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Full Text

Preamble

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Image Segmentation of FCM Algorithm Based on Kernel Function and Mahalanobis Distance

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Abstract: To address the problem of low utilization of neighborhood and spatial information in fuzzy clustering algorithms, which makes them vulnerable to noise, this paper proposes a fuzzy C-means algorithm combining kernel function with Mahalanobis distance, called the FCMKM algorithm. First, image pixels are nonlinearly mapped from low-dimensional space to high-dimensional space through a kernel function. Then, Mahalanobis distance is used to replace the original Euclidean distance as the distance metric in high-dimensional space. Finally, the improved algorithm is applied to image segmentation. To verify the performance of FCMKM, five evaluation metrics are selected as criteria for comparative experiments: Bezdek partition coefficient, Xie-Beni coefficient, reconstruction error rate, running time, and iteration count. Experimental results demonstrate that compared with traditional FCM, kernel-based FCM, and Mahalanobis distance-based FCM algorithms, FCMKM can effectively improve the noise robustness of fuzzy clustering algorithms.

Keywords: kernel function; Mahalanobis distance; image segmentation; fuzzy clustering; neighborhood information; spatial information

0 Introduction

Image segmentation plays a crucial role in the preprocessing stage of image analysis and serves as the foundation for subsequent image processing. It involves partitioning an image into multiple non-overlapping regions with consistent attributes based on various image properties, where the union of all regions constitutes the entire image. The internal similarity within each region should be high, while the differences between regions should be pronounced.

Fuzzy clustering algorithms integrate fuzzy theory with clustering methods, achieving appropriate clustering results through fuzzy partitioning of datasets. Dunn et al. proposed the fuzzy C-means (FCM) algorithm, and Bezdek et al. [?] first applied fuzzy clustering to image segmentation. In recent years, numerous scholars have made significant contributions to image segmentation based on fuzzy clustering. To enhance noise robustness, Hafiane et al. [?] introduced a neighborhood membership function to strengthen the influence of neighborhoods on pixel membership determination, but this approach suffered from loss of spatial information when applied to natural and medical images. Namburu et al. [?] incorporated the concept of soft sets to develop an improved soft set fuzzy clustering segmentation algorithm; however, due to the characteristics of CT images, the experimental results exhibited obvious boundary noise and poor anti-noise performance.

To mitigate the impact of noise on boundary segmentation, Zhang et al. [?] proposed a multi-objective evolutionary fuzzy clustering algorithm. However, due to the inherent characteristic of fuzzy clustering algorithms being overly dependent on membership functions, this method could not effectively solve boundary noise problems, resulting in segmentation errors. Zhang et al. [?] introduced pixel correlation into the fuzzy factor to improve noise robustness, but this approach suffered from low processing efficiency and high computational complexity. Wu et al. [?] integrated Markov random field functions, which effectively improved resistance to simulated noise but showed limited improvement for natural noise in images. Gharieb et al. [?] employed neighborhood relative entropy to calculate membership degrees, which enhanced noise robustness but increased computation time and degraded overall algorithm performance. Choy et al. [?] used a generalized Gaussian distribution-based fuzzy clustering algorithm for boundary refinement, yet the segmented boundaries deviated from actual boundaries with lost boundary information, still failing to resolve noise issues.

Hussain et al. [?] proposed a kernel function-based FCM with local information (KWFLICM) algorithm, which effectively improved noise robustness using kernel functions but exhibited incomplete segmentation when processing natural images. Zhao et al. [?] proposed a Mahalanobis distance-based FCM (FCM-M) algorithm, which effectively improved clustering performance for discrete data and enhanced noise robustness but could not effectively segment target regions when applied to image segmentation.

To address the noise vulnerability of fuzzy clustering algorithms, this paper proposes a fuzzy C-means algorithm combining kernel function with Mahalanobis distance (FCMKM) for image segmentation. By mapping images through kernel functions and replacing Euclidean distance with Mahalanobis distance, the proposed method improves the objective function to enhance the utilization of neighborhood and spatial information, thereby improving noise robustness. Comparative experiments demonstrate that the proposed algorithm effectively solves the noise robustness problem for both synthetic images and Berkeley image database images.

1.1 Kernel Function and Mahalanobis Distance

A kernel function [?] maps pixels from the original space to a high-dimensional feature space through a new feature vector pattern, transforming low-dimensional nonlinear information into high-dimensional linear problems for processing. For all $\mathbf{x}, \mathbf{z} \in X$, if function $K(\mathbf{x}, \mathbf{z}) = \phi(\mathbf{x}) \cdot \phi(\mathbf{z})$, then $K(\mathbf{x}, \mathbf{z})$ is called a kernel function, where $\phi(\mathbf{x})$ is the mapping function and $\phi(\mathbf{x}) \cdot \phi(\mathbf{z})$ represents the inner product of $\mathbf{x}$ and $\mathbf{z}$ mapped to the high-dimensional feature space. Since mapping functions are typically complex and difficult to compute directly, kernel functions are used in practice to solve the inner product without increasing computational complexity. The mapping function serves merely as a logical mapping representing the relationship between input space and high-dimensional feature space.

Commonly used kernel functions include linear, polynomial, Gaussian, Sigmoid, and string kernels. This paper adopts the Gaussian kernel function for experiments, as shown in Equation (1):

$$K(\mathbf{x}, \mathbf{z}) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{z}\|^2}{2\sigma^2}\right)$$

where σ represents the standard deviation of the data, which provides good anti-interference capability against noise but exhibits strong locality. To improve the local characteristics of the kernel function, globally-oriented Mahalanobis distance is introduced to enhance the algorithm' s global performance and avoid falling into local optima.

Mahalanobis distance [?], proposed by Indian statistician P. C. Mahalanobis, represents the covariance distance of data. It is an effective method for calculating similarity between two unknown sample sets. Unlike Euclidean distance, Mahalanobis distance introduces a covariance matrix to effectively describe global relationships between sample points, incorporating more neighborhood and spatial information. The Mahalanobis distance calculation formula is shown in Equation (2):

$$d_M(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})^T \Sigma^{-1} (\mathbf{x} - \mathbf{y})}$$

where Σ represents the covariance matrix between $\mathbf{x}$ and $\mathbf{y}$.

1.2 Selection of Cluster Number and Initial Cluster Centers

The selection of cluster number and initial cluster centers is crucial for improving fuzzy clustering algorithm performance. To achieve better performance, this paper improves the selection method by mapping image grayscale values to obtain a grayscale variation line graph. Since significant grayscale variations typically correspond to target or background regions, calculating local extrema of the grayscale variation line graph can effectively identify optimal cluster centers. Local extrema serve as cluster centers, and the number of local extrema determines the cluster number. The specific calculation steps are: (a) compute the grayscale histogram of the target image and obtain the grayscale variation line graph; (b) calculate and record local extrema of the grayscale variation line graph; (c) use local extrema as cluster centers and their count as the cluster number.

1.3 Evaluation Metrics

To prevent subjective evaluation bias, five objective metrics are introduced: Bezdek partition coefficient [?], Xie-Beni coefficient [?], reconstruction error rate [?], running time, and iteration count.

The Bezdek partition coefficient measures classification fuzziness—higher values indicate clearer classification while lower values indicate fuzzier classification. Its calculation formula is shown in Equation (3):

$$P_c = \frac{1}{n}\sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^2$$

where c is the cluster number, n is the sample count, and u_{ij} are elements of the membership matrix $\mathbf{U}$.

The Xie-Beni coefficient, shown in Equation (4), comprises compactness I_{xb} (evaluating intra-class cohesion) and separation L_{xb} (evaluating inter-class coupling). Good clustering should minimize coupling while maximizing cohesion. A smaller Xie-Beni coefficient indicates higher intra-class cohesion, lower inter-class coupling, and better classification:

$$V_{xb} = \frac{\sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^m \|\mathbf{x}_j - \mathbf{v}_i\|^2}{n \cdot \min_{i \neq k} \|\mathbf{v}_i - \mathbf{v}_k\|^2}$$

where u_{ij} are membership matrix elements, $\mathbf{v}_i, \mathbf{v}_j, \mathbf{v}_k$ are cluster center matrix elements, $\mathbf{x}_j$ are experimental object elements, m is the fuzzification coefficient, n is the sample count, and c is the cluster number.

Reconstruction error rate characterizes the difference between the reconstructed segmented image and the original image, with good segmentation algorithms producing results as similar to the original as possible. Thus, the reconstruction error rate should approach zero, as calculated by Equation (5):

$$R = \frac{1}{n}\sum_{i=1}^{n}\left\|\mathbf{x}_i - \sum_{k=1}^{c} u_{ki}\mathbf{v}_k\right\|^2$$

where u_{ki} are membership matrix elements, c is the cluster number, $\mathbf{x}_i$ are experimental object elements, and n is the sample count.

Running time measures the total time from image input to segmented output, effectively evaluating algorithm time complexity. Iteration count measures the number of updates to membership degrees and cluster centers, with each update incrementing the counter, effectively evaluating algorithm convergence.

1.4 FCMKM Algorithm

By improving the fuzzy clustering objective function (Equation (6)) and integrating kernel function with Mahalanobis distance, the FCMKM algorithm enhances utilization of neighborhood and spatial information to improve noise robustness. The proposed objective function is shown in Equations (7) and (8):

$$J_{old} = \sum_{i=1}^{c} \sum_{k=1}^{n} u_{ik}^{m} \|\mathbf{x}_k - \mathbf{v}_i\|^2$$

$$J_{FCMKM} = \sum_{i=1}^{c} \sum_{k=1}^{n} u_{ik}^{m} \cdot MK(\mathbf{x}_k, \mathbf{v}_i)$$

$$MK(\mathbf{x}_k, \mathbf{v}_i) = K(\mathbf{x}_k, \mathbf{x}_k) + K(\mathbf{v}_i, \mathbf{v}_i) - 2K(\mathbf{x}_k, \mathbf{v}_i)$$

where $MK(\mathbf{x}_k, \mathbf{v}_i)$ represents the Mahalanobis distance in high-dimensional space between the k-th element and the i-th cluster center.

Using the Lagrange multiplier method, the membership update formula is derived as Equation (9) and the cluster center update formula as Equation (10):

$$u_{ij} = \frac{\left(MK(\mathbf{x}_j, \mathbf{v}_i)\right)^{-1/(m-1)}}{\sum_{k=1}^{c} \left(MK(\mathbf{x}_j, \mathbf{v}_k)\right)^{-1/(m-1)}}$$

$$\mathbf{v}_i = \frac{\sum_{j=1}^{n} u_{ij}^{m} \cdot \frac{\partial K(\mathbf{x}_j, \mathbf{v}_i)}{\partial \mathbf{v}_i}}{\sum_{j=1}^{n} u_{ij}^{m} \cdot \frac{\partial^2 K(\mathbf{x}_j, \mathbf{v}_i)}{\partial \mathbf{v}_i^2}}$$

The FCMKM algorithm proceeds as follows:

a) Initialize the image to be segmented. Using the improved method, determine cluster number c and initial cluster centers $\mathbf{V}^{(0)}$, set iteration termination threshold ε and iteration counter $b = 0$.

b) Calculate or update the membership matrix $\mathbf{U}^{(b)}$ using Equation (9) and update cluster centers $\mathbf{V}^{(b+1)}$ using Equation (10). If $\|G^{(b)} - G^{(b+1)}\| < \varepsilon$ or maximum iteration count is reached, terminate iteration; otherwise, increment counter b and repeat this step.

c) After termination, compute the membership matrix $\mathbf{U}$ and cluster center matrix $\mathbf{V}$ that minimize the objective function (Equation (7)) as the optimal solutions.

d) For each pixel $\mathbf{x}_i$, find the maximum element u_{ij} in its corresponding row of the optimal membership matrix $\mathbf{U}$ (i.e., the i-th row). Assign the grayscale value of the optimal cluster center $\mathbf{v}_j$ corresponding to u_{ij} as the new grayscale value of pixel $\mathbf{x}_i$. Repeat until all pixel values are updated, then output the segmented image.

The FCMKM algorithm flow is illustrated in Figure 1.

[Figure 1: see original paper]

2 Experimental Results and Analysis

To validate the effectiveness of the proposed algorithm, experiments employ two image categories: synthetic images and images from the Berkeley segmentation database. Performance comparisons are conducted against traditional FCM, kernel-based FCM (KWFLICM), and Mahalanobis distance-based FCM (FCM-M). The experimental environment consists of a PC with Intel(R) Core(TM) i5 processor (2.6 GHz), 8 GB RAM, Windows 10 system, and MATLAB (R2016b) implementation.

2.1 Parameter Setting

To select the optimal fuzzy weighting exponent m, values ranging from 1.5 to 2.1 are tested. Using FCMKM to segment Figure 2(a) with different m values yields results shown in Figures 2(b)-(h). Objective evaluation using Bezdek partition coefficient, Xie-Beni coefficient, and reconstruction error rate for different m values is presented in Table 1.

[Figure 2: see original paper]

Visual results show poor segmentation performance at $m = 1.7$ and $m = 1.8$, thus these values are excluded. Based on Table 1 data, selecting the value with larger Bezdek coefficient and smaller Xie-Beni coefficient, $m = 1.9$ is chosen as the optimal fuzzy weighting exponent, balancing both factors with maximum partition coefficient and relatively low Xie-Beni coefficient. The iteration termination threshold is set to $\varepsilon = 10^{-4}$, with maximum iteration count of 100 to prevent excessive iterations.

2.2 Synthetic Images

Four synthetic grayscale non-uniform images shown in Figure 3 are used for experiments: (a) figure-eight pattern, (b) simulated noise image, (c) spherical pattern, and (d) composite shape.

[Figure 3: see original paper]

Segmentation results using traditional FCM, KWFLICM, FCM-M, and FCMKM algorithms are compared in Figure 4. Experimental results demonstrate that traditional FCM, KWFLICM, and FCM-M cannot effectively separate targets from backgrounds. The proposed method successfully separates targets from backgrounds while significantly reducing noise interference. Objective analysis using Bezdek partition coefficient, Xie-Beni coefficient, and reconstruction error rate is presented in Tables 2-4.

[Figure 4: see original paper]

Comparative data analysis reveals that FCMKM improves the Bezdek partition coefficient by an average of 24.2%, reduces the Xie-Beni coefficient by an average of 53.9%, and decreases reconstruction error rate by an average of 97.29%. The proposed algorithm demonstrates significant segmentation improvement, effectively utilizing neighborhood and spatial information to enhance noise resistance, particularly for noisy images.

2.3 Noise Robustness Experiments

To evaluate noise robustness, noise simulation experiments are conducted on Figure 3(a) by adding Gaussian noise, salt-and-pepper noise, and white noise. Segmentation results of noisy images are compared to verify anti-noise performance, as shown in Figure 5.

[Figure 5: see original paper]

Figure 5 shows: (a), (f), (k) are images with added Gaussian, salt-and-pepper, and white noise respectively; (b), (g), (l) show segmentation results of conventional algorithms; (c), (h), (m) show KWFLICM results; (d), (i), (n) show FCM-M results; (e), (j), (o) show FCMKM results. Comparative analysis demonstrates that FCMKM effectively improves noise robustness, completely segmenting targets from images corrupted by all three noise types while maintaining clean backgrounds identical to noise-free segmentation results. Experiments confirm that FCMKM significantly enhances anti-noise capability.

2.4 Berkeley Database Image Segmentation Experiments

The Berkeley image database is commonly used for evaluating segmentation algorithms. To verify segmentation effectiveness, three images (#159091, #3063, #15088) are selected as test images, shown in Figures 6(a)-(c). All four algorithms are applied to these images, with results presented in Figures 6(d)-(o).

[Figure 6: see original paper]

Traditional FCM can separate targets but retains most background content, preventing effective target-background separation. KWFLICM and FCM-M achieve better separation but cause target blurring, compromising target integrity. In contrast, the proposed FCMKM algorithm effectively separates targets from backgrounds, preserving only target images to facilitate subsequent processing. Objective evaluation using Bezdek partition coefficient, Xie-Beni coefficient, and reconstruction error rate is presented in Table 5.

Data comparison shows that FCMKM improves the Bezdek partition coefficient by an average of 48.3%, reduces the Xie-Beni coefficient by an average of 53.5%, and achieves reconstruction error rates approaching zero. The analysis demonstrates that FCMKM enables effective clustering partition, increasing intra-class cohesion, reducing inter-class coupling, and minimizing reconstruction error.

2.5 FCMKM Algorithm Performance Experiments

To verify segmentation efficiency, running time and iteration count comparisons are conducted on Figures 3(a), 3(b), 6(a), and 6(b), with results presented in Table 6.

The proposed algorithm significantly reduces both iteration count and running time compared to other methods, achieving an average 77.3% reduction in running time and 68.3% reduction in iteration count, demonstrating substantial performance improvement.

3 Conclusion

This paper proposes a fuzzy clustering algorithm combining kernel function with Mahalanobis distance (FCMKM). By mapping images through kernel functions, pixels are nonlinearly transformed from low-dimensional to high-dimensional space, becoming linearly separable. Replacing traditional Euclidean distance with Mahalanobis distance improves utilization of neighborhood and spatial information. Comparative experiments demonstrate that the proposed algorithm effectively separates backgrounds from targets while enhancing noise robustness and segmentation efficiency. However, the algorithm has limitations: it employs a single Gaussian kernel function, causing loss of some detailed information in noisy target regions during segmentation. Future work will focus on adopting adaptive kernel functions to improve segmentation performance.

References

[1] Xu Qiuye, Li Yu, Lin Wenjie. Remote sensing image segmentation based on information clustering [J]. Journal of China University of Mining & Technology, 2017, 46(1): 209-214.

[2] Daisy V R, Nirmala S. Stability-integrated Fuzzy C-means segmentation for spatial incorporated automation of number of clusters [J]. Sādhanā, 2018, 43(3): 1-16.

[3] Bezdek J C, Hathaway R J, Sabin M J. Convergence theory for fuzzy c-means: Counterexamples and repairs [J]. IEEE Trans on Systems Man & Cybernetics, 1987, 17(5): 873-877.

[4] Hafiane A, Zavidovique B, Chaudhuri S. A modified FCM with optimal Peano scans for image segmentation [C]//Proc of IEEE International Conference on Image Processing. Genova, Italy: IEEE Press, 2005.

[5] Namburu A, Samay S K, Edara S R. Soft fuzzy rough set-based MR brain image segmentation [J]. Applied Soft Computing, 2017, 54: 200-212.

[6] Zhang Mengxuan, Jiao Licheng, Ma Wenping, et al. Multi-objective evolutionary fuzzy clustering for image segmentation with MOEA/D [J]. Applied Soft Computing, 2016, 48(C): 621-637.

[7] Zhang Xiaofeng, Sun Yujuan, Wang Gang, et al. Improved fuzzy clustering algorithm with non-local information for image segmentation [J]. Multimedia Tools & Applications, 2016, 76(6): 1-27.

[8] Wu Chengmao, Shangguan Ruoyu. Neutrosophic fuzzy clustering segmentation algorithm based on HMRF [J]. Journal of Xidian University, 2017, 44(6): 113-118.

[9] Gharieb R R, Gendy G, Abdelfattah A. C-means clustering fuzzified by two membership relative entropy functions approach incorporating local data information for noisy image segmentation [J]. Signal Image & Video Processing, 2017, 11(3): 541-548.

[10] Choy S K, Lam S Y, Yu K W, et al. Fuzzy model-based clustering and its application in image segmentation [J]. Pattern Recognition, 2017, 68(8): 141-157.

[11] Hussain K M, Lee D H. Generalised kernel weighted fuzzy C-means clustering algorithm with local information [J]. Fuzzy Sets and Systems, 2018, 340: 91-108.

[12] Zhao Xuemei, Li Yu, Zhao Quanhua. Mahalanobis distance based on fuzzy clustering algorithm for image segmentation [J]. Digital Signal Processing, 2015, 43: 8-16.

[13] Pham T X, Siarry P, Oulhadj H. Integrating fuzzy entropy clustering with an improved PSO for MRI brain image segmentation [J]. Applied Soft Computing, 2018, 65: 230-242.

[14] González-Carrato R R D L H, Kalogirou S A, Christodoulides P. Wind farm monitoring using Mahalanobis distance and fuzzy clustering [J]. Renewable Energy, 2018, 123: 526-540.

[15] Fan Jiulun, Wu Chengmao. Clustering validity function based on partition coefficient combined with total variation [J]. Acta Electronica Sinica, 2001, 29(11): 1561-1563.

[16] Pu Yunwei, Jin Weidong, Zhu Ming, et al. Kernelized Xie-Beni index and its performance [J]. Control and Decision, 2007, 22(7): 829-832.

[17] Zhang Xiaofeng. Research of medical image segmentation based on fuzzy clustering algorithms [D]. Jinan: Shandong University, 2014.

Note: Figure translations are in progress. See original paper for figures.