

# Joint User Location and Spatial Distance Based Pilot Allocation Scheme for Massive MIMO (Postprint)

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## Abstract

Pilot contamination is a bottleneck problem in massive MIMO (multiple-input and multiple-output) systems. This paper proposes a pilot allocation scheme that jointly considers user location and spatial distance to mitigate pilot contamination. The scheme first classifies all users based on whether their angles of arrival overlap, categorizing users with non-overlapping angles of arrival into the first  $n$  sets and the remaining users into the  $(n+1)$ -th set. For the first  $n$  user sets,  $n$  groups of orthogonal pilots are allocated for reuse within each set, whereas for users in the  $(n+1)$ -th set, an interference function is defined to perform allocation by constructing a weighted graph during pilot selection. Simultaneously, the uplink spectral efficiency is derived, and the pilot overhead as well as algorithmic complexity are analyzed. Simulation results demonstrate that the proposed scheme not only greatly reduces algorithmic complexity and pilot overhead, but also significantly improves the system spectral efficiency (SE).

## Full Text

### Preamble

#### Pilot Allocation Scheme Based on Joint User Location and Spatial Distance in Massive MIMO

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**Abstract:** Pilot contamination represents a fundamental bottleneck in massive multiple-input multiple-output (MIMO) systems. This paper proposes a novel pilot allocation scheme that jointly exploits user location and spatial distance

information to mitigate pilot contamination. The scheme first classifies all users according to whether their angles of arrival (AoAs) overlap, grouping users with non-overlapping AoAs into the first  $n$  sets, while the remaining users are assigned to the  $(n+1)$ th set. For the first  $n$  user sets,  $n$  orthogonal pilot sequences are allocated for reuse within each set. For users in the  $(n+1)$ th set, an interference function is defined to construct a weighted graph for pilot allocation. Additionally, the uplink spectral efficiency is derived, and both pilot overhead and algorithmic complexity are analyzed. Simulation results demonstrate that the proposed scheme significantly reduces computational complexity and pilot overhead while substantially improving system spectral efficiency (SE).

**Keywords:** massive MIMO; user classification; angle of arrival; spatial distance; graph coloring

## 0 Introduction

Massive MIMO systems have attracted considerable attention due to their potential to significantly enhance spectral and energy efficiency. In time division duplex (TDD) massive MIMO systems, downlink channel state information can be obtained through channel reciprocity, which necessitates accurate uplink channel estimation as a prerequisite for designing detection and precoding matrices [1]. However, achieving perfect channel state information requires completely orthogonal pilot sequences among users. Constrained by channel coherence time, pilot sequences must inevitably be reused across adjacent cells, leading to interference known as pilot contamination. When the number of base station antennas becomes very large, intra-cell interference and uncorrelated noise vanish, yet the interference caused by reusing non-orthogonal pilots does not diminish with increasing antenna count. Consequently, pilot contamination constitutes a critical bottleneck limiting massive MIMO system performance [2].

Extensive research has been conducted to mitigate pilot contamination. Reference [1] proposed reducing inter-cell pilot contamination by redesigning pilot transmission protocols, though this approach requires complex control mechanisms to coordinate pilot transmission timing across cells. Reference [2] introduced a smart pilot allocation algorithm with a pilot reuse factor of 1, ensuring orthogonal pilots for users in cells with poor channel quality while allowing users with good channel quality to reuse the same pilots. This method performs pilot allocation for target cells under the assumption of random pilot assignment in interfering cells, but its limitation lies in considering only the target cell's pilot contamination rather than the entire system. Reference [3] proposed a pilot contamination mitigation scheme combining channel estimation with user AoAs, which requires knowledge of channel covariance matrices. References [4,5] constructed interference graphs based on large-scale fading coefficients for pilot allocation, evaluating interference through edge weights in inter-cell user graphs. While this approach reasonably considers system-wide pilot allocation, its complexity becomes a concern when the number of users per cell is large.

Reference [6] introduced machine learning for pilot allocation, where the optimal pilot assignment obtained through exhaustive search serves as training data to learn a pilot allocation model. When pilot allocation is needed, the model determines optimal pilots based on users' large-scale fading coefficients. However, this method suffers from high complexity in obtaining training data. Considering partial pilot reuse, Reference [7] partitioned cells into central and edge user regions, employing a pilot reuse factor of 1 for central users and 3 for edge users to prioritize mitigating pilot contamination for edge users. This method applies different pilot reuse factors based on contamination severity but requires a large number of orthogonal pilot sequences.

This paper proposes a joint user location and spatial distance-based pilot selection scheme. The fundamental idea is to first classify users according to whether their AoAs overlap, grouping users with non-overlapping AoAs into  $n$  sets, while users that cannot satisfy this condition are assigned to the  $(n+1)$ th set. The first  $n$  sets are randomly assigned  $n$  pilot groups for reuse within each set. For the  $(n+1)$ th set, allocating mutually orthogonal pilots to all users would incur unacceptable overhead; therefore, a weighted graph approach is employed for pilot selection.

## 1 System Model

The system model operates in TDD mode, consisting of  $L$  hexagonal cells, each with  $K$  uniformly distributed single-antenna users and a base station equipped with  $M$  antennas ( $M \gg K$ ) at the cell center. When the number of paths  $P$  is large, the uplink channel from the  $k$ th user in the  $j$ th cell to the  $l$ th base station is given by [3,8]:

$$h_{jkl} = \sum_{p=1}^P a_{jklp} \beta_{jkl}^{1/2} a(\theta_{jklp})$$

where  $a(\theta_{jklp})$  is the steering vector. For a uniform linear array (ULA) at the base station, the steering vector [3] is:

$$a(\theta) = \frac{1}{\sqrt{M}} [1, e^{-j2\pi D \cos(\theta)/\lambda}, \dots, e^{-j2\pi D \cos(\theta)(M-1)/\lambda}]^T$$

Here,  $\theta \in [0, 2\pi)$  represents the AoA of the signal path,  $\lambda$  is the signal wavelength, and  $a_{jklp} \sim \mathcal{CN}(0, 1)$  denotes the complex Gaussian gain on the  $p$ th path [8].  $D$  is the antenna spacing [3].  $\gamma$  is the path loss exponent, and  $\beta_{jkl}$  represents the large-scale fading coefficient (including path loss and shadowing).  $x_{jkl}$  and  $x_l$  denote the positions of user  $k$  in cell  $j$  and base station  $l$ , respectively, with  $\|\cdot\|$  representing the Euclidean norm.  $z$  is a constant determined by the cell-edge SNR, cell radius  $R$ , and noise variance  $\delta^2$  [3].

During the uplink pilot transmission phase, the signal received at base station  $l$  is:

$$Y_l = \sum_{j=1}^L \sum_{k=1}^K h_{jkl} \phi_{jk}^T + N_l$$

where  $\phi_{jk} \in \mathbb{C}^{\tau \times 1}$  is the pilot sequence used by user  $k$  in cell  $j$  (with  $\|\phi_{jk}\|^2 = 1$ ),  $\tau$  is the pilot sequence length, and  $N_l \in \mathbb{C}^{M \times \tau}$  is additive white Gaussian noise (AWGN) with i.i.d.  $\mathcal{CN}(0, \delta^2)$  elements.

The AoA range for user  $k$  in cell  $j$  at base station  $l$  is  $[\theta_{jkl}^{\min}, \theta_{jkl}^{\max}]$ , where:

$$\theta_{jkl}^{\min} = \arctan\left(\frac{[x_{jkl}]_2 - [x_l]_2}{[x_{jkl}]_1 - [x_l]_1}\right) - \arcsin\left(\frac{r}{\|x_{jkl} - x_l\|}\right)$$

$$\theta_{jkl}^{\max} = \arctan\left(\frac{[x_{jkl}]_2 - [x_l]_2}{[x_{jkl}]_1 - [x_l]_1}\right) + \arcsin\left(\frac{r}{\|x_{jkl} - x_l\|}\right)$$

where  $r$  represents the user's scattering radius.

[Figure 1: see original paper] illustrates the user AoA model for two cells with single users. According to Theorem 1 in [3], when the AoAs of interfering signals do not overlap at all with the AoA of the desired signal, no interference occurs even with non-orthogonal pilot sequences [3]. That is:

$$\lim_{M \rightarrow \infty} \frac{1}{M} a^H(\theta_i) a(\theta_j) = \begin{cases} 0, & \cos(\theta_i) \neq \cos(\theta_j) \\ 1, & \cos(\theta_i) = \cos(\theta_j) \end{cases}$$

Therefore, users are classified based on whether their AoAs overlap. Let there be  $n + 1$  user sets  $U_1, U_2, \dots, U_n, U_{n+1}$ , where  $U_1$  to  $U_n$  contain users with mutually non-overlapping AoAs, and  $U_{n+1}$  contains users with fully or partially overlapping AoAs. For users in  $U_{n+1}$ , allocating orthogonal pilots to all would be prohibitively expensive. Thus, even when AoAs overlap, we desire sufficiently large spatial distances among users. This paper defines a cost function to evaluate interference magnitude and performs pilot selection through interference graph construction.

Let  $a(\theta_{jkl}^p)$  be the steering vector for the  $p$ th path of target user  $k$  in cell  $j$  at base station  $l$ , and  $a(\theta_{ilk}^p)$  be the steering vector for the  $p$ th path of interfering user  $k$  in cell  $i$  at base station  $l$ . The spatial distance between them is  $d_{ij} = \|x_i - x_j\|$  where  $x_i, x_j \in \mathbb{R}^2$ .

The cost function is defined as:

$$J(d_{ij}, \theta_i, \theta_j) = \left| \frac{1}{M} a^H(\theta_i) a(\theta_j) \right|^2 = \begin{cases} \left| \frac{1 - e^{j2\pi D(\cos \theta_i - \cos \theta_j)M/\lambda}}{M(1 - e^{j2\pi D(\cos \theta_i - \cos \theta_j)/\lambda})} \right|^2, & \cos(\theta_i) \neq \cos(\theta_j) \\ 1, & \cos(\theta_i) = \cos(\theta_j) \end{cases}$$

The uplink channel estimate is:

$$\hat{h}_{jkl} = \frac{1}{\tau} Y_l \phi_{jk}^* = h_{jkl} + \sum_{(i,s) \neq (j,k)} h_{isl} \frac{\phi_{is}^T \phi_{jk}^*}{\tau} + \frac{1}{\tau} N_l \phi_{jk}^*$$

This analysis shows that channel estimation accuracy is impaired by interference from users in adjacent cells using non-orthogonal pilot sequences.

## 2 Pilot Allocation

This section presents a pilot allocation scheme based on joint user location and spatial distance to mitigate pilot contamination. The scheme first classifies users according to AoA overlap, then applies different pilot selection strategies to each user set.

[Figure 2: see original paper] shows the cost function curve versus AoA when  $M = 64$  and  $d = 100$  m. The figure reveals that when the interfering user's AoA falls within a certain range around the target user's AoA, the cost function value is very small, indicating non-overlapping AoAs. Conversely, overlapping AoAs result in larger cost values.

The scheme comprises two stages: user classification and pilot selection. The pseudocode is shown in and .

**User Classification Stage:** 1. **Preparation (Steps 1-4):** Calculate and store each user's AoA, large-scale fading coefficient, and spatial distance in matrices DAR, Beta, and d, respectively. 2. **User Clustering (Steps 5-10):** For each user in the target cell, check whether neighboring cell users have overlapping AoAs. If not, assign them to sets  $U_i$ , resulting in n user sets with mutually non-overlapping AoAs where  $|U_i| \geq 1$  for  $i = 1 \dots n$ . Remaining users that cannot satisfy this condition are assigned to set  $U_{n+1}$ .

**Pilot Selection Stage:** 1. **For the first n sets (Steps 1-4):** Randomly select n columns from the pilot matrix  $\Phi$  and assign them to the n user sets, marking these n pilot groups as unavailable. 2. **For the (n+1)th set (Steps 5-18):** Construct a weighted graph  $G = (V, E)$  for users in  $U_{n+1}$  based on the cost function. Steps 8-10 calculate interference weights for all users in  $U_{n+1}$ . Steps 11-12 select the user with maximum interference weight for subsequent pilot assignment. Steps 13-15 assign the pilot group that minimizes pilot contamination from remaining sequences. Step 16 adds the assigned user to set M.

### 3 Performance Analysis

#### 3.1 Uplink Spectral Efficiency

During uplink data transmission, the signal received at the base station is:

$$y_l^{ul} = \sum_{j=1}^L \sum_{k=1}^K h_{jkl} x_{jk} + n_l$$

where  $x_{jk}$  is the uplink data symbol from user  $k$  in cell  $j$  with  $E\{|x_{jk}|^2\} = 1$ , and  $n_l \in \mathbb{C}^{M \times 1}$  is AWGN.

Using matched filter reception, the detector is  $a_{lk}^H$ , yielding:

$$\hat{x}_{lk} = a_{lk}^H y_l^{ul} = \underbrace{a_{lk}^H h_{lkl} x_{lk}}_{\text{desired signal}} + \underbrace{\sum_{(j,k) \neq (l,k)} a_{lk}^H h_{jkl} x_{jk}}_{\text{interference}} + \underbrace{a_{lk}^H n_l}_{\text{noise}}$$

The uplink SINR is:

$$\text{SINR}_{lk} = \frac{|E\{a_{lk}^H h_{lkl}\}|^2}{\sum_{j \neq l} |E\{a_{lk}^H h_{jkl}\}|^2 + \sum_{(j,k) \neq (l,k)} \text{Var}\{a_{lk}^H h_{jkl}\} + \delta^2 E\{|a_{lk}|^2\}}$$

The uplink spectral efficiency for cell  $l$  using matched filter reception is:

$$\text{SE}_l = \frac{T - \tau}{T} \sum_{k=1}^K \log_2(1 + \text{SINR}_{lk}) \quad [\text{bits/s/Hz/cell}]$$

where  $T$  is the coherence interval,  $\tau$  is the pilot length, and  $T_1$  represents the downlink data slot duration. The pilot overhead for the proposed scheme is:

$$\tau_{\text{proposed}} = n + |U_{n+1}| < dT$$

where  $|*|$  denotes set cardinality.

#### 3.2 Complexity Analysis

The proposed scheme's main idea is to classify users by AoA overlap and apply corresponding pilot allocation strategies. Complexity can be analyzed from two aspects:

**a) User Classification:** Classifying  $K$  users in  $L$  cells requires traversing all  $LK$  users, resulting in time complexity  $O(LK)$ .

**b) Pilot Configuration:** For sets  $U_1$  to  $U_n$ , pilots are randomly assigned with complexity  $O(n)$ . For users in  $U_{n+1}$ , interference graph construction is required, with complexity  $O(|U_{n+1}|^3)$ . The total complexity is:

$$O(LK) + O(n) + O(|U_{n+1}|^3) = O(LK + |U_{n+1}|^3)$$

compares the complexity of exhaustive search, the scheme in [5], and the proposed scheme. The proposed algorithm has lower complexity because user classification reduces the graph construction complexity from  $O((LK)^3)$  to  $O(|U_{n+1}|^3)$ .

## 4 Simulation Analysis

Monte Carlo simulations evaluate the proposed scheme. The multi-cell massive MIMO system comprises  $L$  hexagonal cells with  $K$  uniformly distributed single-antenna users per cell [3]. Base stations are located at cell centers with no users within 100 m. The coherence block has  $T = 200$  (coherence time 1 ms, coherence bandwidth 200 kHz). Simulation parameters are listed in .

[Figure 4: see original paper] shows user classification for  $K = 10$  users, where black triangles represent base stations, black circles denote  $U_{n+1}$  users, and same-colored markers indicate users in the same set. The pilot overhead is 10: 7 orthogonal pilot groups for colored users and 3 for  $U_{n+1}$  users through inter-cell pairing.

[Figure 5: see original paper] compares spectral efficiency versus base station antenna count among conventional random allocation, the scheme in [5], and the proposed scheme. Random allocation reaches an SE ceiling as antenna count increases because intra-cell interference and noise vanish, leaving only pilot contamination. The scheme in [5] achieves higher SE by optimizing pilot selection based on large-scale fading. The proposed scheme outperforms both by reducing pilot overhead and mitigating contamination through differentiated treatment. At  $M = 450$ , the proposed scheme achieves approximately 4 bits/s/Hz/cell improvement over [5] and about 8 bits/s/Hz/cell gain over random allocation at  $M = 500$ .

[Figure 6: see original paper] shows the cumulative distribution function (CDF) of uplink SINR for  $M = 256$  antennas. The proposed scheme provides higher SINR by weakening pilot contamination. For the first  $n$  user sets, no pilot contamination exists either within or between sets due to non-overlapping AoAs and orthogonal pilots, respectively. Contamination only originates from  $U_{n+1}$ , where graph coloring-based pilot selection further suppresses interference. Consequently, the proposed scheme achieves significant uplink SINR improvement.

## 5 Conclusion

This paper proposes a pilot allocation scheme based on joint user location and spatial distance. Users are classified by AoA overlap, enabling different pilot selection strategies for each class. This classification effectively separates non-interfering users into  $n$  sets and potentially interfering users into the  $(n+1)$ th set. The first  $n$  sets are allocated  $n$  orthogonal pilot groups for intra-set reuse. For the  $(n+1)$ th set, users with sufficiently large spatial distances share pilots, while others use orthogonal pilots. This approach substantially reduces pilot overhead and computational complexity while improving spectral efficiency, making it a practical low-complexity pilot selection solution.

[Figure 6: see original paper] The CDF of SINR

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