

## Postprint of Air Route Network Delay Optimization Based on Improved Hungarian Algorithm

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**Date:** 2019-01-03T00:00:00+00:00

### Abstract

Addressing the issue that air route network flow allocation in China relies on controller experience and lacks quantitative models for decision support, this study investigates delay optimization models for air route networks and their solution algorithms. First, a flow assignment model is established with an objective function of minimizing total flight time and subject to capacity constraints. Second, to overcome the limitation of the classical Hungarian algorithm being applicable only to small and medium-scale problems, an improvement is proposed in the efficiency matrix calculation by maximizing the probability that the number of circled zero elements equals the order of the efficiency matrix. Third, based on the characteristic that air route operational efficiency varies with time and traffic flow, an M/M/C queuing theory model is utilized to construct the cost function for air routes, replacing constant efficiency values in the efficiency matrix with variable costs to enable the matrix to vary with time and flow. Finally, actual operational data from November 2016 for partial airspace of the Central South and Southwest Air Traffic Management Bureaus are employed as a case study. The results indicate that in terms of capacity optimization, the algorithm can improve arrival capacity by 8.372% and departure capacity by 8.999%; in delay optimization, it can reduce the average delay per aircraft; and in algorithmic performance, it exhibits advantages of fewer iterations and shorter solution time compared to the classical Hungarian algorithm, making it more suitable for practical air traffic control operations.

### Full Text

### Preamble

**Vol. 37 No. 3**

**Application Research of Computers  
ChinaXiv Cooperative Journal**

## Optimization of En-Route Network Delay Based on Improved Hungarian Algorithm

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**Abstract:** To address the issue that China's en-route network flow allocation relies heavily on controller experience and lacks quantitative models to assist decision-making, this paper investigates delay optimization models and their solution algorithms for en-route networks. First, a traffic flow assignment model with capacity constraints was established, aiming to minimize total flight time. Second, to overcome the limitation that the classical Hungarian algorithm is only suitable for small and medium-sized problems, the efficiency matrix calculation was improved to maximize the probability that the number of circled zero elements equals the order of the efficiency matrix. Third, considering that route operational efficiency varies with time and traffic flow, an M/M/C queuing theory model was used to construct the route cost function, replacing constant efficiency values in the efficiency matrix with variable costs that change over time and flow. Finally, actual operational data from November 2016 for airspace under the administration of Central and Southwest Air Traffic Management Bureaus was used as a case study. Results show that the algorithm can increase arrival capacity by 8.372% and departure capacity by 8.999%; in terms of delay optimization, it reduces average delay per aircraft; and in terms of algorithm performance, it requires fewer iterations and shorter solution time compared to the classical Hungarian algorithm, making it more suitable for practical air traffic control operations.

**Keywords:** air transportation; delay optimization; Hungarian algorithm; en-route network; assignment problem; cost function

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## 0 Introduction

The en-route network serves as the operational carrier of the air traffic system, and its efficiency directly determines the overall system performance. Controllers currently manage air traffic flow based on personal experience, lacking quantitative models to provide decision support for allocating traffic to various routes. This approach easily leads to controller fatigue in complex airspace with high traffic volumes, necessitating a method that can characterize airspace features to assist controller decision-making. Air traffic flow management (ATFM) is a classic problem in air traffic control, for which experts worldwide have established various optimization models with different objective functions and constraints, designing different algorithms based on model characteristics.

Existing short-term ATFM research primarily employs mathematical programming or computer simulation methods. Mathematical programming methods

are mostly based on network flow theory, but can only precisely solve small-scale en-route networks. For large-scale problems, heuristic algorithms or problem decomposition and simplification must be used to reduce computational time. Whether optimizing for minimum ground delay, minimum airborne delay, minimum flight time, or minimum fuel consumption and emissions, en-route network operational optimization is essentially a combinatorial optimization problem of assigning and allocating limited time and space resources. Although modern computing power has improved significantly, the combinatorial scale of large-scale ATFM prevents many algorithms from being applied in practical control operations.

This paper addresses the high time complexity of network flow algorithms for ATFM and the excessive iterations of the classical Hungarian algorithm, which is only suitable for small and medium-sized assignment problems. The contributions are: (a) establishing a 0-1 integer programming model with capacity constraints and transforming it into an assignment problem for solution; (b) improving the efficiency matrix calculation required by the Hungarian method to maximize the probability that the number of circled zero elements equals the order of the efficiency matrix, thereby reducing iterations; and (c) proposing to replace constant coefficients in the efficiency matrix with variable impedance functions that change with traffic flow and other factors, based on the characteristic that en-route network operational efficiency varies with traffic flow. Finally, the efficiency and computational quality of the model and algorithm were validated using actual data from November 2016 for airspace under the administration of Central and Southwest Air Traffic Management Bureaus.

## 2.1 Objective Function

The objective of short-term traffic flow management in en-route networks is to minimize the total navigation cost of all aircraft in the network. Therefore, the objective function can be set as:

For en-route network analysis, in a multi-origin network with nodes, where and represent the number of nodes, China's altitude allocation principle of "east odd, west even" allows the network to be treated as a planar directed weighted connected graph. Let the directed graph be where: represents the total system navigation time, with the goal of minimizing the sum of costs across all routes in the network, i.e., minimizing total navigation time, where the cost varies with time and air traffic flow.

## 2.2 Constraints

Under fixed safety separation, each route segment can only accommodate one traffic flow at a time. For an en-route network with nodes, where represents the set of route intersections, represents the set of origin-destination airports, and represents the set of route segments (with  $m$  as the route origin and  $n$  as the route destination), the weight matrix method is used to describe the en-

route network to facilitate computer solution of the assignment problem. The conversion method can be found in reference [13].

## 1.2 Capacity and Influencing Factors

The basic capacity of an en-route network is defined as the maximum traffic volume that controllers can handle under established network structure and control operation rules, given certain traffic flow arrival patterns and distribution characteristics. Capacity is influenced by many factors, including weather, military activities, terrain, route conflicts at intersections, restricted areas, and other boundary constraints [1,3]. A decision variable is introduced, which can only take values of 1 or 0, where:

Previous ATFM research often treated navigation cost as a constant [6,8,10], which is appropriate for non-congested network flow management but yields results that significantly deviate from reality for congested networks. Actual en-route networks experience severe congestion, where cost increases with traffic flow. Therefore, a cost function that reflects en-route network operational characteristics must be designed.

Among numerous factors, those most closely related to aircraft assignment are time (whether flight delays can be reduced), distance (whether en-route network efficiency can be improved), and associated costs (whether benefits can be maximized). Different flight types prioritize these factors differently. Thus, the cost function is defined as:

where: represents the flow coefficient, representing the impedance impact of traffic flow, which can be calibrated from actual data; represents the node service rate; represents the air traffic flow from node  $r$  to node  $s$  at time  $t$  in the terminal area; is an adjustment coefficient that can be calibrated from actual data; represents the time required for an aircraft to fly route  $rs$ , where is the length of route segment  $rs$  and is the average speed of the air traffic flow.

With the decision variable and route cost function determined, the en-route network capacity model can be considered. Literature has proven that aircraft arrivals per unit time follow a Poisson distribution, and airport node service follows a negative exponential distribution [14]. Therefore, average delay can be calculated using Little' s formula from the M/M/c queuing theory model [15]:

Each traffic flow can only pass through one route in time period. When assigning traffic flows to routes, both route capacity and the capacity of the next airport node must be considered. Let represent the capacity of route  $rs$  and represent the capacity of airport node  $s$ . Capacity constraints can thus be added:

Methods for determining airport capacity and route capacity have been introduced in many references [1] and are not repeated here.

### 3.1 Shortcomings of Classical Hungarian Algorithm

The classical Hungarian algorithm can effectively solve certain engineering application problems (specific steps can be found in reference [15]). However, for ATFM problems, it has the following limitations:

- a) The air route network is a complex giant system. Converting route costs into an efficiency matrix results in high computational time complexity, making it unsuitable for practical control operations.
- b) As air traffic flow increases, the en-route network becomes congested, and efficiency values (congestion levels) of each route change. The classical Hungarian method treats the efficiency matrix as constant, which is unreasonable for traffic flow assignment in air route networks. The efficiency function mentioned above must be introduced.

### 3.2 Algorithm Improvement

#### 1) Improvement Philosophy

To address the first limitation in Section 3.1, the improvement philosophy is to minimize algorithm iterations by maximizing the probability that the number of circled zero elements in step 2 equals the order  $r$  of the efficiency matrix. Improvements are made in steps a) and b) below.

To address the second limitation in Section 3.1, the improvement philosophy is to use the cost function defined above to describe the matrix, i.e., the values in the efficiency matrix are variable and change with traffic flow and time.

#### 2) Detailed Steps of Improved Algorithm

Based on the above philosophy and considering the complex structure of air route networks and time-varying traffic flows that change route capacity, the improved Hungarian algorithm steps are as follows:

- a) Initialize the en-route network to obtain initial impedance. Let be the traffic flow to be allocated in time period  $i$ , and set iteration count.
- b) Efficiency matrix preprocessing. Compare the number of minimum elements per row with the number of minimum elements per column in the efficiency matrix. If , iterate by column; if , iterate by row.
- c) Flow assignment. Assign traffic flow using steps 3-5 of the classical Hungarian algorithm to obtain the assignment result for time period  $i$ . If flow is assigned to route  $r_s$  and , then the unassigned remaining flow enters the next iteration. If capacity constraints are not met, the route still has remaining capacity.
- d) Update route impedance. Use the assigned flow to obtain new route impedance and convert it to a new efficiency matrix.

- e) Convergence check. Let be the total flow to be allocated in each time period. If , stop iterating; otherwise, return to step a).

## 4.1 Case Study Construction

The airspace under the administration of Central and Southwest Air Traffic Management Bureaus is among China' s busiest. Therefore, this airspace was selected to validate the model and algorithm. Case data was obtained from traffic statistics provided by the two bureaus for November 2016. MATLAB 2017a was used as the computational platform to verify the model and algorithm.

### 4.2.1 Capacity Enhancement

Based on 2016 statistics, abnormal traffic data such as military activities and hazardous weather were removed. The daily average arrival and departure capacities of six major civil airports in the simulated airspace were calculated: Chengdu Shuangliu, Chongqing Jiangbei, Guilin Liangjiang, Nanning Wuxu, Zhangjiajie Hehua, and Guiyang Longdongbao (see Tables 1 and 2). Tables 1, 2, and historical data indicate that Chengdu Shuangliu and Chongqing Jiangbei airports face significant operational pressure.

Using the improved Hungarian algorithm to solve for these six airports, the optimized capacities shown in Tables 3 and 4 were obtained. The capacities of the two busy airports (Chengdu Shuangliu and Chongqing Jiangbei) improved significantly. Although Guilin Liangjiang, Nanning Wuxu, Zhangjiajie Hehua, and Guiyang Longdongbao airports themselves were not congested, the entire airspace network is a system. Optimizing congested airspace enabled more aircraft to avoid ground delays, resulting in increased traffic at these four airports. Taking the two busy airports as examples, the algorithm increased arrival capacity by 8.372% and departure capacity by 8.999%.

### 4.2.2 Delay Reduction Optimization

Using equations (4) and (5) to calculate en-route network delays, the delay levels before and after optimization for the two busiest airports in the simulated airspace are shown in Figures 1 [Figure 1: see original paper] and 2 [Figure 2: see original paper]. Chengdu Shuangliu Airport' s average delay decreased by 3 minutes, while Chongqing Jiangbei Airport' s average delay decreased by 2 minutes. ICAO defines a delay as a departure delay exceeding 15 minutes. The algorithm controlled average delay per aircraft during peak periods mostly within 15 minutes.

### 4.2.3 Computation Time Reduction

The improved Hungarian algorithm was implemented in MATLAB 2017a on a desktop computer with an Intel Core i5-4590 3.30 GHz CPU and 4 GB RAM. By gradually increasing the number of nodes in the simulated airspace, the

solution time and iteration count of the improved algorithm were compared with the original algorithm.

As shown in Table 5 , as the number of nodes increases and network complexity grows, the efficiency advantage of the improved Hungarian algorithm becomes increasingly significant.

## 5 Conclusion

Based on the characteristics of large-scale en-route networks with congestion levels varying by traffic flow and time, a 0-1 integer programming model for traffic flow assignment was established. Rules were constructed to simplify efficiency matrix computation, effectively reducing algorithm iterations. Variable cost functions were used to replace constant coefficients in the efficiency matrix. Using actual data from airspace under Southwest and Central Air Traffic Management Bureaus as a case study, historical data was used to calculate capacities of major airports in the simulated airspace. The established model and improved Hungarian algorithm were applied to optimize the airspace. Simulation results demonstrate that the optimization model and improved Hungarian algorithm can effectively enhance en-route network capacity, reduce average delay per aircraft, and offer efficiency advantages over the classical Hungarian algorithm, making them more suitable for practical control operations.

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