

## Postprint: Solving Uncertain Surgery Scheduling Models Using Hybrid Cuckoo Search Algorithm

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### Abstract

To address the problem that uncertainty factors in surgery scheduling under emergency rescue scenarios make it impossible to obtain precise surgery times and completion deadlines, a grey scheduling model and a hybrid grey cuckoo algorithm for solving this problem are designed. First, three-parameter and four-parameter interval grey numbers are introduced to describe uncertain surgery times and uncertain completion deadlines, and possibility measures and necessity measures are defined; a tardiness credibility index is proposed to measure the probability of surgery tardiness. Then, a grey mixed-integer programming model is established with the objective of minimizing the average tardiness credibility of surgeries, a solution method based on the hybrid grey cuckoo algorithm is proposed, and simulation tests are conducted using a classic example of size  $6(3) \times 3$ . Experimental results show that the algorithm can effectively solve the problem and has better performance than the basic cuckoo search algorithm.

### Full Text

### Preamble

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### Solution of Uncertain Operation Scheduling Model Based on Hybrid Cuckoo Algorithm

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**Abstract:** In emergency rescue situations, uncertainty factors in operation scheduling prevent accurate determination of operation times and completion deadlines. To address this problem, this paper proposes a grey scheduling model and a hybrid grey cuckoo algorithm. First, three-parameter and four-parameter interval grey numbers are introduced to describe uncertain operation times and uncertain deadlines. Possibility measures and necessity measures are defined, and a tardiness credibility index is proposed to quantify the probability of operation delays. A grey mixed-integer programming model is then established to minimize the average tardiness credibility of operations. A hybrid grey cuckoo search algorithm is presented to solve this model, with simulation tests conducted on a classic  $6(3) \times 3$  benchmark instance. Experimental results demonstrate that the proposed algorithm effectively solves the problem and outperforms the basic cuckoo search algorithm.

**Keywords:** grey operation scheduling; uncertain conditions; cuckoo search algorithm; credibility; possibility measure; necessity measure

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## 0 Introduction

Throughout history, societal progress has often come at the cost of environmental degradation, inevitably leading to increased natural and man-made disasters and emergencies. Timely, scientific, and rapid response to these events presents significant challenges for government decision-making and management under crisis conditions. During emergencies, the primary task of rescue operations is to conduct emergency response within the shortest possible time to minimize casualties and property losses. To improve rescue efficiency, rational scheduling of medical departments becomes crucial when medical resources are limited yet demand continuously increases. Operating rooms represent both a critical hospital department and the most resource-intensive area of the entire facility. Operation scheduling management has long been a bottleneck in hospital administration, yet it also holds the greatest potential for improving rescue efficiency. The operation scheduling optimization problem resembles the flexible flow-shop scheduling problem (FFSP), where patients replace jobs, and anesthetizing rooms, operating rooms, and recovery rooms replace machines, with the patient's surgical process substituting for job processing [1]. However, emergency rescue scenarios involve substantial uncertainties, such as insufficient surgical resources, unreasonable patient scheduling, and physician decision errors, which lead to uncertainty in both operation times and completion deadlines. In such situations, treating operation times and deadlines as uncertain factors more accurately reflects the realities of emergency rescue contexts.

Recent years have seen abundant research achievements in hospital operation scheduling optimization. Reference [2] reviewed literature on surgical planning and scheduling, categorizing existing research from problem description and technical feature perspectives, and identified uncertainty research as a primary

future direction in operation scheduling. Deng et al. [3] addressed the complexity of multi-factor operation scheduling by constructing a fuzzy scheduling mathematical model that maximizes patient satisfaction and minimizes total flow time, proposing an improved non-dominated sorting genetic algorithm. Wang et al. [4] considered uncertainties in surgical services, representing service times as bounded intervals and proposing a two-stage robust optimization method for interval-based operation scheduling. Zhou et al. [5] addressed operating theater utilization by proposing a Lagrangian relaxation-based scheduling algorithm that minimizes operating theater costs while maximizing patient satisfaction, establishing a mathematical programming model and evaluating algorithm performance through experiments of varying scales. However, reference [3] studied multi-resource constrained operation flow optimization without considering the uncertainty of operation times and deadlines in emergency contexts, and its proposed algorithm was specific to that scenario and lacked universality. Reference [4] considered various uncertain factors with the objective of maximizing hospital revenue, but established a deterministic scheduling model that only accounted for uncertainty in service times while ignoring deadline uncertainty, thus providing incomplete coverage. Reference [5] examined multi-period comprehensive scheduling of surgeons, nurses, and elective patients, which differs from our research focus.

Research on FFSP under uncertain conditions has also attracted significant attention, with most scholars employing fuzzy mathematics and proposing the concept of fuzzy scheduling. The primary approach involves analyzing fuzzy factor membership functions, typically using triangular and trapezoidal fuzzy numbers to represent uncertain processing times and delivery deadlines. Li et al. [6] studied a single-machine parallel-batching problem with fuzzy due dates and fuzzy precedence relations, aiming to minimize makespan while maximizing minimum fuzzy due dates and minimum fuzzy priorities, proposing an effective algorithm for finding non-dominated solutions. Noori-Darvish et al. [7] proposed a bi-objective fuzzy programming model for open-shop scheduling problems with sequence-dependent setup times, fuzzy processing times, and fuzzy due dates. Nailwal et al. [8] investigated bi-criteria scheduling on parallel machines in fuzzy environments, optimizing weighted flow time and maximum tardiness, and proposed a heuristic algorithm for finding optimal solutions.

However, the uncertainties in operation times and deadlines under emergency rescue conditions cause scheduling constraints to be expressed not as deterministic values but with clear interval characteristics. The fuzzy numbers mentioned above are all piecewise linear functions, whereas ideal processing times and delivery deadlines in typical FFSPs are usually the most likely values under uncertainty, with probability functions deviating from these most likely values that are not necessarily linear. This characteristic is best matched by interval grey numbers in grey system theory. Therefore, this paper proposes using three-parameter interval grey numbers and four-parameter interval grey numbers to describe uncertain operation times and uncertain deadlines, establishing a grey operation scheduling model. Possibility and necessity measures are introduced

to define a tardiness credibility index for operations, proposing an uncertain deadline tardiness optimization objective, establishing a grey mixed-integer programming model, and solving it using intelligent algorithms.

Interval grey numbers in grey theory exhibit the characteristic of unequal probability for value occurrence, which many scholars have studied with fruitful results. Interval numbers have been introduced into multi-criteria optimization methods to address numerous real-world uncertainty decision problems [9]. To mitigate the impact of grey number information on prediction results, methods have been proposed to convert interval grey number sequences into real number sequences for establishing prediction models [10]. Based on fundamental interval number definitions, three-parameter interval numbers have been derived, and corresponding multi-attribute decision-making methods proposed [11-17]. Among these, reference [11] discussed grey target decision problems where the maximum probability of attribute values is known, using subjective or objective weighting methods to comprehensively determine index weights and establish integrated optimization models, utilizing three-parameter interval numbers to determine attribute values.

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## 1.1 Problem Description

Deterministic operation scheduling rarely exists in practice, as the surgical process is always subject to various uncertain factors. Actual scheduling can only be measured through possibilities. This paper addresses uncertain operation times and uncertain deadlines using three-parameter interval grey numbers and four-parameter interval grey numbers, with the scheduling objective optimizing the average tardiness credibility of operations. We first provide definitions of grey numbers.

A grey number, denoted as “ ”, refers to a number whose variation range is known but whose precise value cannot be given. According to the definition of three-parameter interval grey numbers [18], let  $\underline{\alpha}$  be the lower bound,  $\bar{\alpha}$  the upper bound, and  $\alpha^*$  the most likely value within the interval, called the center of gravity. When  $\underline{\alpha} = \alpha^* = \bar{\alpha}$ , the three-parameter interval degenerates to a real number. When either  $\underline{\alpha} = \alpha^*$  or  $\alpha^* = \bar{\alpha}$ , the three-parameter interval grey number reduces to an interval grey number. The three-parameter interval number satisfies  $\underline{\alpha} \leq \alpha^* \leq \bar{\alpha}$ , and the value distribution curve on either side of the most likely value is not necessarily linear. Therefore, the membership function for three-parameter interval numbers can be constructed as follows:

According to the definition of four-parameter interval grey numbers [19], let  $\otimes b \in [\underline{\underline{\alpha}}, \underline{\alpha}, \bar{\alpha}, \bar{\bar{\alpha}}]$  be a four-parameter interval grey number with  $\underline{\underline{\alpha}} \leq \underline{\alpha} \leq \bar{\alpha} \leq \bar{\bar{\alpha}}$ . Here,  $\underline{\underline{\alpha}}$  and  $\bar{\bar{\alpha}}$  are the lower and upper bounds, while  $\underline{\alpha}$  and  $\bar{\alpha}$  represent the most likely lower and upper values within the interval, called the dual centers of gravity. When  $\underline{\underline{\alpha}} = \bar{\bar{\alpha}}$ , the four-parameter interval grey number degenerates

to a three-parameter interval grey number. The membership function for four-parameter interval numbers can be constructed as:

Based on the above descriptions, for operation scheduling involving only three stages—anaesthesia, surgery, and recovery—the grey operation scheduling problem is described as follows:

- a) Let  $P = \{p_1, p_2, \dots, p_n\}$  represent the set of  $n$  patients, where  $p_i$  denotes the  $i$ -th patient,  $i = 1, 2, \dots, n$ .
- b) Let  $M$  represent the set of  $m$  operation devices, which can be expressed as  $M = \{\{m_{11}, m_{12}, \dots, m_{1q}\}, \{m_{21}, m_{22}, \dots, m_{2r}\}, \{m_{31}, m_{32}, \dots, m_{3s}\}\}$ , where  $n \geq q + r + s$ . Here,  $m_{1j}$ ,  $m_{2k}$ , and  $m_{3l}$  represent multiple parallel devices for operation stages 1, 2, and 3, respectively.
- c) Each patient must undergo operations in three stages: anaesthesia, surgery, and recovery. The sequence of operation stages is identical for all patients. One operation device can process at most one patient at a time. At least one operation stage has multiple devices, such as anesthetizing rooms, operating rooms, or recovery rooms.
- d) The operation time set for patient  $i$  is  $PT_i = \{\otimes a_{i1}, \otimes a_{i2}, \dots, \otimes a_{im}\}$ , where  $\otimes a_{ik} \in [\underline{\alpha}_{ik}, \alpha_{ik}^*, \bar{\alpha}_{ik}]$  is a three-parameter interval grey number representing the time required for the  $k$ -th operation stage of patient  $i$ . Here,  $\alpha_{ik}^*$  is the most likely operation time,  $\underline{\alpha}_{ik}$  represents the shortest possible time, and  $\bar{\alpha}_{ik}$  the longest possible time. The function  $\mu_{ik}(t)$  represents the possibility of completing the  $k$ -th operation stage of patient  $i$  at time  $t$ .
- e) The operation deadline for patient  $i$  is represented by a four-parameter grey number  $\otimes D_i = \otimes(d_i, d_i, d_i, d_i)$ , where the interval  $[d_i, d_i]$  represents the most likely operation completion window for patient  $i$ .

Other constraints of the scheduling problem are essentially consistent with those of the classical FFSP. Since patient operation deadlines better reflect demand variations in emergency rescue contexts, the ultimate objective of this research is to find a feasible scheduling scheme that minimizes the average tardiness of patient operations.

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## 1.2 Grey Number Operators

For a queue sequence of patients awaiting surgery on device  $k$ , if patient  $i$  has preceding patients  $1, 2, \dots, q$  in the queue, adding their operation times yields the completion time for patient  $i$ . Therefore, we define addition operations between two grey numbers.

**Definition 1** From reference [20], for three-parameter interval grey numbers

$\otimes a_{ik} \in [\underline{\alpha}_{ik}, \alpha_{ik}^*, \bar{\alpha}_{ik}]$  and  $\otimes a_{jk} \in [\underline{\beta}_{jk}, \beta_{jk}^*, \bar{\beta}_{jk}]$ , the addition operation yields:

$$\otimes a_{ik} + \otimes a_{jk} \in [\underline{\alpha}_{ik} + \underline{\beta}_{jk}, \alpha_{ik}^* + \beta_{jk}^*, \bar{\alpha}_{ik} + \bar{\beta}_{jk}]$$

From Definition 1, the grey operation completion time for patient  $i$  is:

$$\otimes T_i = \bigotimes_{q=1}^n (\otimes a_{qk})$$

where  $\otimes a_{qk}$  is the grey operation time of patient  $q$  on device  $k$ , and  $q = 1, 2, \dots, n$ .

To obtain patient operation tardiness, we also need subtraction operations for four-parameter interval grey numbers.

**Theorem 1** For the membership function of four-parameter interval numbers in Equation (2), there exists... [Theorem proof continues with mathematical expressions]

**Definition 2** For two four-parameter interval grey numbers  $\otimes b_i \in [\underline{\alpha}_i, \alpha_i, \bar{\alpha}_i, \bar{\alpha}_i]$  and  $\otimes b_j \in [\underline{\alpha}_j, \alpha_j, \bar{\alpha}_j, \bar{\alpha}_j]$ , reference [21] shows that subtraction yields a four-parameter interval grey number:

$$\otimes b_i - \otimes b_j \in [\underline{\alpha}_i - \bar{\alpha}_j, \alpha_i - \bar{\alpha}_j, \bar{\alpha}_i - \alpha_j, \bar{\alpha}_i - \underline{\alpha}_j]$$

Since patient operation completion times are three-parameter interval grey numbers while operation deadlines are four-parameter interval grey numbers, we can treat three-parameter interval grey numbers as a special class of four-parameter interval grey numbers during operations, i.e.,  $\otimes a_{ik} \in [\underline{\alpha}_{ik}, \alpha_{ik}^*, \bar{\alpha}_{ik}] \Leftrightarrow \otimes a_{ik} \in [\underline{\alpha}_{ik}, \alpha_{ik}^*, \bar{\alpha}_{ik}]$ . From Definition 2, the grey operation tardiness for patient  $i$  in the device waiting area is:

$$\otimes T_i = \otimes a_i - \otimes D_i$$

**Definition 3 [21]** Let  $\otimes b_i$  and  $\otimes b_j$  be two four-parameter interval numbers. If  $\underline{\alpha}_i \geq \underline{\alpha}_j$ ,  $\alpha_i \geq \alpha_j$ ,  $\bar{\alpha}_i \geq \bar{\alpha}_j$ , and  $\bar{\alpha}_i \geq \bar{\alpha}_j$ , then  $\otimes b_i \geq \otimes b_j$ . Similarly, if  $\underline{\alpha}_i \leq \underline{\alpha}_j$ ,  $\alpha_i \leq \alpha_j$ ,  $\bar{\alpha}_i \leq \bar{\alpha}_j$ , and  $\bar{\alpha}_i \leq \bar{\alpha}_j$ , then  $\otimes b_i \leq \otimes b_j$ .

Thus, if patient  $i$ 's uncertain operation tardiness  $\otimes T_i > 0$ , patient  $i$  definitely experiences operation tardiness; otherwise, no tardiness occurs. Therefore, possibility and necessity measures can be used to express uncertain operation tardiness.

### 1.3 Objective Function

**Definition 4** Let sets  $A, B \subseteq F(X)$  with membership functions  $\mu_A(x)$  and  $\mu_B(x)$  for  $x \in X$ . The possibility measure is:

$$Pos(A \geq B) = \sup_{x \in X} \min\{\mu_A(x), \mu_B(x)\}$$

and the necessity measure is:

$$Nec(A \geq B) = \inf_{x \in X} \max\{1 - \mu_A(x), \mu_B(x)\}$$

**Definition 5** Assuming patient  $i$ 's grey operation tardiness is  $\otimes T_i = \otimes(\underline{\alpha}_i, \underline{\alpha}_i, \bar{\alpha}_i, \bar{\alpha}_i)$ , the operation tardiness credibility is defined as the weighted sum of its possibility and necessity measures, denoted as  $Con_i(\otimes T_i \geq 0)$ :

$$Con_i(\otimes T_i \geq 0) = \delta \cdot Pos_i(\otimes T_i \geq 0) + (1 - \delta) \cdot Nec_i(\otimes T_i \geq 0)$$

where  $\delta \in [0, 1]$  is the tardiness credibility coefficient.

**Property 1** If patient  $i$ 's grey operation tardiness is  $\otimes T_i = \otimes(\underline{\alpha}_i, \underline{\alpha}_i, \bar{\alpha}_i, \bar{\alpha}_i)$ , then the operation tardiness credibility is:

$$Con_i(\otimes T_i \geq 0) = \begin{cases} 0, & \bar{\alpha}_i \leq 0 \\ \frac{\delta \bar{\alpha}_i}{\bar{\alpha}_i - \underline{\alpha}_i}, & \underline{\alpha}_i \leq 0 < \bar{\alpha}_i \\ \delta + \frac{(1-\delta)(\bar{\alpha}_i - \underline{\alpha}_i)}{\bar{\alpha}_i - \underline{\alpha}_i}, & \underline{\alpha}_i \leq 0 < \underline{\alpha}_i \\ 1, & \underline{\alpha}_i > 0 \end{cases}$$

**Proof:** Since the patient operation tardiness credibility coefficient satisfies  $0 \leq \delta \leq 1$ , for the piecewise function in Equation (4), we clearly have...[Proof continues with mathematical derivations]

Assume the grey operation completion time, grey deadline, and grey tardiness for patient  $i$  in the scheduling are  $\otimes a_{ik}$ ,  $\otimes D_i$ , and  $\otimes T_i$ , respectively. Let  $S$  be the set of all feasible operation schedules. If the scheduling objective function is known, the grey mixed-integer programming model can be established from Equation (4) as follows:

The model's objective is to obtain an optimal scheduling scheme that minimizes the objective function. In this paper, the objective function is the average value of all patients' operation tardiness credibility. Constraint (6) ensures each patient occupies a unique position in the device queue sequence without simultaneously occupying multiple positions. Constraint (7) ensures each position in the device queue sequence can accommodate only one patient. Constraint (8) defines the grey operation time for patient  $i$  at position  $j$  in device  $k$ 's queue sequence. Constraint (9) defines patient  $i$ 's grey operation tardiness. Constraint (10)

defines the indicator variable  $\omega_{ikjm}$ , which equals 1 when patient  $i$  is at position  $j$  in device  $k$ 's queue sequence, and 0 otherwise.

**Theorem 2** If patient  $i$ 's grey operation tardiness  $\otimes T_i = \otimes(\underline{\alpha}_i, \alpha_i, \bar{\alpha}_i, \bar{\bar{\alpha}}_i)$  satisfies  $\underline{\alpha}_i < \alpha_i \leq \bar{\alpha}_i < \bar{\bar{\alpha}}_i$ , then the possibility and necessity measures of tardiness occurrence are...[Theorem statement continues]

**Theorem 3** In a scheduling sequence  $S$ , if operation device  $k$  has several patients in its queue, with patient  $i$ 's grey operation completion time and operation time being  $\otimes D_i$  and  $\otimes a_{ik}$ , and patient  $j$ 's grey operation completion time and operation time being  $\otimes D_j$  and  $\otimes a_{jk}$ , where  $i, j = 1, 2, \dots, r$  are patient indices in device  $k$ 's queue, and  $r_k$  is the number of patients in device  $k$ 's queue. If  $\otimes D_i < \otimes D_j$ , then patient  $i$ 's operation sequence must precede patient  $j$ .

**Proof:** Assuming operation times are greater than 0, in an optimized scheduling sequence  $S$ , the average operation tardiness credibility for all patients in device  $k$ 's queue is...[Proof continues with mathematical derivations showing that swapping positions does not improve the solution]

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## 2.1 Introduction to Cuckoo Search Algorithm

The Cuckoo Search (CS) algorithm [22] is a bio-inspired swarm intelligence algorithm proposed by Yang from Cambridge University and Deb from Raman Engineering University in 2009 through studying cuckoo nest-laying behavior. Since its introduction, CS has attracted widespread attention from researchers and achieved numerous outstanding results. In 2010, Yang et al. [23] applied CS to multi-objective optimization problems. In 2011, Valian et al. [24] proposed an improved CS algorithm using feedforward neural network feedback. In 2015, Wang et al. [25] proposed a cuckoo algorithm with chaotic maps and variable value patterns.

The fundamental concept of CS originates from cuckoo reproduction behavior and the Lévy flight patterns of birds. Lévy flight is a random process combining long-term small-scale local search with occasional large-scale exploration, typically generating non-uniform random walks with large jumps. This approach effectively avoids attraction to local optima and demonstrates excellent global optimization capability, making CS suitable for solving the grey mixed-integer programming model proposed in this paper.

The CS algorithm is well-suited for small-scale experiments but not for computationally expensive optimization problems. Its search relies entirely on random walks, consuming excessive computational resources during optimization without guaranteeing rapid convergence. However, integrating CS with problem-specific characteristics can enhance its search capability and achieve better computational performance. This paper presents a Hybrid Grey Cuckoo Search (HGCS) algorithm.

## 2.2 Encoding Design

For discrete patient operation sequencing problems, CS cannot be applied directly. This paper adopts an operation sequence-based encoding rule. Each nest's position represents a feasible scheduling solution. For  $m$  operation devices and  $n$  patients, a feasible solution can be encoded using  $m \times n$  nest positions, representing a permutation of operation sequences where each patient appears exactly once. For example, in a  $4 \times 3$  problem with 4 patients and 3 devices, if the position encoding is 132143123442, the corresponding patient operation sequence is  $(J_{1,1}, J_{3,1}, J_{2,1}, J_{1,2}, J_{4,1}, J_{3,2}, J_{1,3}, J_{2,2}, J_{3,3}, J_{4,2}, J_{4,3}, J_{2,3})$ , where  $J_{i,j}$  represents the  $j$ -th operation stage of patient  $i$ . That is, patient 1 first undergoes anesthesia, followed by patient 3, then patient 2, then patient 1 undergoes surgery, and so on, until patient 2 completes recovery, as shown in [Figure 1: see original paper].

## 2.3 Initial Population Generation

In CS, nest positions must first be initialized randomly, without utilizing any problem-specific knowledge, resulting in low-quality initial populations. To improve initial population quality, this paper combines the shortest operation time first rule with the earliest deadline first rule, leveraging their characteristics to propose a grey heuristic algorithm for generating partial initial nest positions.

Let  $\otimes w = (\underline{\alpha}_w, \bar{\alpha}_w, \bar{\alpha}_w, \bar{\alpha}_w)$  be the grey weight,  $\delta$  the patient tardiness credibility coefficient, and  $\otimes a_{ik} = (\underline{\alpha}_{ik}, \alpha_{ik}^*, \bar{\alpha}_{ik})$  the grey operation time for patient  $i$ . The shortest operation time and earliest deadline can be expressed as...[Mathematical expressions continue]

The grey heuristic algorithm for obtaining initial population solutions proceeds as follows:

- a) Establish grey-marked sets for patients, operation devices, and deadlines:  $\{(\text{sign}(\otimes a_{1k}), (\text{sign}(\otimes a_{2k})), \dots, (\text{sign}(\otimes a_{nk}))\}$  for  $k \in [1, m]$  and  $\{(\text{sign}(\otimes D_1)), (\text{sign}(\otimes D_2)), \dots, (\text{sign}(\otimes D_n))\}$ .
- b) Assign patient  $i$  to device  $k$  for operation, updating sets  $P'_k = P_k - \{i\}$  and  $M' = M - \{k\}$ .
- c) Repeat step b) until each operation device is assigned patients.

To maintain population diversity, one-third of the population is generated using the above procedure, while the remainder is generated randomly. The basic CS parameters are set as: search space dimension  $D = m \times n$ , number of nests  $N = 30$ , probability of host discovering alien eggs  $P_a = 0.25$ , and maximum iterations  $MaxT = 1000$ . After initializing nest positions, they are converted

to operation sequence permutations, and the objective function values are calculated (minimizing the average tardiness credibility across all patients). The best nest position is identified.

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## 2.4 Candidate Population Generation

First, the previous generation's best nest position is retained, and each nest's fitness function is calculated as  $\text{fitness}(x_i) = Q - f(x_i)$ , where  $Q$  is an appropriate positive constant and  $f(x_i)$  is the objective function. Candidate populations are then generated through exploratory paths and position updates using Lévy flights. Lévy flight direction is arbitrary, with step size  $s$  following a Lévy distribution. Equation (18) generates candidate populations:

$$x_i^{t+1} = x_i^t + \alpha \cdot s \cdot \text{Levy}(\lambda)$$

where  $x_i^t$  represents the  $i$ -th nest's position at generation  $t$ ,  $\alpha$  is the step size control coefficient (typically 1),  $\otimes$  denotes point-to-point multiplication, and  $\text{Levy}(\lambda)$  represents the random search path of Lévy flights with directions following a uniform distribution.

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## 2.5 Selection

Theorem 3 is applied to improve all nest positions in the current population by comparing patient operation completion times: if  $\otimes D_i < \otimes D_j$ , then patient  $i$ 's operation sequence must precede patient  $j$ . CS's selection operator compares parent and candidate offspring one-to-one based on fitness values, with superior individuals retained for the next generation. If the previous generation's maximum fitness exceeds the current generation's maximum, the previous best nest replaces the current worst nest, following the rule in Equation (19).

This greedy selection strategy tracks the optimal solution found during evolution, preventing degradation during iteration and accelerating convergence.

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## 2.6 Random Migration

Random migration resembles mutation in genetic algorithms. When eggs are discovered, basic CS assumes cuckoos randomly search for new nests, which is ineffective and prone to local optima. Instead, we can utilize undiscovered nest positions through crossover operations. The crossover method proceeds as: first, randomly shuffle the  $N$  host nest positions and store the result; repeat this operation; then subtract the two results to obtain a crossover step size for finding new host nests.

If the probability  $P_a$  of alien eggs being discovered exceeds a uniformly distributed random number  $r \in [0, 1]$ , the crossover method searches for new host nests. After boundary checking, better nests replace inferior ones; otherwise, host nests remain unchanged. This operation preserves relatively good nests, eliminates inferior individuals, and increases population diversity.

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## 2.7 Termination Condition

After selection and random migration, the optimal nest position and fitness value are evaluated against stopping criteria (reaching maximum iterations  $MaxT$  or required precision). If satisfied, the global optimum and corresponding position are output; otherwise, the process returns to selection and random migration for further iteration. Finally, decoding the global optimal position yields the desired scheduling scheme.

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## 3 Experiments and Results

To validate the HGCS algorithm, a classic  $6(3) \times 3$  benchmark instance is used for simulation testing. This instance, proposed by Sakawa, simulates operation scheduling with uncertain operation times and deadlines. Experimental parameters for HGCS and CS [26] are: number of nests  $N = 30$ , discovery probability  $P_a = 0.25$ , and maximum iterations  $MaxT = 1000$ . The simulation environment is: Windows 7 OS, 2.0 GHz Intel Core i3-2350M CPU, 2 GB RAM, MATLAB 2014a.

For the  $6(3) \times 3$  grey operation scheduling problem minimizing average patient operation tardiness credibility, the grey operation times and deadlines are shown in , with patient operation precedence constraints as follows: [TABLE:1 content would appear here]

Patient tardiness credibility coefficients  $\delta$  are selected as 0.1, 0.2, 0.5, 0.7, and 0.8. The objective is the minimum average patient operation tardiness credibility value, with results averaged over 10 random runs shown in .

[TABLE:2 content would appear here]

Table 2 shows that as  $\delta$  increases, the mean objective value also increases, primarily because larger  $\delta$  significantly increases the weight of the possibility measure, thereby increasing patient operation tardiness credibility. For  $\delta = 0.2, 0.5, 0.7$ , and 0.8, CS and HGCS algorithms are compared. Convergence curves for one arbitrary run at each  $\delta$  value are shown in [Figure 2: see original paper] through [Figure 5: see original paper].

[FIGURE:2-5 descriptions and content would appear here]

Table 3 demonstrates that for different  $\delta$  values, HGCS consistently achieves better (lower) average optimal objective values than CS. The convergence curves in Figures 2-5 also show that when curves converge globally, HGCS produces superior optimal values with faster convergence speed.

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## 4 Conclusion

Batch operation scheduling for wounded patients in emergency rescue resembles flexible flow-shop scheduling optimization, but with uncertainties in operation times and deadlines. This paper proposes a grey scheduling model using three-parameter grey numbers for uncertain operation times and four-parameter grey numbers for uncertain deadlines—a more general and practical approach than triangular or trapezoidal fuzzy numbers used in other literature. Additionally, by leveraging properties of grey numbers, optimal grey scheduling characteristics are obtained and used to improve the basic cuckoo algorithm, resulting in the Hybrid Grey Cuckoo Search (HGCS) algorithm.

Simulation tests on the  $6(3) \times 3$  benchmark instance demonstrate that HGCS effectively solves patient operation tardiness problems caused by uncertain operation times and deadlines. Comparison with basic CS shows HGCS significantly outperforms CS, indicating its effectiveness for uncertain operation scheduling problems and its practical application value. However, the current solution addresses only single-objective optimization; multi-objective and resource-constrained scheduling problems represent future research directions.

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