Postprint: Fast Decaying Component Identification for Low-Frequency Oscillations in Power Systems Using an Improved Prony Method

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Abstract

Considering that the traditional Prony algorithm encounters significant difficulties in analyzing fast-decaying components in low-frequency oscillation modes of power systems and is prone to missing such components, an improved method for extracting fast-decaying signals—the Prony piecewise analysis method based on Givens transformation—is proposed. This method divides the sampling time into two intervals: first analyzing the second interval to determine the slowly decaying components, then analyzing the first interval to determine the fast-decaying components. Finally, a numerical example verifies that the algorithm can accurately extract the fast-decaying components in low-frequency oscillation modes while also improving the accuracy of the slowly decaying components.

Full Text

Preamble

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Identification of Fast-Attenuating Components in Power System Low-Frequency Oscillations Based on Improved Prony Method

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Abstract

Traditional Prony algorithms encounter significant difficulties when analyzing fast-attenuating components in power system low-frequency oscillation modes and are prone to losing these rapidly decaying components. To address this issue, this paper proposes an improved method for extracting rapid attenuation signals—the Prony segmented analysis method based on Givens transform. This approach divides the sampling time into two periods: the second period is analyzed first to obtain the slow-attenuation components, followed by analysis of the first period to extract the fast-attenuation components. Numerical examples demonstrate that the proposed algorithm can accurately extract fast-attenuating components from low-frequency oscillation modes while simultaneously improving the accuracy of slow-attenuation component identification.

Keywords: low-frequency oscillation; Prony method; fast-attenuation component; Givens transform; segmented analysis; accuracy

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Abstract (English): Considering that the traditional Prony algorithm is very difficult to analyze the fast-attenuating component of power system low-frequency oscillation modes and such components can be easily missed, an improved method for extracting rapid attenuation signals is proposed, called the Prony segment analysis method based on Givens transform. This method divides the data acquisition time into two periods, and the second period is analyzed first to calculate the slow-attenuating component, then the first period is analyzed for calculating the fast-attenuating component. Numerical examples prove that this algorithm can accurately extract fast-attenuating components of low-frequency oscillation modes and also makes the slow-attenuating component more accurate.

Keywords: Low frequency oscillation, Prony method, fast attenuation, Givens transform, segment analysis, accuracy

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The improved algorithm proposed in this paper has been granted a national invention patent (Patent No.: 2016100315119).

1 Introduction

With the introduction of ultra-high voltage transmission networks and the acceleration of cross-regional interconnection in China, low-frequency oscillation has become a critical issue affecting power grid security and stability. Sustained oscillations can damage electrical equipment and trigger cascading accidents, resulting in severe consequences.

Analysis of measured data indicates that methods for identifying oscillation modes include Short-Time Fourier Transform (STFT) [?], wavelet analysis [?], Prony algorithm, and Hilbert-Huang Transform (HHT) [?]. Considering computational complexity, time consumption, and applicability, the Prony algorithm is selected as it can directly obtain signal modal information through fitting in the time domain without requiring frequency response calculations, significantly reducing computational load and demonstrating strong applicability.

To more effectively analyze low-frequency oscillations in power systems and extract complete oscillation characteristics, this paper improves the traditional Prony algorithm by implementing segmented analysis based on Givens transform. The Givens transform approach does not require forming normal equations; instead, it directly solves for characteristic equation coefficients from the eigenvalue equation, avoiding the difficulties and large errors associated with solving normal equations in the original algorithm. Furthermore, after segmentation, the method can accurately identify fast-attenuating components in low-frequency oscillation modes while simultaneously improving the precision of slow-attenuation components.

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2 Prony Algorithm Calculation Steps

The Prony algorithm [?] is a commonly used method for extracting stationary oscillation modes. For equally spaced sampling points, it assumes the model is a linear combination of q exponential functions with arbitrary amplitude, phase, frequency, and decay factor. Its discrete-time function form is:

$$x(t) = A_i e^{\alpha_i t} \cos(2\pi f_i t + \theta_i)$$

where A_i is the amplitude; θ_i is the initial phase (rad); f_i is the oscillation frequency (Hz); α_i is the attenuation factor (damping); and q is the signal order.

Assuming Equation (1) contains q attenuated sinusoidal components, the cosine term can be expressed using Euler' s formula:

$$\cos(2\pi f_i t + \theta_i) = \frac{e^{j(2\pi f_i t + \theta_i)} + e^{-j(2\pi f_i t + \theta_i)}}{2}$$

Let $p = 2q$, then the discrete-time function can be expressed as:

$$\hat{x}(n) = \sum_{k=1}^{p} a_k \hat{x}(n-k) \quad n = p, \ldots, N-1$$

To establish the Prony algorithm, define the error $e(n)$ between the actual measured value $x(n)$ and the estimated value $\hat{x}(n)$:

$$x(n) = \hat{x}(n) + e(n) \quad n = 0, \ldots, N-1$$

Substituting Equation (5) into Equation (6) yields:

$$\hat{x}(n) = \sum_{k=1}^{p} a_k \hat{x}(n-k) + e(n) = \sum_{k=1}^{p} a_k x(n-k) + \sum_{k=1}^{p} a_k e(n-k)$$

Equation (7) shows that this model is an Auto-Regressive Moving Average (ARMA) model ARMA(p,p) excited by error $e(n)$, with identical Auto-Regressive (AR) and Moving Average (MA) parameters (a_i). To make the simulated signal approach the real signal, the principle of minimum squared error can be adopted, where the least squares estimation of $a_1, \ldots, a_{p-1}, a_p$ minimizes $e(n)$, resulting in a set of nonlinear equations that are difficult to solve.

Changing the objective function for error estimation can avoid solving nonlinear equations. Let $\epsilon(n) = \sum_{k=0}^{p} a_k e(n-k)$, then Equation (7) becomes:

$$x(n) = \sum_{k=1}^{p} a_k x(n-k) + \epsilon(n) \quad n = p, \ldots, N-1$$

Therefore, minimizing $e(n)$ is equivalent to minimizing $\epsilon(n)$, which constitutes the extended Prony algorithm. The extended Prony algorithm solves the following matrix equation:

$$\begin{bmatrix} x(p) & x(p-1) & \cdots & x(0) \\ x(p+1) & x(p) & \cdots & x(1) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-1) & x(N-2) & \cdots & x(N-p-1) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \epsilon(p) \\ \epsilon(p+1) \\ \vdots \\ \epsilon(N-1) \end{bmatrix}$$

The key to the Prony method is recognizing that Equation (1) is the homogeneous solution of a linear difference equation with constant coefficients. To minimize the objective function $J(\mathbf{a}) = \sum_{n=p}^{N-1} |\epsilon(n)|^2$, we set $\frac{\partial J(\mathbf{a})}{\partial a_i} = 0$, which yields:

$$\sum_{k=0}^{p} a_k \sum_{n=p}^{N-1} x(n-k)x^*(n-i) = 0 \quad i = 1, \dots, p$$

The corresponding minimum error energy is:

$$\epsilon_p = \sum_{k=0}^{p} a_k \sum_{n=p}^{N-1} x(n-k)x^*(n)$$

Define $r(i,k) = \sum_{n=p}^{N-1} x(n-k)x^*(n-i)$, for $i, j = 0, \dots, p$. The normal equation form of the Prony algorithm is obtained:

$$\begin{bmatrix} r(0,0) & r(0,1) & \cdots & r(0,p) \\ r(1,0) & r(1,1) & \cdots & r(1,p) \\ \vdots & \vdots & \ddots & \vdots \\ r(p,0) & r(p,1) & \cdots & r(p,p) \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \epsilon_p \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Solving Equation (12) yields estimates of parameters $a_1, \dots, a_p$ and minimum error energy ϵ_p. Substituting $a_1, \dots, a_p$ into the characteristic equation and solving further yields characteristic roots z_i, $i = 1, 2, \dots, p$. Equation (3) can then be simplified to a linear equation in terms of b_i, expressed in matrix form as:

$$\mathbf{Z}\mathbf{b} = \hat{\mathbf{x}}$$

where

$$\mathbf{Z} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_p \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_p^{N-1} \end{bmatrix}, \quad \mathbf{b} = [b_1, \dots, b_p]^T, \quad \hat{\mathbf{x}} = [\hat{x}(0), \dots, \hat{x}(N-1)]^T$$

The solution is:

$$\mathbf{b} = [\mathbf{Z}^H\mathbf{Z}]^{-1}\mathbf{Z}^H\hat{\mathbf{x}}$$

where the superscript H denotes conjugate transpose. During the solution process, computational load can be reduced and efficiency improved through the following relationship:

$$\mathbf{Z}^H\mathbf{Z} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1p} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{p1} & \gamma_{p2} & \cdots & \gamma_{pp} \end{bmatrix}$$

where

$$\gamma_{ij} = \frac{(z_i^* z_j)^N - 1}{z_i^* z_j - 1}$$

In Equation (19), "*"represents conjugation. Since the number of frequency components in low-frequency oscillations is unknown beforehand (i.e., p in Equation (9) is uncertain), and to improve algorithm accuracy, the model order can be set larger (i.e., take $p_e > p$), then calculate the effective rank p of the coefficient matrix through appropriate methods.

Using Equation (20), calculate frequency f_i, attenuation factor α_i, phase θ_i, and amplitude A_i:

$$f_i = \frac{\arctan[\mathrm{Im}(z_i)/\mathrm{Re}(z_i)]}{2\pi\Delta t}$$

$$\alpha_i = \frac{\ln|z_i|}{\Delta t}$$

$$\theta_i = \arctan[\mathrm{Im}(b_i)/\mathrm{Re}(b_i)]$$

$$A_i = |b_i|$$

3.1 Traditional Calculation Method

In applying the Prony algorithm to solve power system low-frequency oscillation signals, solving for the characteristic equation coefficients is the core of the entire process. The traditional calculation method uses the normal equation form of the Prony algorithm to obtain the characteristic equation coefficients **a**.

Equation (9) can be abbreviated as:

$$\mathbf{X}\mathbf{a} = \mathbf{d}$$

where **X** is the coefficient matrix formed from sampling data; **a** represents the characteristic equation coefficients; and **d** is the signal error.

The normal equation form of Equation (21) is:

$$\mathbf{X}^T\mathbf{X}\mathbf{a} = \mathbf{X}^T\mathbf{d}$$

Gaussian elimination is used to solve Equation (22) and obtain the characteristic equation coefficients **a**. The above constitutes the traditional calculation method based on the normal equation form [**?**].

3.2 Segmented Analysis Method

The Givens method is now selected to solve for the characteristic equation coefficients. Using Givens transform [**?**] to solve the equations and obtain characteristic equation coefficients does not require forming normal equations; instead, it can directly solve Equation (21). The Givens transform method can accurately calculate signal parameter values when processing signals without fast-attenuating components. However, when processing signals containing fast-attenuating components, especially those with two or more rapid decay components, the Givens method and other traditional methods may not achieve satisfactory results and sometimes even lose the faster decaying components.

To address this problem, this paper proposes an improved method based on the Givens transform—segmented analysis method. The specific implementation steps are as follows:

(1) **Read sampling data**: Let the sampling interval be Δt and sampling duration be T. To accurately extract fast-attenuating components, the sampling frequency should be set around 0.01s. The sampling data thus has interval Δt, duration T, and number of samples $T/\Delta t$.

(2) **Time period division**: Divide the total time length into two periods. The first period has duration t_1, and the remaining portion constitutes the second period with duration $T - t_1$. The division point between the first and second periods is t_s.

(3) **Second period Prony low-frequency oscillation analysis**: This period analyzes slow-attenuation components. The sampling interval Δt_2 is set to t_b, starting from time t_s and taking one sample value from the sampling data every interval t_b (i.e., selecting one sample every $t_b/\Delta t$ data points). The specific analysis process is: Starting from t_s, take one sample every t_b until time T ends; Filter the denoised sampling data to remove high-frequency components; First select order q_e greater than the effective rank p, then form an extended-order matrix $\mathbf{R}_e$ from the filtered sampling data, use singular value decomposition to compute all singular values of $\mathbf{R}_e$, and determine its effective rank p; Re-select order n slightly larger than effective rank p but smaller than q_e; Use Givens transform to solve for coefficients **a** in Equation (21); Substitute the

coefficients from step into the characteristic equation to solve for characteristic roots z_i; Substitute the characteristic roots into Equation (20) to obtain frequencies and attenuation factors; Use Equations (17)-(19) to solve for parameters b_i; Substitute b_i into Equation (20) to obtain amplitudes and initial phases; Finally, substitute the four parameters—frequency, attenuation factor, amplitude, and initial phase—into Equation (1) to obtain the expression for the slow-attenuation component signal $x_2(t)$.

(4) **First period sampling data processing**: The first period contains both fast- and slow-attenuation components. Let the first period sampling interval Δt_1 be t_a, sampling from 0s until time t_s. Substitute these time points into the slow-attenuation component expression $x_2(t)$ obtained in step (3) to get the slow-attenuation data corresponding to each time point in this period, then subtract these slow-attenuation data from the first period sampling data to obtain the fast-attenuation data.

(5) **First period Prony low-frequency oscillation analysis**: Similarly, using the same method as in step (3) to analyze the obtained fast-attenuation data yields the four modal information parameters of the fast-attenuation components.

The figure below shows the flowchart of the low-frequency oscillation mode analysis using the Prony segmented analysis method proposed in this paper. After adopting the segmented analysis method, the Givens method' s effectiveness in solving low-frequency signals containing fast-attenuating components is greatly improved.

[FIGURE:N] Flow chart of low frequency oscillation mode analysis by Prony method

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4 Example Analysis

This section first compares the improved algorithm with the Prony segmented analysis algorithm based on normal equation form, then compares the improved algorithm with the non-segmented Givens transform method through example analysis, finally verifying that the improved algorithm has excellent extraction performance and can accurately extract fast-attenuating components.

The constructed input signal is:

$$
\begin{aligned}
x(t) = {} & 10e^{-0.0027t}\cos(2\pi\cdot 0.423t + 110^\circ) \\
& + 2e^{-0.2652t}\cos(2\pi\cdot 0.42t + 13^\circ) \\
& + 10e^{-0.0311t}\cos(2\pi\cdot 0.2473t + 20^\circ) \\
& + 1e^{-0.2936t}\cos(2\pi\cdot 1.0349t + 60^\circ) \\
& + 5.8e^{-45.8788t}\cos(2\pi\cdot 2.4t + 10^\circ) \\
& + 6.3e^{-55.1156t}\cos(2\pi\cdot 0.9t + 60^\circ)
\end{aligned}
$$

The signal model order n is uniformly set to 15. The above signal contains 6 sinusoidal components, equivalent to 12 complex exponential components. One value is calculated every 0.01s for this signal, totaling 1,000 values to simulate sampling data. Thus, the sampling data has an interval of 0.01s, duration of 10s, and 1,000 samples. The parameters of each signal component are shown in Table 1 .

4.1 Comparison Between Improved Algorithm and Prony Segmented Analysis Based on Normal Equation Form

To verify that the improved Prony segmented analysis method proposed in this paper can accurately extract low-frequency oscillation signals containing fast-attenuating components, the signal segmentation point is set at 1s here. The first period analyzes fast-attenuating components with a sampling interval of 0.01s; the second period analyzes slow-attenuating components with a sampling interval of 0.1s. Table 2 compares the calculation results of the segmented analysis methods based on Givens transform and normal equation form.

As shown in Table 2, for components 1-4 (slow-attenuation components), both the Givens-transform-based segmented analysis and the normal-equation-form-based segmented analysis can accurately extract modal information. However, for fast-attenuating components, the latter identifies component 6 with a frequency of 1.5285 Hz versus the theoretical value of 0.9000 Hz, and an amplitude of 2.4217 versus the theoretical value of 6.3001, showing serious deviation. The Givens-transform-based segmented analysis method identifies fast-attenuating components with high precision and minimal error from theoretical values.

4.2 Comparison Between Segmented and Non-Segmented Givens Transform Methods

The signal segmentation point is selected at 1s. Table 3 compares the calculation results of segmented and non-segmented analysis based on the Givens transform method.

As shown in Table 3, the Givens-transform-based segmented method can accurately extract modal information for low-frequency signals with various oscillation characteristics. When not segmented, the method fails to identify fast-attenuating component 6. This example demonstrates that the segmented method proposed in this paper achieves good results and can accurately identify fast-attenuating components in signals.

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5 Conclusion

(1) In cases where oscillation modes do not contain fast-attenuating components, traditional Prony algorithms can identify oscillation modal information with relatively high accuracy. When oscillation modes contain two or more fast-attenuating components, traditional Prony algorithms, due to their fixed sampling frequency, cannot capture rapidly decaying components, which subsequently affects the identification accuracy of slow-attenuating components.

(2) The improved algorithm in this paper combines the advantages of Givens transform and segmented Prony method, offering unique benefits for processing denoised low-frequency oscillation signals. Using Givens transform avoids the difficulties of solving normal equations in traditional algorithms, improves numerical stability, and reduces computational load. Employing segmented analysis with different resolutions (sampling frequencies) avoids losing fast-attenuating components that occurs when using a single sampling frequency. This algorithm requires small memory, can accurately extract fast-attenuating components, and is suitable for identifying power system low-frequency oscillation modes.

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Note: Figure translations are in progress. See original paper for figures.