Multi-Attribute Decision-Making Method Based on Pythagorean Fuzzy Heronian Operator: Postprint

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Abstract

To address the information aggregation problem in multi-attribute decision-making under Pythagorean fuzzy environments, where the attributes of alternatives are interrelated and mutually influential, a multi-attribute decision-making method based on the Pythagorean fuzzy Heronian operator is proposed. First, by combining Pythagorean fuzzy numbers with the Heronian operator, the Pythagorean fuzzy Heronian operator and the Pythagorean fuzzy weighted Heronian operator are proposed, and the properties of these operators are discussed with corresponding proofs provided. On this basis, a multi-attribute decision-making method based on the Pythagorean fuzzy Heronian operator is proposed. Finally, it is applied to the service quality evaluation of four domestic airlines to demonstrate the effectiveness and feasibility of the proposed operator.

Full Text

Preamble

Multi-Attribute Decision Making Method for Pythagorean Fuzzy Heronian Operators

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Abstract: This paper addresses the information aggregation problem in multi-attribute decision making within Pythagorean fuzzy environments, where attributes of alternatives are interrelated and mutually influential. A novel multi-attribute decision-making method based on Pythagorean fuzzy Heronian operators is proposed. First, by combining Pythagorean fuzzy numbers with Heronian operators, we introduce the Pythagorean fuzzy Heronian operator and the

Pythagorean fuzzy weighted Heronian operator, discuss their properties, and provide corresponding proofs. Building upon this foundation, a multi-attribute decision-making method based on Pythagorean fuzzy Heronian operators is established. Finally, the method is applied to evaluate the service quality of four domestic airlines, demonstrating the effectiveness and feasibility of the proposed operators.

Keywords: Pythagorean fuzzy numbers; Heronian operators; Pythagorean fuzzy Heronian aggregation operators; multiple-criteria decision making

0 Introduction

Since Zadeh [1] proposed fuzzy set theory as a powerful tool for describing uncertainty and vagueness, numerous extensions have been developed to better capture real-world uncertainties. These include interval-valued hesitant fuzzy sets (IVHFS), type-2 fuzzy sets (T2FS), fuzzy multisets (FMS), and intuitionistic fuzzy sets (IFS). Among these, Atanassov' s intuitionistic fuzzy set theory [2], introduced in 1986, represents one of the most significant extensions of classical Zadeh fuzzy sets, yielding extensive research 成果 and profound influence in the field. Intuitionistic fuzzy sets characterize concepts using both membership and non-membership degrees, enabling more comprehensive and nuanced representation of fuzzy and abstract concepts. However, in intuitionistic fuzzy decision-making processes, situations may arise where the sum of membership and non-membership degrees for an alternative' s attribute exceeds 1.

To address this limitation, Yager [4] extended intuitionistic fuzzy sets by proposing Pythagorean fuzzy sets, which satisfy the condition that the sum of membership and non-membership degrees may exceed 1, but their squared sum does not exceed 1. This allows decision-makers to more accurately and meticulously model real-world situations without modifying their original membership and non-membership values. Since their inception, Pythagorean fuzzy sets have generated substantial research. For instance, Zhang [10] proposed a hierarchical QUALIFLEX method for Pythagorean fuzzy decision-making; Peng et al. [11] investigated interval-valued Pythagorean fuzzy sets and their decision-making applications; Ren et al. [12] developed the Pythagorean fuzzy TODIM method for multi-attribute decision making; Guo et al. [13] established Pythagorean fuzzy functions and studied their properties including continuity, derivability, and differentiability; Yager [4] introduced Pythagorean aggregation operators including the Pythagorean fuzzy weighted average (PFWA) and ordered weighted geometric (PFOWG) operators; Liu et al. [14] examined aggregation operators in Pythagorean fuzzy decision environments, proposing quasi-weighted geometric and quasi-ordered weighted geometric operators; Zhang et al. [5] defined operations and distances for Pythagorean fuzzy numbers and developed a TOPSIS method based on Pythagorean fuzzy sets; and Ding et al. [15] studied multi-attribute group decision-making in Pythagorean fuzzy environments, proposing the Pythagorean fuzzy power weighted average (PFPWA) operator.

However, the aforementioned studies on Pythagorean fuzzy aggregation operators and decision-making methods assume attribute independence. In practical decision-making, attributes often exhibit interdependencies such as complementarity, redundancy, or preference relationships. Therefore, investigating multi-attribute decision-making problems with interrelated attributes holds significant theoretical importance. Aggregation operators serve as the foundation for many decision-making methods, making their study particularly crucial in Pythagorean fuzzy environments. While most existing research assumes attribute independence, real-world complexity often involves various degrees of correlation among attributes. These relationships can be identified either through decision-makers' experience and knowledge (e.g., product cost and price typically correlate positively) or through hidden patterns in the evaluation values themselves. In the airline service quality evaluation context considered in this paper, booking/ticketing procedures may influence check-in processes, while cabin service and responsiveness may also interact. To address such interdependent multi-attribute decision-making problems, we employ the Heronian mean (HM) operator [7~9], which is specifically designed to handle attribute correlations. HM-based research has advanced rapidly: Liu [16] combined power mean and Heronian operators in interval-valued intuitionistic fuzzy environments to propose the IVIFPHA operator and corresponding decision models; Liu et al. [17] developed intuitionistic uncertain linguistic Heronian mean operators for multi-attribute group decision-making; and Yu [18] proposed intuitionistic fuzzy geometric Heronian mean operators for multi-criteria decision-making with correlated attributes.

In summary, research on Pythagorean fuzzy aggregation operators for correlated attributes remains in its exploratory stage, with existing operators exhibiting two main limitations: (a) they fail to consider potential attribute interdependencies, and (b) they lack parameter flexibility to reflect decision-makers' preferences. To address these gaps, this paper integrates Pythagorean fuzzy sets with Heronian operators, proposing Pythagorean fuzzy Heronian operators and Pythagorean fuzzy weighted Heronian operators. These operators incorporate parameters p and q that allow decision-makers to flexibly express their preferences, thereby providing greater dynamism and subjectivity. Furthermore, we develop a multi-attribute decision-making method based on these operators.

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1 Preliminary Knowledge

This section briefly reviews fundamental concepts of intuitionistic fuzzy sets, Pythagorean fuzzy sets, and Heronian operators. Pythagorean fuzzy sets generalize intuitionistic fuzzy sets, enabling more nuanced real-world modeling, while Heronian operators handle attribute correlations in aggregation.

Definition 1 [2]. Let X be a universe of discourse. An intuitionistic fuzzy set

A on X is defined as $A = \{ x, \ (x), \ (x) \ | \ x \ \ X\}$, where $\ : X \to [0,1]$ and $\ :$ $X \to [0,1]$ represent the membership and non-membership degrees of element x belonging to A, respectively, with the constraint that $0 \leq \ (x) + \ (x) \leq 1$ for all $x \ \ X$.

Definition 2 [3,4]. Let X be a universe of discourse. A Pythagorean fuzzy set A on X is defined as $A = \{ x, \ (x), \ (x) \ | \ x \ \ X\}$, where $\ : X \to [0,1]$ and $\ :$ $X \to [0,1]$ represent the membership and non-membership degrees of element x belonging to A, respectively, with the constraint that $0 \leq (\ (x))^2 + (\ (x))^2 \leq 1$ for all $x \ \ X$. The hesitation degree is defined as $\pi \ (x) = \sqrt{}(1 - (\ (x))^2 - (\ (x))^2)$. The pair $\ (x), \ (x)$ is called a Pythagorean fuzzy number [5], and the set of all Pythagorean fuzzy numbers is denoted by PFN.

Definition 3 [5]. Let $\alpha = \ , \$ be a Pythagorean fuzzy number. The following operations are defined:

$\alpha_1 \quad \alpha_2 = \sqrt{}({}_1{}^2 + {}_2{}^2 - {}_1{}^2 {}_2{}^2), \ {}_1 {}_2$
$\alpha_1 \quad \alpha_2 = \ {}_1 {}_2, \sqrt{}({}_1{}^2 + {}_2{}^2 - {}_1{}^2 {}_2{}^2)$
$\lambda\alpha = \sqrt{}(1 - (1 - \ ^2)\hat{}\lambda), \ \hat{}\lambda \ , \lambda > 0$
$\alpha\hat{}\lambda = \ \hat{}\lambda, \sqrt{}(1 - (1 - \ ^2)\hat{}\lambda) \ , \lambda > 0$

For ranking Pythagorean fuzzy numbers, we define score and accuracy functions:

Definition 4 [5]. Let $\alpha = \ , \$ be a Pythagorean fuzzy number. Its score function is defined as $S(\alpha) = \ ^2 - \ ^2$, where $S(\alpha) \ $ [-1,1].

Definition 5 [6]. Let $\alpha = \ , \$ be a Pythagorean fuzzy number. Its accuracy function is defined as $h(\alpha) = \ ^2 + \ ^2$, where $h(\alpha) \ $ [0,1].

Definition 6 [6]. Let $\alpha_1 = \ {}_1, \ {}_1$ and $\alpha_2 = \ {}_2, \ {}_2$ be two Pythagorean fuzzy numbers. Then: - If $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 > \alpha_2$; - If $S(\alpha_1) = S(\alpha_2)$, then: - If $h(\alpha_1) > h(\alpha_2)$, then $\alpha_1 > \alpha_2$; - If $h(\alpha_1) = h(\alpha_2)$, then $\alpha_1 \sim \alpha_2$.

Definition 7 [7~9]. Let a (i = 1,2,···,n) be a collection of non-negative real numbers, with p, $q \geq 0$ and not simultaneously zero. The Heronian mean (HM) operator is defined as:

$$HM^{p,q}(a_1, a_2, ..., a_n) = \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} a_i^p a_j^q$$

2 Pythagorean Fuzzy Heronian Operators

This section defines Pythagorean fuzzy Heronian operators, establishes their properties, and introduces the weighted variant.

Definition 8. Let p, $q \geq 0$ (not simultaneously zero) and $\alpha = \ , \$ (i = 1,2,···,n) be a collection of Pythagorean fuzzy numbers. The Pythagorean fuzzy Heronian mean (PFHM) operator is defined as:

$$PFHM^{p,q}(\alpha_1, \alpha_2, ..., \alpha_n) = \left(\frac{2}{n(n+1)} \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} (\alpha_i^p \otimes \alpha_j^q) \right)^{\frac{1}{p+q}}$$

Theorem 1. Let p, $q \geq 0$ (not simultaneously zero) and $\alpha \;=\; ,\;$ (i = 1,2,···,n) be Pythagorean fuzzy numbers. The aggregated result using the PFHM operator remains a Pythagorean fuzzy number:

$$PFHM^{p,q}(\alpha_1, \alpha_2, ..., \alpha_n) = \left\langle \sqrt{1 - \left(1 - \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \mu_i^p \mu_j^q \right)^{\frac{2}{p+q}} \right)^{\frac{1}{2}}}, \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \nu_i^p \nu_j^q \right)^{\frac{1}{p+q}} \right\rangle$$

Proof. The proof follows from Definition 8 and the operational laws in Definition 3, showing that the aggregated result satisfies the Pythagorean fuzzy number condition.

The PFHM operator satisfies three fundamental properties:

Property 1 (Idempotency). Let $\alpha \;=\; ,\;$ (i = 1,2,···,n) be a collection of Pythagorean fuzzy numbers. If all $\alpha \;= \alpha$, then $PFHM^{p,q}(\alpha_1, \alpha_2, ..., \alpha_n) = \alpha$.

Proof. Substituting $\alpha \;= \alpha$ into Theorem 1 yields the result directly.

Property 2 (Monotonicity). Let $\alpha \;=\; ,\;$ and $\beta \;=\; ',\; '$ (i = 1,2,···,n) be two collections of Pythagorean fuzzy numbers. If $\;\leq\; '$ and $\;\geq\; '$ for all i, then $PFHM^{p,q}(\alpha_1, ..., \alpha_n) \leq PFHM^{p,q}(\beta_1, ..., \beta_n)$.

Proof. Since $\;\leq\; ',\;\; \geq\; '$, and p, $q \geq 0$, the monotonicity of the aggregation functions ensures the inequality holds.

Property 3 (Boundedness). Let $\alpha \;=\; ,\;$ (i = 1,2,···,n) be a collection of Pythagorean fuzzy numbers, with $\alpha^- = \min\{\alpha\}$ and $\alpha^+ = \max\{\alpha\}$. Then:

$$\alpha^- \leq PFHM^{p,q}(\alpha_1, \alpha_2, ..., \alpha_n) \leq \alpha^+$$

Proof. The boundedness follows from the idempotency and monotonicity properties.

Definition 9. Let p, $q \geq 0$ (not simultaneously zero), $\alpha \;=\; ,\;$ (i = 1,2,···,n) be a collection of Pythagorean fuzzy numbers, and $\omega = (\omega_1, \omega_2, \cdots, \omega\;)$ be a weight vector with $\omega\;$ [0,1] and $\Sigma\;\; _1{}^n\; \omega\; = 1$. The Pythagorean fuzzy weighted Heronian mean (PFWHM) operator is defined as:

$$PFWHM^{p,q}(\alpha_1, \alpha_2, ..., \alpha_n) = \left(\frac{2}{n(n+1)} \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} ((\omega_i \alpha_i)^p \otimes (\omega_j \alpha_j)^q) \right)^{\frac{1}{p+q}}$$

Theorem 2. Let p, $q \geq 0$ (not simultaneously zero), α = , be Pythagorean fuzzy numbers, and ω be a weight vector as defined above. The aggregated result using the PFWHM operator remains a Pythagorean fuzzy number:

$$PFWHM^{p,q}(\alpha_1, ..., \alpha_n) = \left\langle \sqrt{1 - \left(1 - \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} (1-(1-\mu_i^2)^{\omega_i})^p (1-(1-\mu_j^2)^{\omega_j})^q \right)^{\frac{2}{p+q}} \right)^{\frac{1}{2}}}, \left(\frac{2}{n(n \right.$$

Proof. The proof is analogous to Theorem 1 and is omitted for brevity.

The PFWHM operator also satisfies idempotency, monotonicity, and boundedness (Properties 4-6), with proofs similar to Properties 1-3.

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3 Multi-Attribute Decision Making Method Based on Pythagorean Fuzzy Weighted Heronian Operators

This section presents a Pythagorean fuzzy multi-attribute decision-making method using the PFWHM operator, where attribute evaluations are expressed as Pythagorean fuzzy numbers.

Consider a decision problem with m alternatives $A = \{A_1, A_2, \cdots, A\}$ and n attributes $C = \{C_1, C_2, \cdots, C\}$, with attribute weight vector $\omega = (\omega_1, \omega_2, \cdots, \omega)$ where ω [0,1] and $\Sigma_1^n\ \omega = 1$. The decision-maker provides evaluation values α = , (Pythagorean fuzzy numbers) for alternative A with respect to attribute C, forming the decision matrix $Q = (\alpha)$. Based on this information, we rank the m alternatives.

The decision procedure is as follows:

Step 1: Standardization. Normalize the decision matrix $Q = (\alpha)$ to obtain the standardized matrix $G = (g)$, where:

$$g_{ij} = \begin{cases} \alpha_{ij} & \text{if } C_j \text{ is a benefit attribute} \\ \alpha_{ij}^c & \text{if } C_j \text{ is a cost attribute} \end{cases}$$

with α being the complement of α.

Step 2: Aggregation. Use the PFWHM operator to aggregate the evaluation values in the standardized matrix:

$$g_i = PFWHM^{p,q}(g_{i1}, g_{i2}, ..., g_{in})$$

Step 3: Score calculation. Compute the score function $S(g)$ for each alternative using Definition 4.

Step 4: Ranking. Rank alternatives according to Definition 6 and identify the optimal solution.

Step 5: Termination.

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4 Case Study

This section demonstrates the practical application of the proposed PFWHM operator and decision model through a real-world airline service quality evaluation problem, validating its effectiveness and feasibility.

China' s domestic aviation market has experienced rapid development alongside intensifying competition. Beyond price wars, service quality has become a critical factor for airlines to maintain competitiveness. Over four decades of reform and opening-up, China' s sustained economic growth and rising living standards have made air travel accessible to ordinary families, transforming it from a luxury to a preferred transportation mode. In this information age, airlines must leverage technology to develop distinctive services beyond homogeneous basic offerings. Facing immense pressure from both domestic and international competitors, analyzing service quality is essential for survival and growth in this monopolistically competitive service industry. Service quality represents a crucial component of competitiveness and a key source of competitive advantage in civil aviation, which is intimately connected to people' s daily lives. Improving service quality and passenger satisfaction is therefore imperative.

In this study, we evaluate the service quality of four domestic airlines (A_1, A_2, A_3, A_4) after expert screening [17]. Through expert discussion, four evaluation criteria are identified: C_1 (booking/ticketing service), C_2 (check-in procedures), C_3 (cabin service), and C_4 (responsiveness). To authentically capture expert assessments, evaluations are expressed as Pythagorean fuzzy numbers. The attribute weight vector is $\omega = (0.15,\ 0.25,\ 0.35,\ 0.25)$, with parameters $p = 1$ and $q = 1$. To ensure scientific rigor, experts from research institutions, government agencies, and non-profit organizations participated in the evaluation. Given time constraints, limited reference materials, and bounded personal experience, experts preferred using Pythagorean fuzzy numbers to express their judgments, facilitating the representation of uncertainty. The evaluation results are presented in Table 1 .

Table 1. Pythagorean Fuzzy Decision Matrix

Alternative	C_1	C_2	C_3	C_4
A_1	(0.9, 0.3)	(0.7, 0.6)	(0.5, 0.8)	(0.6, 0.3)
A_2	(0.4, 0.7)	(0.9, 0.2)	(0.8, 0.1)	(0.5, 0.3)
A_3	(0.8, 0.4)	(0.7, 0.5)	(0.6, 0.2)	(0.7, 0.4)
A_4	(0.7, 0.2)	(0.8, 0.2)	(0.8, 0.4)	(0.6, 0.6)

Analysis:

a) All criteria are benefit attributes, requiring no standardization.

b) Aggregate evaluations using the PFWHM operator:

$$g_i = PFWHM^{1,1}(\alpha_{i1}, \alpha_{i2}, \alpha_{i3}, \alpha_{i4})$$

The aggregated results are: - g_1 = (0.9837, 0.1562) - g_2 = (0.9849, 0.0609) - g_3 = (0.9857, 0.0879) - g_4 = (0.9890, 0.0803)

c) Compute score functions using Definition 4:

- $S(g_1) = 0.9430$
- $S(g_2) = 0.9664$
- $S(g_3) = 0.9639$
- $S(g_4) = 0.9716$

d) Ranking alternatives by score values yields: A_4 A_2 A_3 A_1, making A_4 the optimal choice.

The above analysis assumes $p = 1$, $q = 1$. Table 2 and Table 3 show how aggregation results, score values, and rankings vary with different parameter values. Table 3 reveals that while rankings differ slightly across parameter settings, they converge as p and q increase, demonstrating that results are subjectively influenced by parameter selection, allowing experts to choose parameters according to their preferences.

Table 2. Aggregation Results of PFWHM Operator Under Different Parameters

$(p,\ q)$	A_1	A_2	A_3	A_4
(1,1)	(0.9837, 0.1562)	(0.9849, 0.0609)	(0.9857, 0.0879)	(0.9890, 0.0803)
(2,2)	(0.9780, 0.1787)	(0.9795, 0.0730)	(0.9804, 0.1053)	(0.9846, 0.0959)
(5,5)	(0.9438, 0.1975)	(0.9452, 0.0897)	(0.9470, 0.1294)	(0.9564, 0.1163)
(10,10)	(0.9700, 0.1974)	(0.9689, 0.1001)	(0.9693, 0.1455)	(0.9734, 0.1279)
(50,50)	(0.9815, 0.1646)	(0.9765, 0.1033)	(0.9733, 0.1577)	(0.9701, 0.1219)

Table 3. Ranking Results Under Different Parameters

$(p,\ q)$	Score Values	Ranking
(1,1)	(0.9430, 0.9664, 0.9639, 0.9716)	A_4 A_2 A_3 A_1
(2,2)	(0.9247, 0.9540, 0.9501, 0.9627)	A_4 A_2 A_3 A_1
(5,5)	(0.8518, 0.8854, 0.8801, 0.9013)	A_4 A_2 A_3 A_1
(10,10)	(0.9020, 0.9288, 0.9183, 0.9312)	A_4 A_2 A_3 A_1
(50,50)	(0.9363, 0.9429, 0.9225, 0.9263)	A_2 A_4 A_1 A_3

The analysis with $p = q$ has limitations. Table 4 examines asymmetric parameter values, showing that rankings vary with different p and q combinations, with optimal solutions being either A_2 or A_4. As p and q increase, attribute interactions strengthen. In practice, simpler parameter values can be chosen to reduce computational complexity while maintaining flexibility to model real decision environments.

Table 4. Score Values and Rankings for Asymmetric Parameters

$(p,\ q)$	Score Values	Ranking
(5,1)	(0.9184, 0.9490, 0.9399, 0.9511)	A_4 A_2 A_3 A_1
(1,5)	(0.9234, 0.9504, 0.9438, 0.9536)	A_4 A_2 A_3 A_1
(2,10)	(0.9069, 0.9395, 0.9257, 0.9386)	A_2 A_4 A_3 A_1
(10,2)	(0.9130, 0.9403, 0.9299, 0.9409)	A_2 A_4 A_3 A_1
(3,20)	(0.9098, 0.9378, 0.9182, 0.9308)	A_2 A_4 A_3 A_1
(20,3)	(0.9169, 0.9380, 0.9225, 0.9326)	A_2 A_4 A_3 A_1
(4,30)	(0.9156, 0.9398, 0.9172, 0.9284)	A_2 A_4 A_1 A_3
(30,4)	(0.9224, 0.9398, 0.9211, 0.9297)	A_2 A_4 A_1 A_3
(5,40)	(0.9210, 0.9422, 0.9183, 0.9278)	A_2 A_4 A_1 A_3
(40,5)	(0.9273, 0.9422, 0.9216, 0.9288)	A_2 A_4 A_1 A_3

Decision-makers can select appropriate p and q values based on specific problem characteristics.

Comparative Analysis:

Table 5 compares our method with existing approaches: PFWA operator [4], GPFOWA operator [14], TOPSIS method [5], and PFPWA operator [15].

Table 5. Comparative Analysis Results

Method	Score Values	Ranking	Optimal
Literature [4] (PFWA)	(0.0850, 0.4250, 0.3250, 0.3650)	A_2 A_4 A_3 A_1	A_2
Literature [14] (GPFOWA)	(0.0013, 0.2122, 0.1441, 0.2174)	A_4 A_2 A_3 A_1	A_4
Literature [5] (TOPSIS)	(-0.2466, 0.3695, 0.2921, 0.5630)	A_4 A_2 A_3 A_1	A_4
Literature [15] (PFPWA)	(0.9430, 0.9664, 0.9639, 0.9716)	A_4 A_2 A_3 A_1	A_4
Our method	(0.9430, 0.9664, 0.9639, 0.9716)	A_4 A_2 A_3 A_1	A_4

Our ranking aligns with GPFOWA [14] and PFPWA [15], while PFWA [4] and TOPSIS [5] identify A_2 as optimal. The differences arise because methods [4] and [5] assume attribute independence, whereas our PFWHM-based method explicitly accounts for attribute interdependencies. The TOPSIS method [5] also suffers from computational complexity. In service quality evaluation, attributes are inherently interrelated, making our approach more reasonable and effective.

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5 Conclusion

Real-world multi-attribute decision-making problems often involve interdependent attributes. To address this complexity, this paper integrates Pythagorean fuzzy numbers with Heronian operators, proposing PFHM and PFWHM operators that overcome limitations of existing Pythagorean fuzzy aggregation operators. The developed decision-making method considers attribute correlations and allows flexible parameter selection based on decision-maker preferences. The application to airline service quality evaluation demonstrates the method' s validity and feasibility, providing a new approach for multi-attribute decision-making research.

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Note: Figure translations are in progress. See original paper for figures.