seeks non-negative basis matrix $W \in \mathbb{R}_+^{n\times r}$ and non-negative feature matrix $H \in \mathbb{R}_+^{r\times m}$ such that $V \approx WH$. This decomposition can be understood as: each column vector of the original matrix V is a weighted sum of all column vectors in the left matrix W, where the weight coefficients are the elements of the corresponding column vector in the right matrix H. Therefore, W is called the basis matrix and H is the coefficient matrix or feature matrix. Here, r represents the rank of the matrix to be decomposed, which is typically chosen to be much smaller than n and m, satisfying $(n+m)r < nm$. In this case, using the feature matrix to replace the original matrix achieves dimensionality reduction, yielding a reduced-dimensional matrix of data features. The NMF matrix decomposition optimization process aims to minimize the difference between the original matrix V and the product of matrices W and H. The key to the algorithm lies in the objective function definition and iterative rule selection, defined as follows:

The objective function is given by $\min_{W,H} \|V - WH\|^2$ subject to $W, H \geq 0$. The iterative rules are:

$$H \leftarrow H \odot \frac{W^T V}{W^T W H}$$

$$W \leftarrow W \odot \frac{V H^T}{W H H^T}$$

Unlike common signal transformation methods such as PCA, FA, and ICA, all components after NMF decomposition are non-negative and achieve nonlinear dimensionality reduction. This non-negativity constraint leads to a certain degree of sparsity in the corresponding representation, better reflecting the essence of intelligent data processing, making data description more convenient and reasonable, while also suppressing the impact of external variations on feature extraction to some extent. Consequently, NMF decomposition offers several advantages over traditional algorithms, including implementation simplicity, interpretability in decomposition form and results, and reduced storage space requirements.

1.2 Singular Value Decomposition

Singular Value Decomposition (SVD) is the most famous and widely used matrix decomposition method. After SVD, the corresponding orthogonal matrices represent the geometric structure of the image, while the singular matrix represents brightness information. Due to its strong stability, this decomposition algorithm is extensively applied in image processing. For a digital image matrix $A \in \mathbb{R}^{N\times N}$, there exist orthogonal matrices $U \in \mathbb{R}^{N\times N}$ and $V \in \mathbb{R}^{N\times N}$ such that A can be expressed as:

$$A = USV^T$$

where U represents the left singular orthogonal matrix and V represents the right singular orthogonal matrix. S is a diagonal matrix with all off-diagonal elements equal to 0 and diagonal elements satisfying $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_r > 0$, where r is the rank of matrix A. The diagonal elements λ_i are uniquely determined by the decomposition and are denoted as the singular values of matrix A, with the decomposition formula referred to as the singular value decomposition of A.

After image enhancement using SVD, a one-to-one correspondence exists between the image and singular value vectors. However, different images can share identical singular values without any structural relationship between the images. This drawback causes false alarm problems during watermark extraction, where watermark information may be extracted from images that never contained any watermark, leading to erroneous detection.

1.3 Boost Normed Singular Value Decomposition

Boost Normed Singular Value Decomposition (BN-SVD) [13] is a novel decomposition algorithm proposed based on SVD theory, which modifies the singular value matrix by introducing parameter β. Its purpose is to balance the grayscale intensity along the diagonal direction of the singular value matrix. For a digital image A of size $M \times M$, the boost normed singular value decomposition yields:

$$A = US^{\beta}V^T$$

where U is the left singular matrix, V is the right singular matrix, and S^{β} is a diagonal matrix. BN-SVD performs power operations on the diagonal matrix

S obtained from conventional SVD decomposition. When processing different digital images, the parameter can be reasonably adjusted based on the image' s inherent information to achieve optimal attack resistance. The advantages of BN-SVD include: singular values are amplified, reducing the sensitivity of the image matrix to attacks and thereby improving algorithm robustness. Moreover, in addressing the false alarm errors inherent in SVD, BN-SVD' s special processing of singular value vectors establishes a one-to-one relationship between vectors and images, allowing singular values to represent image features. This approach provides a better solution to the false alarm problem.

1.4 Adaptive Beetle Antennae Search Algorithm

The Beetle Antennae Search (BAS) algorithm [14], proposed by Li Shuai et al. in 2017, is an optimization algorithm inspired by beetle foraging behavior. Similar to particle swarm and genetic algorithms, it achieves optimization without requiring specific function forms or gradient information. Due to the simplicity of individual beetles, the algorithm eliminates extensive computational overhead while improving optimization speed. The biological theory [15] states that beetles locate food based on odor intensity, using their two antennae to randomly search surrounding areas. By comparing odor intensity values received by the left and right antennae, the beetle flies toward the side with stronger concentration, effectively finding food through this simple principle.

The algorithm principle is as follows:

a) Initialize random beetle position X_0 and random direction vector $\vec{b}$ for antennae orientation, normalize them, and set maximum iteration count T. Let d_0 be the initial distance between antennae, where rand() is a random function and k represents spatial dimension.

b) The beetle' s two antennae collect odor intensity from two nearby points:

$$\begin{cases} X_{rt} = X_t + \frac{d_t}{2} \times \vec{b} \\ X_{lt} = X_t - \frac{d_t}{2} \times \vec{b} \end{cases}$$

where X_{rt} and X_{lt} are the right and left antenna coordinates at iteration t, X_t is the beetle' s centroid coordinate, and d_t is the distance between the two antennae.

c) Iteratively update the beetle' s position:

$$X_{t+1} = X_t + \delta_t \times \vec{b} \times \text{sign}(f(X_{rt}) - f(X_{lt}))$$

where δ_t is the step size factor at iteration t and sign() is the sign function.

d) Since parameter β has a value range of [0,1], the odor intensity function $f(X)$ in this experiment requires piecewise constraints.

e) The odor intensity at points near the beetle' s two antennae (X_{rt}, X_{lt}) is determined using the current spatial position and an odor intensity measurement function (fitness function) to calculate the beetle's individual position.

f) Through the measurement function $f()$, the optimal odor intensity at the beetle' s antennae positions is found.

g) The beetle moves toward the position with the strongest odor intensity:

$$\begin{cases} f_{\text{best}} = \max(f) \\ X_{\text{best}} = X_{\max(f)} \end{cases}$$

h) Repeat steps b) through f) for iterative optimization to find the optimal solution. At each iteration' s end, judge the odor intensity. If the current intensity is better than the previous iteration, execute step g). The entire process terminates when reaching the maximum iteration count T, yielding the optimal result β_{best}.

2.1 Watermark Image Preprocessing

Arnold transformation [15], commonly known as cat map, is a mapping from regular to random positions and represents a traditional chaotic system. For an image of size $N \times N$, Arnold scrambling is defined as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \mod N$$

where (x, y) and (x', y') represent the positions of pixels before and after transformation, N is the image matrix order, and $\mod$ denotes the modulo operation. The transformation essentially changes pixel positions to weaken correlation between adjacent pixels and improve image security. Due to its periodicity, the original image can be recovered after a finite number of iterations.

The Logistic map [16] offers advantages including high ergodicity, strong pseudo-randomness, and high sensitivity to initial values. It is defined as:

$$x_{i+1} = \mu x_i (1 - x_i)$$

where μ is the control parameter of the Logistic chaotic system and x_i is the chaotic sequence. When $3.569456 < \mu < 4$, the Logistic map operates in a chaotic state, meaning sequences generated from initial conditions $x_0 \in (0, 1)$ under the Logistic map are non-periodic. When μ is close to 4, the iteratively generated values exhibit pseudo-random distribution (chaotic state). For other μ values, the generated values converge to specific numbers after certain iterations.

2.2 Zero Watermark Construction

This paper employs discrete wavelet transform as the foundation, performing a two-level DWT on the original image. Since the original image' s energy is highly concentrated in the LL2 region of the low-frequency band while detail information is distributed primarily in high-frequency bands, and low-frequency coefficients exhibit high stability, we choose to embed watermark information in this region to ensure strong robustness and significantly enhanced attack resistance.

[Figure 1: see original paper]

a) Perform 2-level DWT on an $M \times M$ original grayscale image I. Divide the low-frequency approximation subband LL2 obtained from the 2-level DWT transformation into non-overlapping sub-blocks A_i of size $n \times n$, and decompose each A_i using NMF with rank r:

$$A_i \approx W_i H_i$$

where W_i and H_i are non-negative matrices of dimensions $m \times r$ and $r \times n$, respectively.

b) Apply boost normed singular value decomposition to the feature matrix H_i:

$$H_i = U_i S_i^{\beta} V_i^T$$

where U_i and V_i are orthogonal matrices and S_i is a diagonal matrix. The parameter β is optimally determined using the BAS algorithm described in Section 1.4.

c) Extract the first singular value from each diagonal matrix S_i, yielding a total of $(M/2n) \times (M/2n)$ singular values.

d) Calculate the mean value mean(η) of all singular values. Construct a feature vector F based on the relationship between each block' s maximum singular value η_i and the global mean:

$$F_i = \begin{cases} 1 & \text{if } \eta_i > \text{mean}(\eta) \\ 0 & \text{otherwise} \end{cases}$$

e) Perform K_1 iterations of Arnold scrambling on the original binary watermark image W to obtain scrambled watermark W_1, then apply Logistic mapping to W_1 to produce doubly-encrypted watermark W_2 and key K_2.

f) XOR the feature vector F with the scrambled watermark W_2 to generate the zero watermark:

$$M = F \oplus W_2$$

2.3 Zero Watermark Detection

Watermark detection is the inverse process of watermark construction, as shown in Figure 2 [Figure 2: see original paper].

a) Perform 2-level DWT on the original grayscale image I', divide the obtained low-frequency approximation subband LL2 into $n \times n$ non-overlapping sub-blocks A_i', and decompose each A_i' using NMF with rank r:

$$A_i' \approx W_i' H_i'$$

b) Apply boost normed singular value decomposition to feature matrix H_i' to obtain diagonal matrix S_i'.

c) Extract the first singular value from each diagonal matrix S_i', yielding a total of $(M/2n) \times (M/2n)$ singular values.

d) Calculate the mean value $\text{mean}'(\eta)$ and construct feature vector F' based on the relationship between each η_i' and $\text{mean}'(\eta)$.

e) XOR the zero watermark image M with feature vector F' to obtain the encrypted scrambled watermark W_2':

$$W_2' = M \oplus F'$$

f) Finally, apply inverse Logistic mapping and K_1 iterations of inverse Arnold scrambling to W_2' to recover the original watermark image W.

3 Simulation Results and Analysis

Experiments were conducted on the Matlab 2016a platform. Standard grayscale images Lena, Baboon, and Plane of size 512×512 pixels were selected as test images (Figures 3(a)-(c)), and a 32×32 binary watermark image "LiaoNing Tech University" was used (Figure 3(d)). The BAS optimization iteration count was set to 20. While conventional watermarking algorithms are evaluated by imperceptibility and robustness, zero-watermarking algorithms require only robustness as an evaluation metric since watermark information is not embedded in the carrier image. The Normalized Correlation (NC) function was employed to evaluate similarity between extracted and original watermarks:

$$\text{NC}(x_1, x_2) = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} x_1(i,j)x_2(i,j)}{\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N} x_1(i,j)^2}\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N} x_2(i,j)^2}}$$

where x_1 represents the original watermark and x_2 represents the extracted watermark.

3.1 Robustness Experiments

To verify the robustness of the proposed algorithm, NC values were used as the evaluation standard. For 32×32 watermarks, the BAS algorithm obtained optimal parameter β values of 0.4289, 0.5070, and 0.1382 for Lena, Baboon, and Plane images, respectively, achieving strong robustness. Both geometric and non-geometric attacks were conducted on three grayscale images. Figure 4 [Figure 4: see original paper] shows experimental results for Lena image under various attacks from Table 1 . Table 1 lists NC values for extracted 32×32 watermarks under different attacks. The data demonstrate that the proposed algorithm effectively resists various attacks, with average NC values above 0.9984. While NC values are affected by increasing attack intensity and hybrid attacks, the lowest NC value of 0.9988 occurs under 10-degree rotation attack, confirming effective resistance to rotation. For cropping attacks, NC values remain above 0.9984 even as cropping area increases. Under noise, filtering, scaling, compression, and hybrid attacks, extracted watermark NC values approach 1.

3.2 Security Analysis

Security is crucial throughout the algorithm. The proposed scheme applies Arnold transformation and Logistic mapping to the watermark image, achieving double encryption with keys K_1 and K_2. Even if zero-watermark information is leaked, the watermark cannot be recovered without the correct keys, demonstrating high security. Figure 5 [Figure 5: see original paper] shows extracted watermark information under correct and incorrect key conditions: (a) with only key K_2 incorrect, (b) with only key K_1 incorrect, and (c) with both keys incorrect. The results confirm that watermark information cannot be extracted with incorrect keys, validating the algorithm' s high security.

3.3 False Positive Rate Experiment

To verify whether the proposed algorithm effectively solves SVD false alarm errors, comparative experiments were conducted with reference [17] and traditional SVD algorithms. Figure 6 [Figure 6: see original paper] shows: (a) original grayscale image, (b) forged watermark image, and (c) watermark extracted by the proposed algorithm. Experiments demonstrate that after introducing parameter β, a one-to-one correspondence exists between images and singular value vectors. Without knowing the correct β value, watermark information cannot be properly extracted.

Table 2 compares three algorithms using discrete wavelet low-frequency blocking with different matrix decomposition methods. Reference [17] employs Schur decomposition. The results show that both Schur and the proposed algorithm achieve NC values below 0.5 for extracted watermarks compared to original watermarks, while the SVD algorithm exhibits significantly higher false alarm rates. The proposed algorithm achieves NC values below 0.4, demonstrating superior capability in handling false alarm problems.

3.4 Comparison with References [10,18,19]

To further validate the algorithm' s advantages, the proposed method applies NMF decomposition to the second-level low-frequency region after DWT, then performs BN-SVD on the feature matrix to balance diagonal grayscale intensity, achieving strong robustness. The BAS algorithm adaptively determines optimal parameter β for maximum robustness. Using Lena as the test image, results were compared with algorithms from references [10,18,19]. Reference [10] uses DWT low-frequency blocking followed by DCT and BN-SVD to construct feature vectors, with cellular neural networks determining reversible logical operations for zero-watermarking. Reference [18] constructs feature matrices using non-uniform NURP in the time domain and employs SURF for geometric correction. Reference [19] performs two-level redundant DWT on secure regions with approximate lossless pixels, then applies SVD to construct feature matrices. The proposed algorithm uses two-level DWT low-frequency blocking followed by NMF decomposition and BN-SVD with BAS optimization, demonstrating strong robustness. Comparative results are shown in Table 3 .

The data in Table 3 indicate that the proposed algorithm' s robustness against rotation attacks significantly outperforms reference [10], with average NC values reaching 0.9998. For cropping attacks, resistance decreases with increasing crop area but remains superior to [10,18,19], achieving NC values between 0.9999 and 1. For noise and filtering attacks, the proposed algorithm significantly outperforms [10,18,19] with NC values approaching 1. Under JPEG compression attacks, the proposed algorithm' s resistance decreases at quality factor 70 but remains superior to [10,18,19]. For scaling attacks, the proposed algorithm' s resistance decreases more than reference [19] with increasing scaling intensity. Overall, the algorithm demonstrates strong robustness across most attack types.

4 Conclusion

This paper analyzes the false alarm errors and weak robustness issues in traditional SVD-based watermarking algorithms and proposes an adaptive zero-watermarking algorithm combining block NMF with boost normed singular value decomposition. Arnold transformation and Logistic mapping provide double encryption for enhanced watermark security. NMF decomposition offers linear independence and sparsity properties that effectively express image data features and structures. Combined with BN-SVD and BAS optimization for adaptive parameter determination, the algorithm effectively resolves false alarm errors and robustness issues. Experimental results demonstrate that NC values remain below 0.4 for false alarm problems, while extracted watermark NC values exceed 98% under both geometric and non-geometric attacks, confirming strong robustness. However, further improvement is needed for compression and scaling attacks.

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Note: Figure translations are in progress. See original paper for figures.