All-Terminal Reliability Analysis of Hypernetworks: Postprint

Authors: Zhang Ke, Zhao Haixing, Ye Zhonglin, Zhu Yu

Date: 2018-12-13T00:00:00+00:00

Abstract

Network reliability is an important field in complex network research. Hypernetworks, which can effectively characterize certain complex systems, fall within the research scope of complex networks. Based on the topological structure of hypernetworks—the hypergraph, we propose a definition of all-terminal reliability for hypernetworks under edge failures and present two fundamental methods for calculating reliability: the state enumeration method and the factorization method. We further employ the factorization method to simplify certain hypernetworks with special structures. As an application of hypernetwork reliability, we investigate the number of connected spanning subnetworks. Through comparison with ordinary complex networks, it is evident that the reliability study of hypernetworks cannot be replaced by the reliability research of their converted ordinary complex network counterparts. This study represents a preliminary exploration into hypernetwork reliability research, which possesses broad research space and application prospects.

Full Text

Analysis for All-Terminal Reliability of Hypernetworks

ZHANG Ke[1,2,3], ZHAO Haixing[1,2,3]†, YE Zhonglin[2,3], , ZHU Yu[1,2,3]
[1]School of Computer Science, Qinghai Normal University, Xining 810016, China
[2]Key Laboratory of Tibetan Information Processing & Machine Translation of Qinghai Province, Xining 810008, China
[3]Key Laboratory of Tibetan Information Processing of Ministry of Education, Xining 810008, China
School of Computer Science, Shaanxi Normal University, Xi' an 710062, China

Abstract: Network reliability is an important field of complex network research. Hypernetworks, which can effectively characterize certain complex systems, belong to the scope of complex network research. Based on the topological structure of hypernetworks—hypergraphs—this paper presents the definition of all-terminal reliability of hypernetworks under edge failure and proposes two fundamental methods for calculating reliability: state enumeration and factorization. According to the factorization method, some hypernetworks with special structures can be simplified. As an application of hypernetwork reliability, the number of connected spanning sub-networks is studied. Compared with ordinary complex networks, we conclude that the reliability of hypernetworks cannot be replaced by the reliability of ordinary complex networks transformed from corresponding hypernetworks. This research represents a preliminary exploration of hypernetwork reliability, which has broad research spaces and application prospects.

Key words: reliability; all-terminal; hypernetwork; factorization; hypergraph

---

0 Introduction

Various complex systems in the real world are often abstracted as network models for study, where the network topology is represented by ordinary graphs. Particularly in the past two decades, network science has achieved considerable development, and the reliability of corresponding systems has become an important research direction [1]. For instance, faults, natural disasters, or deliberate attacks can degrade the functionality of power networks and even cause large-scale blackouts. Similarly, a small local interruption in communication networks may ultimately affect communication services across entire regions. Solving such problems depends on the research and development of network reliability.

In evaluating the connectivity reliability of network systems, the probability that a network remains connected is used as its reliability metric. Network reliability models fall into three categories: node failure, edge failure, and both node and edge failure [2]. Network reliability research encompasses two aspects: reliability analysis and design [2]. Abstracted network graphs can be directed or undirected, and various models exist for their study, including 2-terminal, k-terminal, and all-terminal models, each further divided into active and passive types [3]. Research methods mainly include: (a) classical analytical methods such as state enumeration, factorization, and inclusion-exclusion principle; (b) approximate and simulation methods such as bounding methods, Monte Carlo methods, and intelligent algorithms; and (c) other methods related to network structure and specific requirements. Rich research results from different perspectives have substantially advanced network reliability theory.

With further development of network science, researchers have discovered that some networks feature diverse vertices, relationships between vertices that are not limited to simple pairwise connections, and structures exhibiting multi-layer or nested characteristics [4]. To address these new problems encountered in network research, the concept of hypernetworks has been proposed, broadening and enriching the scope of complex network research. Generally, hypernetworks are divided into two categories: graph-based hypernetworks and hypergraph-based hypernetworks. The reliability of the former has been studied [5]; the latter is the focus of this paper, referring to complex networks modeled using hypergraphs. If these networks were modeled using ordinary graphs, issues such as vertex heterogeneity or loss of network information would arise, making hypergraph modeling a more effective approach. For example, hypergraph descriptions of scientific collaboration networks can more comprehensively reflect cooperative relationships among researchers [6], and hypergraph descriptions of food competition networks can clearly reflect competitive relationships among populations [7].

Over the past decade, hypernetworks have attracted increasing attention from researchers. Studies on hypernetworks have mainly focused on model establishment [6,8,9], topological metrics [10,11], and applications [12] [4]. In terms of applications, since a single link in a hypernetwork model can contain two or more vertices, hypergraph-based modeling of complex networks offers significant advantages over ordinary graphs. On one hand, since graphs are special types of hypergraphs, any complex system that can be modeled using ordinary complex networks can also be modeled using hypernetworks. On the other hand, hypernetworks can accurately characterize more diversified connection relationships in complex systems, thereby exploring undiscovered laws in network science [4]. As evident from the development of hypernetworks, they belong to the category of complex network research. Network reliability, as a research hotspot with major theoretical significance and application value, has yielded many relevant results in ordinary complex networks. With more complex systems being described using hypernetworks, the need for hypernetwork reliability research has become increasingly apparent. For instance, in express delivery hypernetworks [13], overloading a few logistics companies can significantly reduce the circulation efficiency of the hypernetwork. However, in the context of hypernetworks, this research area has barely been explored domestically or internationally to date, and related research results are scarce.

In 1987, Chen Tinghuai et al. [14] applied hypergraph connectivity to the design of fault-tolerant multibus systems. In 1997, Cao Qiguo et al. [15] designed highly reliable multibus structures in communication networks from a hypergraph perspective. Recently, Ma Xiujuan et al. [13] analyzed cascading failures in express delivery hypernetworks and electronic component hypernetworks.

This paper primarily presents a preliminary exploration of hypernetwork reliability. We first define the all-terminal reliability of hypernetworks and propose two basic methods for calculating reliability. Based on these calculation methods, we simplify reliability computations for hypernetworks with special structures. Using hypernetwork reliability, we analyze the number of connected spanning sub-hypergraphs and their relationship with reliability. Finally, we conduct a comparative study between ordinary graphs and hypergraphs using reliability, identifying hypergraphs whose reliability differs from that of ordinary graphs.

We also list several directions for further research.

---

1 Basic Concepts and Terminology of Hypergraphs

This paper studies the reliability of hypernetworks. Here, hypernetworks are modeled using finite, undirected, connected hypergraphs; that is, the objects of study in hypernetworks correspond to the vertices of hypergraphs, and the edges of hypergraphs have the same connectivity patterns as the links in hypernetworks. In the following, we do not distinguish between hypernetworks and hypergraphs. A finite, undirected, connected hypergraph with n vertices and m hyperedges (simply called edges) is denoted as H = (V,), where the vertex set is V = {v , v , ···, v } and the edge set is = {E , E , ···, E }. This paper considers only the case where vertices never fail but edges may fail, with edge failures being mutually independent. Let p denote the failure probability of the i-th edge E . We adopt the all-terminal reliability as the reliability metric for the corresponding hypernetwork, which generalizes the all-terminal reliability in ordinary complex networks.

Definitions and terminology related to graphs can be found in reference [16], and those related to all-terminal network reliability can be found in reference [2]. Below we briefly introduce concepts and terminology related to hypergraphs; for unmentioned concepts, please refer to [17].

A hypergraph H is a binary pair (V,), where V is the set of vertices of H, and is a collection of nonempty subsets of V. An element of set is a hyperedge of H, or simply an edge of H. The hypergraphs studied in this paper contain no isolated vertices (vertices not contained in any edge). The degree of a vertex v in H is denoted as d_H(v), representing the number of edges in H that contain vertex v. In the edge set, if any edge contains two particular vertices, these two vertices are called adjacent; if the intersection of two edges is nonempty, these two edges are called adjacent. If for any i, j, |E E | 1, then hypergraph H is called linear. If |E | = r for all i, then H is called an r-uniform hypergraph. Thus, a simple 2-uniform hypergraph is an ordinary graph, and vice versa.

For u, v V, a sequence of alternating vertices and edges (v , E , v , E , ···, v , E , v) is called a path from u to v, where u = v , v = v , and v , v E for i = 1, 2, ···, k. The length of this path is k. The shortest length among all paths between u and v is denoted as d(u, v), representing the distance between u and v.

If any two vertices in hypergraph H are connected by a path, H is called connected; otherwise, it is disconnected. If H is disconnected, the number of its connected components is denoted as (H). The edge connectivity of H is denoted as (H), representing the minimum number of edges whose removal disconnects H. This definition generalizes the corresponding definition in graphs.

Trees are important objects of study in graph theory, and the number of spanning trees in a graph is closely related to the edge reliability of the corresponding network [18]. Generalizing the definition of trees to hypergraphs yields structures called hypertrees (simply called trees). Compared with the definition of trees, the definition of hypertrees is much more complex, and different definitions are not equivalent [19,20]. The definition of hypertrees adopted in this paper is: a hypergraph in which removing any edge results in a disconnected sub-hypergraph.

A spanning hypertree of hypergraph H is a spanning sub-hypergraph of H that is also a hypertree. Like trees in graphs, hypertrees are fundamental structures in hypergraphs and play an important role in reliability research on hypernetworks with edge failures.

The following conclusion characterizes the structure of tight hypertrees [21]. From the definitions of trees and hypertrees, this conclusion applies to trees as well.

Let H be a connected hypergraph with n vertices. If H is a tight hypertree, then m = (_{i=1}^m |E |) - n + 1.

Viewing this conclusion from another perspective: if H is a tight hypertree, removing any edge E results in exactly |E | connected components. In mathematical notation: if H is a tight hypertree, then for any edge E , (H - E) = |E |.

For r-uniform hypergraphs, reference [22] provides a related conclusion:

Let H be a connected r-uniform hypergraph with n vertices. If H is connected, then m (n - 1)/(r - 1). Equality holds if and only if removing any edge E from H yields exactly r connected components.

This conclusion gives a lower bound on the number of edges in a connected r-uniform hypergraph and indicates that an r-uniform hypergraph achieving this lower bound is a tight r-uniform hypertree.

---

2 Hypernetwork Reliability and Basic Calculation Methods

Even after nearly two decades of development in network science, computing network reliability remains extremely challenging. Networks such as transportation networks, telecommunication networks, social networks, and power systems have higher reliability requirements, making their reliability computation more difficult due to high complexity.

In recent years, due to its combinatorial 合理性 (rationality) and application effectiveness, an increasing number of researchers have attempted to model complex systems using hypergraphs. With further development of hypergraph design theory, reliability research and applications of hypergraphs will receive more attention. However, research on hypernetwork reliability remains a rarely explored field.

Among various models for studying network reliability, all-terminal reliability has received significant attention as a common reliability metric. For networks such as power grids, transportation networks, and computer networks, all-terminal reliability is a particularly important reliability measure [23]. The object of study in this paper is the all-terminal reliability of finite, undirected hypergraphs, i.e., the probability that the hypergraph remains connected under edge failures.

Since only edge unreliability is considered, each edge has only two states: failed or operational. The state vector of the edge set is defined as X = (x , x , ···, x), where the state indicator variables x are binary Boolean variables:

x = { 1, if E is operational; 0, if E has failed }, for i = 1, 2, 3, ···, m.

Thus, under the assumption that all indicator variables are random variables, all spanning sub-hypergraphs of an undirected hypergraph H form a random system. Clearly, there are 2^m possible states. The number of connected spanning sub-hypergraphs of H is denoted as (H). This paper assumes that the m random variables are mutually independent, with the following probability distribution:

P{x = 1} = 1 - p , P{x = 0} = p , for i = 1, 2, 3, ···, m.

For the hypernetwork shown in Figure 1 [Figure 1: see original paper], we calculate its reliability using the factoring method. Selecting edge E for contraction and deletion, as shown in Figure 2 [Figure 2: see original paper], yields the same all-terminal reliability as obtained by the state enumeration method.

2.1 State Enumeration Method

Based on our assumptions, the state enumeration method for calculating hypernetwork reliability can be summarized in the following theorem:

Theorem 1. The all-terminal reliability of a finite, undirected hypergraph H is:

R(H, p) = _{k=1}$^{\{2}$m} [(_{i=1}^m x_i^{(k)} p_i$^{\{1-x_i}${(k)}} (1-p_i)$^{\{x_i}${(k)}}) · (H_k)]

The right side of equation (3) is an m-th degree homogeneous polynomial in p and (1-p), with (H_k) {0,1}, containing 2^m terms, where each term contains only p_i or (1-p_i). This is called the standard reliability polynomial of hypergraph H.

The enumeration method is the most intuitive approach for calculating hypernetwork reliability. The enumeration method for ordinary networks is a special case of this method. Consider the hypernetwork with 4 links shown in Figure 1. To calculate its reliability using enumeration, we need to examine 16 subsets of the edge set, of which 8 correspond to connected spanning sub-networks of the network.

Figure 1. A 3-uniform hypergraph with 5 vertices and 4 edges
Figure 2. The factoring theorem applied to a simple 3-uniform hypergraph

Its all-terminal reliability is:

R(p , p , p , p) = (1-p)(1-p)(1-p)(1-p) + p (1-p)(1-p)(1-p) + (1-p)p (1-p)(1-p) + (1-p)(1-p)p (1-p) + (1-p)(1-p)(1-p)p + p p (1-p)(1-p) + p (1-p)p (1-p) + p (1-p)(1-p)p + (1-p)p p (1-p) + (1-p)p (1-p)p + (1-p)(1-p)p p + p p p (1-p) + p p (1-p)p + p (1-p)p p + (1-p)p p p + p p p p .

2.2 Factoring Theorem

The factoring theorem for computing ordinary network reliability was first proposed by Moskowitz [24] and Mine [25]. Currently, many researchers utilize the factoring theorem to compute ordinary network reliability [26,27]. Below, we extend the factoring theorem for ordinary networks to hypernetworks. In hypernetworks, edges subject to contraction or deletion can be hyperedges, thereby expanding its application scope in complex network reliability research. We present the factoring theorem for computing hypernetwork reliability and prove its correctness.

Theorem 2. For a hypernetwork H where a hyperedge E has only two states (failed or operational), the states of H can be divided into two categories corresponding to these states, and its all-terminal reliability can be expressed as:

R(H) = (1-p) · R(H · E) + p · R(H - E)

where H · E and H - E represent the hypernetworks obtained by contracting and deleting edge E in H, respectively.

Proof. Among all spanning sub-hypergraphs of H, the probability that a connected spanning sub-hypergraph includes E as one of its edges is (1-p) · R(H · E), and the probability that a connected spanning sub-hypergraph does not include E is p · R(H - E). Therefore, the total probability of connected spanning sub-hypergraphs of H is (1-p) · R(H · E) + p · R(H - E).

---

3 Simplification of Hypernetwork Reliability

For a hypernetwork H with m hyperlinks, the complexity of computing its reliability using the factoring theorem from Section 2.2 is O(2^m). Based on the computational principle of the factoring method, if a hypernetwork has one fewer hyperlink, the complexity of computing its reliability is reduced by half. Therefore, equivalent simplification of hypernetworks for reliability computation is of great significance.

Many real-world hypernetworks exhibit structural characteristics such as series, parallel, and degree-1 vertices, making such equivalent simplifications feasible.

3.1 Equivalent Simplification for Hypernetworks with Degree-1 Vertices

Theorem 3. Let E be a hyperlink in hypernetwork H that contains a degree-1 vertex, with failure probability p . Then the reliability of H is:

R(H) = (1-p) · R(H - E)

Proof. By the factoring theorem, R(H) = (1-p) · R(H · E) + p   · R(H - E). Since hyperlink E contains a degree-1 vertex, the sub-hypernetwork H · E is disconnected, so R(H · E) = 0. Therefore, R(H) = (1-p) · R(H - E).

Figure 3 [Figure 3: see original paper] provides an example of applying the factoring theorem to compute the reliability of a hypernetwork with a degree-1 vertex. This also demonstrates that the computational time complexity of the factoring theorem for hypernetwork reliability depends on edge selection.

3.2 Series Hypernetworks and Parallel Hypernetworks

Many large-scale hypernetworks can ultimately be decomposed into basic hypernetworks, or these networks are built upon some basic hypernetworks. Below we study two types of basic hypernetworks: series hypernetworks and parallel hypernetworks.

3.2.1 Series Hypernetworks In ordinary complex networks, the topological structure of series networks is a path in graph theory. Series forms in hypernetworks exhibit complex and diverse characteristics, all sharing the property that failure of any single edge disconnects the series hypernetwork. Thus, the reliability of a series hypernetwork is:

R(H) = _{i=1}^m (1-p)

If all links have the same failure probability p, then:

R(H) = (1-p)^m

Figure 4 [Figure 4: see original paper] shows all series hypernetworks with 5 vertices and 2 (hyper)links, all having reliability (1-p)(1-p).

3.2.2 Parallel Hypernetworks Parallel hypernetworks fail only when all hyperlinks fail. Their reliability equals 1 minus the product of all link failure probabilities:

R(H) = 1 - _{i=1}^m p

If all links have the same failure probability p, then:

R(H) = 1 - p^m

This strict requirement leads to a single form of parallel hypernetworks: every parallel link must contain all vertices in the network. Figure 5 [Figure 5: see original paper] shows two hypernetworks with 5 vertices and 2 links, where hypernetwork (a) is parallel and hypernetwork (b) is not.

(a) Parallel
(b) Not Parallel

Figure 5. Two hypernetworks with 5 vertices and 2 edges.

---

4 Relationship Between Hypergraph Reliability and Number of Connected Spanning Sub-hypergraphs

Assume every link in a hypernetwork has the same failure probability p, and edge failures are mutually independent. Then the standard form of reliability mentioned in Section 2 is an m-th degree homogeneous polynomial in p and (1-p), expressed as:

R(H, p) = _{i=0}^m s (H) · p^{m-i} (1-p)^i

where s (H) is the number of connected spanning sub-hypergraphs of H with i edges.

To more accurately describe certain properties of hypernetwork H reflected by reliability and to deepen understanding of H' s reliability polynomial, we further assume that the topological structure of hypernetwork H is an r-uniform hypergraph. The conclusions obtained can be generalized to general cases.

From the definition of spanning hypertrees, tree structures in hypernetworks refer to spanning sub-networks with minimal number of links. Among these, spanning sub-networks with the minimum number of links are called primary spanning sub-networks, with corresponding topological structures called primary spanning hypertrees. For example, the hypernetwork shown in Figure 1 has 4 spanning sub-networks forming the set:

S_T = {{E , E , E , E }, {E , E , E }, {E , E , E }, {E , E , E }}

Among these, there are 3 primary spanning sub-networks forming the set:

M_ST = {{E , E , E }, {E , E , E }, {E , E , E }}

From the reliability polynomial of hypernetwork H, when hyperedge failure probability is large (approaching 1), the number of primary spanning sub-networks of H plays a decisive role in network reliability.

Let the topological structure of hypernetwork H be a connected hypergraph with n vertices and m links. Based on the reliability polynomial, we can derive the following main results about the polynomial coefficients:

1. s (H) = 0, s (H) = 0, ···, s_{ (H)-1}(H) = 0

2. s_{ (H)}(H) = (H), where (H) is the number of minimum edge cuts of size (H)
3. s_{m-n+1}(H) = _M(H), where _M(H) is the number of primary spanning hypertrees
4. s_m(H) = 1

If all links have the same failure probability p, then for two r-uniform hypergraphs H and H with the same number of vertices and edges, if (H) > (H), or if (H) = (H) and (H) > (H), then R(H , p) > R(H , p) for all $0 < p < 1$.

---

5 Comparison Between Hypergraphs and Ordinary Graphs from a Reliability Perspective

Assume all vertices in the studied graphs or hypergraphs are reliable, all (hyper)edges are unreliable, and (hyper)edge failure probability is p, with failures being mutually independent.

From a reliability perspective, structural differences between graphs and hypergraphs can be observed. We illustrate this with two examples.

Example 1. No ordinary graph can be found that has the same reliability as the hypergraph in Figure 1.

Proof. Under our assumptions, the reliability of the hypergraph in Figure 1 is the polynomial:

$R(p) = 3p(1-p) \ + 4(1-p)^3p^2 + (1-p)$

If there exists a graph G with the same reliability polynomial R(p), then G must have 5 vertices. When n(G) = 5, a connected graph G has at least 4 edges. If m(G) = 4, then G must be a tree with 5 vertices, which has 3 non-isomorphic structures, all with reliability (1-p) . If m(G) = 5, there are only two non-isomorphic structures for connected graphs with 5 edges, as shown in Figure 5 (G and G). If m(G) = 6, there are three non-isomorphic structures for connected graphs with 6 edges, as shown in Figure 5 (G , G , and G). Their reliability polynomials are:

$R(G , p) = 4(1-p) \ + p(1-p)$
$R(G , p) = 3(1-p) \ + p(1-p)$
$R(G , p) = 5(1-p) \ + 4(1-p) p^2 + (1-p)^3p^3$
$R(G , p) = 3(1-p) \ + 3(1-p) p^2 + (1-p)^3p^3$
$R(G , p) = 4(1-p) \ + 4(1-p) p^2 + (1-p)^3p^3$

None of these equals R(p). When m(G) 7, connected graphs with 4 edges have unique structures, i.e., parallel networks with 4 edges as shown in Figure 5 (G), with reliability $R(G , p) = 4(1-p) \ + 6(1-p) p^2 + 4(1-p) p^3 + (1-p)^3p$, which also does not equal R(p).

Example 2. While there are only 3 non-isomorphic tree structures with 4 edges, among linear 3-uniform hypertrees with tree structures, there are 7 non-isomorphic hypergraphs, as shown in Figure 6 [Figure 6: see original paper] and Figure 7 [Figure 7: see original paper].

These examples demonstrate that subtle differences in edge formation between ordinary graphs and hypergraphs lead to fundamental structural differences. The diversity of vertex connection patterns in hypergraphs makes hypergraph structures inherently more complex and varied. For complex network reliability, substitutive research that converts hypergraph models to ordinary graph models is not completely equivalent. Therefore, new theories specifically for hypergraphs and hypernetworks themselves are needed.

---

6 Conclusion

As real-world networks become increasingly complex, hypernetworks have become powerful tools for exploring network science. Hypernetwork reliability research constitutes an important component of network reliability studies. This paper proposes and studies the all-terminal reliability of hypernetworks, presenting two fundamental calculation methods for all-terminal reliability under edge failures: state enumeration and factorization. We then discuss simplifications for hypernetworks with special structures. Subsequently, we investigate the application of hypernetwork reliability in computing the number of connected spanning sub-hypergraphs. Finally, we conduct a comparative analysis between ordinary network models and hypernetwork models from a reliability perspective, demonstrating that hypernetworks are suitable for more generalized network modeling.

Many problems in hypernetwork reliability remain to be explored, which can be summarized into three major directions:

a) **Research on various reliability calculation methods under different reliability model definitions.** Even for ordinary complex networks with over half a century of development, reliability analysis for larger-scale networks remains challenging, indicating that hypernetwork reliability computation has even broader research space.

b) **Design of hypernetwork reliability under certain conditions.** Due to the diverse characteristics of hypernetwork models, their reliability design will be more challenging.

c) **Research on intrinsic hypernetwork characteristics from a reliability perspective.** Moving beyond the 思维模式 (thinking patterns) of ordinary complex networks to explore the inherent laws of hypernetworks themselves can enable a more comprehensive understanding of the real network world.

References

[1] Wu Jun, Duan Dongli, Zhao Juan, et al. Status and prospects on network reliability [J]. Complex Systems and Complexity Science, 2011, 8(2): 77-86.

[2] Li Xiaoming. Current status and trends in the synthesis of reliable networks [J]. Chinese Journal of Computers, 1990, 13(9): 699-705.

[3] Beichelt F. Reliability preserving network transformation applications and problems [J]. International Journal of Reliability Quality & Safety Engineering, 2008, 14(3): 189-199.

[4] Suo Qi, Guo Jinli. The structure and dynamics of hypernetworks based on hypergraphs [J]. Systems Engineering-Theory & Practice, 2017, 37(3): 720-734.

[5] Ai Yi. Super-network reliability research [D]. Chengdu: Southwest Jiaotong University, 2013.

[6] Hu Feng, Zhao Haixing, He Jiabei, et al. An evolving model for hypergraph-structure-based scientific collaboration networks [J]. Acta Phys. Sin., 2013, 62(19): 198901.

[7] Estrada E, Rodríguez-Velázquez J A. Subgraph centrality and clustering in complex hyper-networks [J]. Physica A, 2006, 364: 581-594.

[8] Liu Shengjiu, Li Tianrui, Hong Shijin, et al. Hypernetwork model and its properties [J]. Journal of Frontiers of Computer Science and Technology, 2017, 11(2): 194-211.

[9] Suo Qi, Guo Jinli. Spreading model and simulation analysis of Internet public opinion in hypernetworks [J]. Application Research of Computers, 2017, 34(9): 2629-2632.

[10] Sun L, Wu J, Cai H. The Wiener index of r-uniform hypergraphs [J]. Bulletin of the Malaysian Mathematical Sciences Society, 2016, 40(3): 1225-1236.

[11] Hu Feng, Li Faxu, Zhao Haixing. The research on scale-free characteristics of hypernetworks [J]. Sci. Sin.: Phys. Mech. Astron., 2017, 47(6): 060501.

[12] Zhang H, Song L, Han Z, Y Zhang. Hypergraph theory in wireless communication networks [M]. Cham: Springer, 2017.

[13] Ma Xiujuan, Zhao Haixing, Hu Feng. Cascading failure analysis in hypernetwork based on the hypergraph [J]. Acta Phys. Sin., 2016, 65(8): 088901.

[14] Chen Tinghuai, Kang Tai, Yao Rong. The connectivity of hypergraph and the design of fault-tolerant multibus systems [J]. Sci. Sin.: Maths. Phys. Astron. Sci Technol., 1987, 17(12): 79-89.

[15] Cao Qiguo, Sun Yugeng. The hypergraph design method of multibus structures of reliable communication networks [J]. Acta Electronica Sinica, 1997, 25(10): 88-90.

[16] Bondy J A, Murty U S R. Graph Theory [M]. New York: Springer, 2008.

[17] Berge C. Hypergraphs: combinatorics of finite sets [M]. Amsterdam: North-Holland, 1989.

[18] Li Xiaoming, Huang Zhenjie. On the number of spanning trees of graphs and applications of networks [M]. Harbin: Harbin Institute of Technology Press, 1999.

[19] Nieminen J, Peltola M. Hypertrees [J]. Applied Mathematics Letters, 1999, 12: 35-38.

[20] Katona G Y, Szabó P G N. Bounds on the number of edges in hypertrees [J]. Discrete Mathematics, 2016, 339(7): 1884-1891.

[21] Xu Xiaoman, Sun Yugeng, Yang Shan. Hypergraph theory with applications [J]. Acta Electronica Sinica, 1994, 22(8): 65-72.

[22] Gu Xiaofeng, Lai Hongjian. Realizing degree sequences with k-edge-connected uniform hypergraphs [J]. Discrete Mathematics, 2013, 313: 1394-1400.

[23] Jain S P, Gopal K. An efficient algorithm for computing global reliability of a network [J]. IEEE Trans on Reliability, 1988, 37(5): 484-492.

[24] Moskowitz F. The analysis of redundancy networks [J]. IEEE Trans on Communications and Electronic, 1958, 39: 627-632.

[25] Mine H. Reliability of physical systems [J]. IEEE Trans on Circuit Theory, 1958, CT-6: 138-151.

[26] Satyanarayana A, Chang M K. Network reliability and factoring theory [J]. Networks, 1983, 13: 107-120.

[27] Page L B, Perry J E. Combining vertex decompositions with factoring in an all-terminal network reliability algorithm [J]. Microelectronics Reliability, 1990, 30(2): 249-262.

Note: Figure translations are in progress. See original paper for figures.