Postprint of Adaptive Integral Backstepping Control Strategy for Inverted Pendulum

Authors: Liu Chengji, Tao Yufan, Guo Siwen, Chen Yihang

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Abstract

To address the stable control problem of a linear single-stage inverted pendulum under model parameter uncertainties and external disturbances, an adaptive integral backstepping control strategy is proposed. The Lagrange equations are employed to establish the kinematic model of the inverted pendulum system. To reduce steady-state error, the error integral term is introduced into the backstepping method, and a controller for the inverted pendulum is designed. For the nonlinear state differential equations of the system containing unknown parameters, an appropriate Lyapunov function is designed to derive the adaptive update law for the unknown system parameters, which mitigates the impact of parameter uncertainties. Simulation results comparing the adaptive integral backstepping control with conventional backstepping control, fuzzy control, and neural network control are presented, and physical experiments are conducted in the LabVIEW development environment. The results demonstrate that the adaptive integral backstepping method can accomplish stable control relatively quickly and accurately, effectively overcoming the effects of system parameter uncertainties and external disturbances, and exhibits strong robustness.

Full Text

Preamble

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Adaptive Integral Backstepping Control Strategy for Inverted Pendulum

Liu Chengji, Tao Yufan, Guo Siwen, Chen Yihang
(School of Mechanical Electronic & Information Engineering, China University

of Mining & Technology, Beijing 100083, China)

Abstract: This paper proposes an adaptive integral backstepping control strategy to address the stability control problem of linear single-stage inverted pendulum systems with uncertain model parameters and external disturbances. The kinematic model of the inverted pendulum system is established using Lagrange equations. To reduce steady-state error, an error integral term is introduced into the backstepping method to design the controller. For the nonlinear state differential equations containing unknown parameters, an appropriate Lyapunov function is designed to derive adaptive update laws for the system's unknown parameters, thereby weakening the influence of parameter uncertainty. Simulation results comparing the proposed adaptive integral backstepping control with conventional backstepping, fuzzy control, and neural network control are presented, along with physical experiments conducted in the LabVIEW development environment. The results demonstrate that the adaptive integral backstepping method can achieve rapid and precise stability control, effectively overcome system parameter uncertainties and external disturbances, and exhibits strong robustness.

Keywords: inverted pendulum; uncertainty; adaptive control; integral backstepping

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0 Introduction

The inverted pendulum system serves as an important research platform in the control field, characterized by nonlinearity, underactuation, instability, and strong coupling. In recent years, numerous scholars have conducted extensive research on stability control of inverted pendulum systems [1~3]. The main control methods include linearization approaches such as PID and LQR; variable structure control methods like sliding mode control; and intelligent control methods such as fuzzy control and neural network control. References [4,5] applied PID and LQR control methods to nonlinear inverted pendulum systems, employing linear control theory on linearized models, but their limitation lies in requiring small linearization errors and simple models. Considering the nonlinear characteristics of inverted pendulum systems, reference [6] parameterized the switching gain in sliding mode control using fuzzy logic, achieving certain control effects. Although fuzzy control does not require precise mathematical models, conventional fuzzy control strategies struggle to establish complete fuzzy rules to guarantee performance and must be combined with other methods such as LQR and PID [7,8]. Therefore, reference [9] incorporated parameter self-tuning into fuzzy control, adjusting fuzzy parameters online based on error and its rate of change, yielding improved results.

Backstepping is a constructive method with strong systematicity, structure, and flexibility, proving highly effective for designing adaptive controllers for a broad class of strict parametric nonlinear systems. As a nonlinear control design tool, it not only avoids canceling useful nonlinearities but also offers great flexibility in selecting stabilization functions and control Lyapunov functions. The construction of feedback control laws and associated control Lyapunov functions is systematic, enabling robust handling of constant parameter uncertainties and general uncertainty disturbances [10]. The design approach decomposes the entire system into multiple subsystems not exceeding the system order, starting from the core of the high-order system and progressively designing control laws for each subsystem to ultimately derive the overall system control law.

This paper first establishes the inverted pendulum system' s kinematic model using Lagrange equations [11]. Unlike conventional backstepping, an error integral term is incorporated into the controller design. By establishing appropriate integral functions for each subsystem and adding integral terms to each error variable, the system' s steady-state error is reduced [12-15]. Subsequently, adaptive control is combined with backstepping. For parameter uncertainties in the controller, the designed adaptive law enables online estimation of uncertain parameters, compensating for system parameter uncertainty effects. When the inverted pendulum experiences external disturbances, the adaptive law adjusts input parameters in real time, making the adaptive integral backstepping method more robust against disturbances than conventional backstepping and significantly enhancing system robustness. Finally, through simulation comparisons with conventional backstepping, fuzzy control, and neural network control, as well as physical experiments under LabVIEW, the superiority of the proposed method is verified.

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1 Inverted Pendulum Kinematic Model

This paper focuses on the linear single-stage inverted pendulum and establishes its kinematic model using Lagrange equations. The Lagrange approach does not require Newton' s laws and eliminates the need to consider constraint forces, thereby simplifying mechanical calculations and avoiding cumbersome computations. The system' s kinematic model is transformed into a second-order motion control model, after which the backstepping method is employed to construct the inverted pendulum system controller. The linear inverted pendulum model is shown in [Figure 1: see original paper].

[Figure 1: see original paper]

In the diagram, F represents the force acting on the cart, M and m denote the masses of the cart and pendulum rod respectively, l is the length from the pendulum' s center of mass to the pivot point, is the angle between the pendulum rod and the vertical direction, x is the cart position, g is gravitational acceleration, and f is the friction between cart and track.

When analyzing the system model, friction at the pendulum-cart joint is negligible due to its minimal impact. The system Lagrange equation is given by:

FMml xgfiqiiiidTTVFdtqqqq TV iq22111{[(cos)]22dTImldt 2[(sin)]}dxldt 2212TMx 1cosVmgl i

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2.1 Controller Design

The inverted pendulum controller design can be divided into four steps:

a) Derive the system state equations (11)~(13) from the inverted pendulum model equations (7)~(10).

b) Define error variables and design integral terms to ensure asymptotic stability, obtaining intermediate virtual control quantities. Define the error variable as e and design the integral term to guarantee asymptotic stability of z, yielding the virtual control law. When e converges to 0, z converges accordingly. The derivative of z is designed to asymptotically converge to 0, where k is the error gain with $k > 0$.

c) Based on system model (12), substitute the virtual control law to obtain the system stabilization controller. To make z converge to 0, design the integral function where k is the gain for control input F. Combining the equations yields the system controller (20).

d) The integral action ensures convergence of the error variables, where k represents the gain for control input F. Equation (20) constitutes the final system controller.

2.2 Adaptive Law Derivation

Adaptive control typically addresses systems with known structure but unknown parameters, requiring minimal prior knowledge about models and disturbances. It continuously extracts model information during system operation to update parameters toward their true values. Here, parameter update laws are designed based on Lyapunov stability theory. Let the estimates of parameters , , be ˆ , ˆ , ˆ respectively. The estimation errors are defined as ˜ = - ˆ .

For the system equation (7) with unknown parameters, substituting the controller expression yields the error dynamics. To derive adaptive update laws for , , , define the Lyapunov function:

222231211231()2222Vzhx 1 2 3

where , , are positive constants. Differentiating V with respect to time and substituting the system dynamics yields:

3311221123ˆˆˆ()Vzzhx 211111ˆ[()sec]dVkzzxkex 2212ˆ(tan)zx 23311113ˆ[()cossin]dzxkexzxx

By Lyapunov stability theory, for asymptotic stability we require V 0. This is satisfied by choosing the adaptive update laws as:

1111^()secdzxkex 221^tanzx 1233111^sin()cosdzxxxkex

These constitute the adaptive update laws for parameters , , .

3.1 Simulation Model and Parameter Settings

Based on the established linear single-stage inverted pendulum kinematic model and the designed adaptive integral backstepping controller, a control system is built using MATLAB/Simulink. The controller and pendulum model components are encapsulated into three main modules: the integral backstepping controller, parameter adaptive estimator, and inverted pendulum plant, as shown in [Figure 2: see original paper].

[Figure 2: see original paper]

The physical parameters of the inverted pendulum system model used in simulation are listed in .

The gains for error e and control input F were determined through iterative testing as $k = 10$ and $k = 2.5$. The constants , , are adaptive gains, all set to 1. Differentiating equation (23) with respect to time and substituting equations (13) and (22) yields the parameter adaptation dynamics. According to Lyapunov stability theory, ensuring V 0 guarantees asymptotic stability, which is achieved through the derived adaptive laws.

3.2 Simulation Results

To verify the rapidity, accuracy, disturbance rejection capability, and parameter uncertainty adaptation of the proposed adaptive integral backstepping method, comparative simulation experiments were conducted against conventional backstepping, fuzzy control, and neural network control under both disturbance-free and disturbed conditions.

Experiment 1: Balance Control Without Disturbance

With an initial angle of 0.1 rad, balance control experiments were performed without disturbance. The results are shown in [Figure 3: see original paper]. The settling times for adaptive integral backstepping, conventional backstepping, fuzzy control, and neural network control were 1.5 s, 2.2 s, 2.5 s, and 3.4 s, respectively. The steady-state errors were 2.5×10 rad, 3×10 rad, 5×10 rad, and 6.5×10 rad, respectively. The overshoot values were 0.007 rad, 0.015 rad, 0.034 rad, and 0.044 rad, respectively. All four methods achieved balance control, but the proposed method exhibited faster convergence (0.7 s, 1.0 s, and 1.3 s quicker than the others), smaller steady-state error than conventional backstepping, and reduced overshoot compared to all alternative methods.

Experiment 2: Balance Control With Disturbance

To validate disturbance rejection capability, pulse disturbances were introduced after system stabilization. A pulse disturbance with 0.1 s width and 0.1 rad amplitude was applied, with response results shown in [Figure 4: see original paper]. The adaptive integral backstepping method stabilized in 4.7 s, which was 0.5 s, 1.2 s, and 2.7 s faster than conventional backstepping, fuzzy control, and neural network control, respectively. It also exhibited the smallest maximum fluctuation amplitude of 0.7 rad, representing a 1.2 rad reduction compared to the neural network control' s 1.9 rad. The proposed method stabilized after a single small oscillation, while other methods required multiple adjustments, demonstrating superior response speed and anti-interference performance.

Experiment 3: Parameter Adaptation Without Disturbance

The adaptive capability for uncertain parameters was tested by setting different initial pendulum angles: 5° (solid line), 10° (dashed line), and 15° (dotted line). Simulation results are shown in [Figure 5: see original paper] through [Figure 10: see original paper]. [Figure 5: see original paper] shows the pendulum angle responses, where larger initial angles produce greater overshoot. [Figure 6: see original paper] displays the virtual control variable z, which converges to 0 due to integral action. [Figure 7: see original paper] illustrates the controller output F response, showing trends similar to z. [Figure 8: see original paper] through [Figure 10: see original paper] present the estimated values of parameters , , and , respectively. Under all three conditions, the parameter estimates converge to constant values within finite time, demonstrating that the adaptive controller can online estimate uncertain system parameters [16] and adjust controller outputs accordingly, enabling adaptation to different unknown parameters.

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4 LabVIEW Experiment

Physical control experiments were conducted in the LabVIEW development environment using the platform shown in [Figure 11: see original paper]. The inverted pendulum control system comprises an angle encoder, stepper motor, pendulum rod and slide rail, data acquisition card, computer, and DC power supply (12V). An OMRON E6B2-CWZ6C encoder transmits pendulum angle signals via two digital channels to the NI USB-6210 data acquisition card. A 57-stepper motor connects via three digital channels: one for frequency signals controlling motor speed and two for direction signals. The data acquisition card interfaces with the computer for data exchange.

[Figure 11: see original paper]

The LabVIEW program consists of input, controller, and output modules. The input module uses DAQ Assistant VI to receive two-channel encoder signals containing angle information, configured with 600 pulses per revolution and an initial angle of 180°. The controller module encapsulates the control algorithm VI. The output module employs DAQmx VI to output three signals: one "CO Pulse Frequency" for motor speed control and two "Digital Output" channels for direction control. Parameters include "Frequency" for initial motor frequency, "Counter(s)" for port selection, "Idle State" for idle-level configuration, and "Initial Delay" for pre-run timing. The control loop executes every 10 ms within a while loop structure, as shown in [Figure 12: see original paper].

[Figure 12: see original paper]

Under disturbance-free conditions with an initial angle of 0.2 rad, the pendulum angle response is shown in [Figure 13: see original paper]. The angle stabilizes to 0 rad within 2 s with overshoot below 0.02 rad, maintaining stable dynamic balance near the vertical. After stabilization, a manual disturbance was applied at 4 s, with results shown in [Figure 14: see original paper]. The system demonstrates strong anti-interference capability, good dynamic performance, and high accuracy. Compared to simulation, the physical system exhibits slightly longer settling times and minor overshoot, but the results are fundamentally consistent.

[Figure 13: see original paper]
[Figure 14: see original paper]

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5 Conclusion

This paper proposes an adaptive integral backstepping control strategy for linear single-stage inverted pendulum systems with parameter uncertainties and external disturbances. By analyzing the system model' s state equations, an integral backstepping controller is designed, with adaptive laws enabling online estimation of uncertain parameters. The integral error term reduces steady-state error compared to conventional backstepping. Simulation and physical control results under both disturbance-free and disturbed conditions demonstrate that the proposed method can rapidly and effectively estimate system parameters, mitigate parameter uncertainty effects, enhance anti-interference capability, and achieve fast, accurate control with strong robustness. Future research will consider neglected factors such as friction terms in the mathematical model and explore combinations with other methods to further improve control precision.

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Note: Figure translations are in progress. See original paper for figures.