Direction-of-Arrival Estimation Based on Improved Block Sparse Bayesian Learning Algorithm (Postprint)

Authors: Wang Shuhao, Ruan Huailin

Date: 2018-12-13T00:00:00+00:00

Abstract

To address the issue that traditional sparse representation-based DOA estimation algorithms solely exploit the spatial sparsity of signals, resulting in degraded sparsity performance at low SNR and affecting sparse signal reconstruction effectiveness, block sparse theory is utilized to perform sparse decomposition of signals. As the number of targets increases and operational missions change, DOA estimation often exhibits characteristics of group direction finding for targets. In order to better utilize the structural and statistical features of signals, a joint spatial-temporal block sparse DOA estimation algorithm is proposed, which employs block sparse theory to excavate the internal structure of signals, fully exploiting both intra-block sparsity and inter-block correlation of signals to improve sparse reconstruction performance, thereby significantly enhancing DOA estimation effectiveness. Simulation results demonstrate that compared with classical DOA methods, the proposed method achieves superior estimation performance.

Full Text

Preamble

Vol. 37 No. 2
Application Research of Computers
ChinaXiv Cooperative Journal

DOA Estimation Based on Improved Block Sparse Bayesian Learning Algorithm

Wang Shuhao, Ruan Huailin
(School of Electronic Countermeasures, National University of Defense Technol-

ogy, Hefei 230037, China)

Abstract: Traditional DOA estimation algorithms based on sparse representation only exploit the spatial sparsity of signals, which leads to degraded sparsity performance at low SNR and affects sparse signal reconstruction. This paper employs block sparse theory for signal decomposition. As the number of targets increases and operational missions change, DOA estimation often exhibits characteristics of target group direction finding. To better utilize the structural and statistical features of signals, we propose a block sparse DOA estimation algorithm based on space-time combination. This method leverages block sparse theory to mine the internal structure of signals, fully utilizing both intra-block sparsity and inter-block correlation to improve sparse reconstruction performance, thereby significantly enhancing DOA estimation effectiveness. Simulation experiments demonstrate that compared with classical DOA methods, the proposed method achieves better estimation performance.

Keywords: spatial-temporal combined; block sparse; sparse Bayesian learning; DOA estimation

---

1.1 MMV Model

Consider a uniform linear array with M elements placed along the axis with equal spacing d. The array reception model is shown in Figure 1 [Figure 1: see original paper]. Assume the signal has center frequency f, bandwidth B, and wavelength λ. For an incident angle θ, the received data at the m-th array element at time t can be expressed as:

$$y_m(t) = \sum_{i=1}^{L} x_i(t - \tau_{mi}) + n_m(t), \quad m = 1, 2, \ldots, M$$

where the time delay is $\tau_{mi} = \frac{(m-1)d\sin(\theta_i)}{c}$, $i = 1, 2, \ldots, L$, and c is the propagation speed.

Considering the entire array, the received signals from all M elements can be represented in matrix form as:

$$\mathbf{Y} = \ \mathbf{X} + \mathbf{N}$$

where $\mathbf{Y} \in \mathbb{R}^{M \times P}$ is the observation matrix composed of P measurement vectors, is the overcomplete basis matrix constructed from dense spatial grid partitioning (referred to as the sensing matrix in the MMV model), $\mathbf{X}$ is the signal source matrix to be reconstructed, and $\mathbf{N}$ represents additive Gaussian noise. The key to DOA estimation in this framework is determining the positions of non-zero signals across different column vectors in $\mathbf{X}$, which correspond to the signal azimuth angles.

Extending this to the Multiple Measurement Vector (MMV) model, we set the number of snapshots to P, meaning we perform P observations of all signals of interest in the spatial domain, obtaining P observation vectors as shown in equation (2). The matrix representation becomes:

$$\mathbf{Y} = \ \mathbf{X} + \mathbf{N}$$

where $\mathbf{Y} \in \mathbb{R}^{M\times P}$, $\ \in \mathbb{R}^{M\times N}$ is the sensing matrix, $\mathbf{X} \in \mathbb{R}^{N\times P}$ is the source signal matrix, and $\mathbf{N}$ is the noise matrix.

1.2 Block Sparse MMV Model

In the MMV model described above, we make the following assumption: the positions of non-zero elements are identical across different column vectors in signal matrix $\mathbf{X}$. Under this assumption, $\mathbf{X}$ exhibits row sparsity [4], providing the foundation for block sparsity. The advantage of block sparsity lies in its ability to fully exploit the internal structural characteristics of signals and the temporal correlation between non-zero blocks, thereby improving sparse reconstruction performance. When applied to DOA estimation, this yields better estimation results and faster convergence.

We establish the corresponding block sparse MMV model as follows. The signal matrix $\mathbf{X}$ is divided into g blocks, with the i-th signal block denoted as $\mathbf{X}_i$, each having size $d_i \times P$ (block sizes may vary, which differs from traditional block sparse reconstruction algorithms). The number of non-zero signal blocks W satisfies $W \ll g$. The block structure is expressed as:

$$\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_g]^T$$

Correspondingly, the sensing matrix is partitioned into g blocks as $= [\ {}_1, {}_2, \cdots, {}_g]$, where ${}_i$ represents the sub-dictionary corresponding to the i-th signal block.

2 Space-Time Joint Block Sparse Bayesian Learning Algorithm

This section presents the core framework for DOA estimation under the space-time joint block sparse Bayesian learning paradigm and provides the corresponding algorithmic steps. We first establish the block sparse model within the Bayesian framework, then propose the space-time joint sparsity concept tailored to the spatial-temporal correlation characteristics of group target signals under multiple measurement conditions [5]. We derive iterative methods for solving temporal characteristic parameters and structural parameters, enabling signal reconstruction and subsequent DOA estimation of target signal sources.

2.1 Construction of Block Sparse Model Under Bayesian Framework

Block sparse models are commonly processed using sparse Bayesian learning methods, as SBL not only facilitates modeling of various physical information but also demonstrates excellent reconstruction performance and robustness to highly correlated sensing matrices [6]. Sparse Bayesian Learning (SBL) was first proposed by Tipping [7] and continuously improved by Wipf et al., garnering increasing attention from researchers in the sparse signal reconstruction field. This method establishes a hierarchical Bayesian model for observations $\mathbf{y}$ and signals $\mathbf{x}$, employing parametric probability models for system solution and providing a flexible mathematical framework for extracting sparse structural information.

Since this method was originally proposed under the Single Measurement Vector (SMV) premise, we convert the block sparse MMV model established above into a block sparse Bayesian model under the SMV framework for analysis. The DOA estimation model for block sparse Bayesian learning transforms the MMV model into SMV form using vectorization. In this model, signal $\mathbf{x}$ is block sparse, with each signal block having size $d_i \times 1$. The signal model schematic is shown in Figure 2 [Figure 2: see original paper].

As illustrated in Figure 1, far-field signals often exhibit multi-target and clustered characteristics, causing target signals to be not only block sparse in the spatial domain (row vectors of the MMV model) but also strongly correlated in the temporal domain (column vectors of the MMV model) [9]. Leveraging both types of information to develop fast, high-performance reconstruction algorithms is the primary concern of this paper. Based on these considerations, we propose the Space-Time Combination Block Sparse Bayesian Learning (STC-BSBL) algorithm, which combines signal block sparsity characteristics, intra-block correlation structure, and multi-observation vector temporal correlation structure to enhance reconstruction performance.

2.2 DOA Estimation Based on STC-BSBL Algorithm

We introduce hyperparameters $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_N]^T$, where γ_i represents the correlation between the i-th row vector of $\mathbf{X}$ and the observation matrix $\mathbf{Y}$. Signal components have larger γ_i values, while noise components have smaller values. During parameter learning, larger γ_i values indicate higher likelihood that $\mathbf{x}_i$ is a signal component, and smaller values suggest it is noise. Once the non-zero positions in γ are determined, the signal azimuth angles are consequently identified. Therefore, learning the hyperparameters γ is central to the algorithm.

For block sparse signals, we must also consider intra-block structure by introducing parameter $\mathbf{B}_i$, defined as the covariance matrix of $\mathbf{x}_i$. $\mathbf{B}_i$ is a positive definite symmetric matrix describing the spatial correlation of signal $\mathbf{X}$. For temporal correlation modeling, we assume all column vectors of signal matrix $\mathbf{X}$ share the same temporal correlation structure Γ, expressed as $\mathbf{X} = \mathbf{A} \otimes \Gamma$, where $\mathbf{A}$ represents spatial correlation. Further assuming row vectors in $\mathbf{X}$ are uncorrelated, equation (9) simplifies to $\mathbf{X} \sim \mathcal{N}(0, \mathbf{B} \otimes \Gamma)$.

Assuming noise follows a zero-mean Gaussian distribution, i.e., $\mathbf{N} \sim \mathcal{N}(0, \lambda \mathbf{I})$, the new sensing matrix becomes excessively large (size $MP \times NP$). In sparse reconstruction algorithms, storing and computing each sensing matrix block consumes significant memory, prompting us to introduce matrix normal distributions to convert the SMV model into a Bayesian learning model under matrix form [10].

Updating equations (10) and (11) yields the prior distribution for $\mathbf{X}$ in the MMV model:

$$\mathbf{X} \sim \mathcal{N}(0, \mathbf{B} \otimes \Gamma)$$

and the observation model:

$$\mathbf{Y} = \ \mathbf{X} + \mathbf{N}$$

Combining the prior probability models (12) and (13), we obtain the likelihood function and posterior probability. After estimating the hyperparameters, the posterior probability density is derived as:

$$p(\mathbf{X}|\mathbf{Y}; \gamma, \mathbf{B}, \lambda) = \mathcal{N}(\mu, \Sigma)$$

where $\mu = \Sigma \ ^T\mathbf{Y}$ and $\Sigma = (\ ^{-1} + \ ^T \ )^{-1}$. This allows recovery of the original signal, and DOA estimation is completed based on the non-zero positions of γ. The entire signal reconstruction and DOA estimation process is described in the flowchart shown in Figure 3 [Figure 3: see original paper].

For hyperparameter λ estimation, it is typically treated as a penalty parameter assigned different values according to SNR [11]. Solving for the mean and covariance is equivalent to estimating hyperparameters γ and $\mathbf{B}$. Correct estimation of these hyperparameters yields the maximum a posteriori solution, enabling reconstruction of $\mathbf{X}$ and obtaining the desired DOA estimate. We construct the log-likelihood function:

$$\mathcal{L}(\gamma, \mathbf{B}, \lambda) = \log p(\mathbf{Y}; \gamma, \mathbf{B}, \lambda)$$

Taking partial derivatives of the likelihood function with respect to each hyperparameter yields update formulas. To simplify calculations, we express this as two cost functions: $J_1(\gamma)$ treating $\mathbf{B}$ as known, and $J_2(\mathbf{B})$ treating γ as known.

By initializing $\mathbf{B}$ and alternately updating using equations (17) and (18) [12], the process iterates until convergence.

For updating parameter $\mathbf{B}$, we can directly update $\mathbf{B}_i$ using:

$$\mathbf{B}_i^{\text{new}} = \frac{1}{P}\mathbf{X}_i\mathbf{X}_i^T$$

For updating the block correlation matrix, the cost function $J_2(\mathbf{B})$ can be written as:

$$J_2(\mathbf{B}) = \log|\mathbf{C}| + \text{Tr}(\mathbf{C}^{-1}\mathbf{S})$$

where $\mathbf{C} = \mathbf{D}_i + {}_i\mathbf{B}_i{}_i^T$ and $\mathbf{S} = \mathbf{Y}\mathbf{Y}^T$. Applying the matrix inversion lemma [13] and minimizing the cost function yields the update formula for $\mathbf{B}_i$.

3.1 DOA Estimation Performance Comparison

We consider a uniform linear array with $M = 30$ elements spaced at half-wavelength intervals. The spatial region is $[-90^\circ, 90^\circ]$ with three true signals arriving from directions $\theta = [0^\circ, 35^\circ, 40^\circ]$. The input SNR is 8 dB with $P = 100$ snapshots. After 1000 Monte Carlo experiments, the DOA estimation results are shown in Figure 4 [Figure 4: see original paper].

For comparison, we evaluate the convex optimization-based Orthogonal Matching Pursuit (OMP) algorithm, the subspace-based MUSIC algorithm, and our proposed STC-BSBL algorithm. With four signals arriving from $\theta = [30^\circ, 45^\circ, 60^\circ, 75^\circ]$ and other conditions unchanged, the simulation results are shown in Figure 5 [Figure 5: see original paper].

The results demonstrate that under the given conditions, all three methods can achieve DOA estimation. However, in terms of estimation accuracy, both STC-BSBL and OMP algorithms exhibit narrower angular estimation ranges and lower amplitude fitness compared to MUSIC.

3.2 DOA Estimation Accuracy Comparison

Besides convex optimization algorithms, sparse Bayesian learning frameworks commonly include Block Sparse Bayesian Learning (BSBL) and Temporal Correlation-based Sparse Bayesian Learning (TM-SBL) [15]. This experiment uses sparsity level K as the independent variable to simulate and compare the DOA estimation accuracy of these three methods with our proposed approach. The correct recognition rate curves under different sparsity levels are shown in Figure 6 [Figure 6: see original paper].

The simulation results indicate that our STC-BSBL algorithm achieves better reconstruction performance under equivalent measurement conditions, demonstrating higher DOA estimation accuracy at the same sparsity level and accommodating a greater number of signal DOAs.

3.3 Simulation Experiment 3: DOA Estimation RMSE Analysis

To further highlight the superiority of our algorithm in DOA estimation, we compare the root mean square error (RMSE) curves of five methods using SNR and snapshot number as independent variables. The RMSE for DOA estimation is defined as [3]:

$$\mathrm{RMSE} = \sqrt{\frac{1}{JK}\sum_{j=1}^{J}\sum_{k=1}^{K}(\hat{\theta}_{jk} - \theta_k)^2}$$

where $\hat{\theta}_{jk}$ is the estimated angle and θ_k is the true angle.

The results are shown in Figures 7 [Figure 7: see original paper] and 8 [Figure 8: see original paper]. As SNR and snapshot number increase, the RMSE of all algorithms decreases. However, at low SNR and with few snapshots, OMP and ISSM algorithms exhibit significantly increased estimation errors. While BSBL and TM-SBL algorithms utilize block structure or temporal correlation to reduce reconstruction errors, STC-BSBL consistently achieves the smallest estimation error by jointly exploiting both spatial and temporal correlations.

4 Conclusion

This paper analyzes the superiority of block sparse theory in sparse signal reconstruction within the sparse Bayesian learning framework. We derive the block sparse Bayesian learning model and construct a block sparse MMV model for narrowband signal DOA estimation. Building upon existing BSBL algorithms, we improve the approach by incorporating temporal correlation across multiple observations while considering intra-block correlation, proposing the Space-Time Combination Block Sparse Bayesian Learning (STC-BSBL) algorithm. Comparative experiments demonstrate that the proposed algorithm achieves superior direction-finding accuracy for radar signal DOA estimation.

References

[1] Li Zigao, Li Shuqiu, Wen Shulin. An improved DOA estimation method based on orthogonal matching pursuit [J]. TT&C Technology, 2017, 36(1): 27-31.

[2] Eldar Y C, Patrick K, Bölcskei H. Block-sparse signals: uncertainty relations and efficient recovery [J]. IEEE Trans on Signal Processing, 2010, 58(6): 3042-3054.

[3] Zhang Zhilin, Rao B D. Sparse signal recovery with temporally correlated source vectors using sparse Bayesian learning [J]. IEEE Journal of Selected Topics in Signal Processing, 2011, 5(5): 912–926.

[4] Wipf D P, Rao B D. An empirical Bayesian strategy for solving the simultaneous sparse approximation problem[J]. IEEE Trans on Signal Processing, 2007, 55(7): 3704-3716.

[5] Zhang Zhilin, Jung T P, Makeig S, et al. Spatiotemporal sparse Bayesian learning with applications to compressed sensing of multichannel physiological signals [J]. IEEE Trans on Neural Systems & Rehabilitation Engineering, 2014, 22(6): 1186-1197.

[6] Tipping M E. Sparse Bayesian learning and the relevance vector machine [J]. Journal of Machine Learning Research, 2001, 1(3): 211-244.

[7] Tipping M E. Sparse Bayesian learning and the relevance vector machine [J]. Journal of Machine Learning Research, 2001, 1(3): 211-244.

[8] Fang Qing, Zhang Gong, Yi De. Multitask compressed sensing reconstruction algorithm based on block sparse Bayesian learning [J]. Acta Physics, 2015, 64(7): 70201-070201.

[9] Ziniel J, Schniter P. Efficient high-dimensional inference in the multiple measurement vector problem [J]. IEEE Trans on Signal Processing, 2013, 61(2): 340-354.

[10] Liu Benyuan. Theory and application of fast block sparse Bayesian learning algorithm [D]. Changsha: National University of Defense Science and Technology, 2015.

[11] Yu L, Sun H, Barbot J P, et al. Bayesian compressive sensing for cluster structured sparse signals [J]. Signal Processing, 2012, 92(1): 259-269.

[12] Zhang Xinyue. Research on DOA estimation of broadband signals based on sparse reconstruction algorithm [D]. Chengdu: Xidian University, 2015.

[13] Roweis Matrix identities[EB/OL]. (1999-06). https://cs.nyu.edu/~roweis/notes/matrixid.pdf.

[14] Wang Biao, Zhu Zhihui and Dai Yuewei. A fast sparse Bayesian learning method for underwater acoustic target azimuth estimation [J]. Acta Acoustica Sinica, 2016, 41(1): 81-86.

[15] Zhang Zhilin, Bhaskar D. Rao. Recovery of block sparse signals using the framework of block sparse Bayesian learning [C]//Proc of IEEE International Conference on Acoustics, Speech and Signal Processing. 2012: 3345.

Note: Figure translations are in progress. See original paper for figures.