Robust Dissipativity of Uncertain Neural Networks with Additive Time-Varying Delays (Postprint)

Authors: Yang Fei, Tang Qian, forest orchard

Date: 2018-11-29T00:00:00+00:00

Abstract

For the delay-dependent robust dissipativity problem of uncertain neural networks with additive time-varying delays, a more generalized activation function is proposed. Unlike previous studies, sufficient information regarding neuronal activation functions and additive time-varying delays is fully considered. An appropriate Lyapunov-Krasovskii functional (LKF) is constructed by employing some novel integral terms, and its derivative is computed utilizing newly developed single integral inequalities, which include special cases of Jensen' s inequality and Wirtinger integral inequality. By employing linear matrix inequality (LMI) technique, a novel delay-dependent less conservative criterion for global asymptotic stability and dissipativity is established. Finally, the effectiveness of the proposed theory is verified through calculations and numerical simulations.

Full Text

Preamble

Vol. 37 No. 1
Application Research of Computers –ChinaXiv Cooperative Journal
Accepted Paper

Research on Robust Dissipativity of Uncertain Neural Networks with Additive Time-Varying Delays

Yang Fei[1], Tang Qian[2], Lin Guoyuan[1]
(1. School of Computer Science & Technology, China University of Mining & Technology, Xuzhou, Jiangsu 221000, China;
2. The 28th Research Institute of China Electronics Technology Group Corporation, Nanjing 210007, China)

Abstract: This paper addresses the delay-dependent robust dissipativity problem for uncertain neural networks with additive time-varying delays by proposing a more generalized activation function. Unlike previous studies, sufficient information regarding neuron activation functions and additive time-varying delays is fully considered. By constructing an appropriate Lyapunov-Krasovskii functional (LKF) with novel integral terms and estimating its derivative using newly developed single integral inequalities—including special cases of Jensen' s inequality and Wirtinger-based integral inequality—a new delay-dependent, less conservative global asymptotic stability and dissipativity criterion is established via linear matrix inequality (LMI) techniques. Finally, the effectiveness of the proposed theory is verified through calculations and numerical simulations.

Keywords: neural network; artificial intelligence; time-varying delays; Lyapunov-Krasovskii functional

0 Introduction

Neural networks (NNs) have attracted increasing research interest due to their widespread applications in signal processing, pattern recognition, combinatorial optimization, and other practical systems [1~4]. All these applications depend on the dynamic performance of the system. On the other hand, time delays commonly affect system behavior, causing instability, poor performance, and oscillations [5~6]. Therefore, it is essential to investigate the dynamics of neural networks with time delays.

Generally, time delays of state variables are assumed to appear in simple or singular forms. In [7], a novel system model was introduced that contains successive time-varying delay components of the state vector in a certain sense. The system model proposed in [7] can be applied in various contexts, such as remote control and networked control systems (NCS). In NCS, signals transmitted between two nodes can sometimes be dual-segmented. Therefore, in [7], two delay components were considered through variable transmission characteristics. Systems with such delays have stronger practical significance than those with single delays. Consequently, the concept of additive time-varying delays has received widespread attention in stability analysis of neural networks [8~10]. In recent years, researchers have dedicated efforts to studying the stability of neural networks with additive time delays and have achieved favorable results. In [10], stability criteria for generalized NNs with Markov jump parameters and additive time delays were proposed using the free-weighting matrix method. In [9], improved stability criteria were developed by introducing an enhanced LKF that utilizes additive time-delay information. Recently, [11] investigated robust delay-dependent stability criteria for uncertain neural networks with two additive time-varying delay components. In [12], new asymptotic criteria for neural networks with additive time-varying delay components were derived by constructing appropriate Lyapunov functionals and employing free-weighting matrix methods. In [13], less conservative stability conditions based on quadratic convex combination were proposed for mode-dependent additive time-varying delays. Most recently, [14] presented an improved stability criterion for neural networks with additive time-varying delay components and leakage delay.

On the other hand, dissipativity analysis represents a hot topic in neural network research, as it is a fundamental property closely related to the intuitive phenomenon of energy loss or dissipation in physical systems. Willems first introduced the concept of dissipative systems in dynamic systems, and since then, the concept has been extended to nonlinear systems in [15]. Dissipativity theory provides an energy-related input/output description model that forms the basic idea for control system design and analysis. It offers a more flexible theoretical foundation for modern control applications such as robotics, active vibration damping, electromechanical systems, internal combustion engines, and circuit theory.

Motivated by the aforementioned studies, this paper employs a more general activation function method and extends this condition to address the dissipativity analysis of neural networks. The main contribution is to reduce the conservatism of previous studies as much as possible. To this end, a new single integral inequality is considered to obtain superior results compared to recent research. This inequality also includes two special cases: Jensen' s inequality and Wirtinger integral inequality. Furthermore, a more in-depth investigation of neuron activation functions is conducted by dividing the bounds of neuron activation functions into two subintervals and defining a parameter σ, where L_i^- and L_i^+ are known real scalar values that can be positive, negative, or zero. This means the obtained activation function can be non-monotonic and more general than typical sigmoid functions and Lipschitz conditions. Moreover, the concept of generalized activation functions is further extended to additive time-varying delay components. Finally, the effectiveness of the proposed method is verified through three numerical examples.

---

1 Problem Description and Preliminaries

We first introduce some notation used throughout this paper. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices, $\|\cdot\|$ denotes the Euclidean norm, $P > 0$ denotes a symmetric positive-definite matrix, and I denotes the identity matrix. diag{...} denotes a block diagonal matrix. For a symmetric matrix A, A^T denotes its transpose. $L_2[0,\infty)$ is the space of n-dimensional square-integrable function vectors.

This paper considers uncertain neural networks with additive time-varying delays:

$$\dot{x}(t) = -Dx(t) + A(t)f(x(t)) + B(t)f(x(t-\tau_1(t)-\tau_2(t))) \quad (1)$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$ represents the state vector associated with n neurons; $D = \mathrm{diag}\{d_1, d_2, \dots, d_n\}$ is a positive diagonal matrix with each $d_i > 0$; A and B are known connection weight matrices and delayed connection weight matrices, respectively; $f(\cdot) = [f_1(\cdot), f_2(\cdot), \dots, f_n(\cdot)]^T \in \mathbb{R}^n$ denotes the neuron activation function; $\tau_1(t)$ and $\tau_2(t)$ represent additive time-varying delays satisfying the following conditions:

$$0 \leq \tau_1(t) \leq \tau_1, \quad 0 \leq \tau_2(t) \leq \tau_2, \quad \dot{\tau}_1(t) \leq \mu_1, \quad \dot{\tau}_2(t) \leq \mu_2 \quad (2)$$

where τ_1, τ_2, μ_1, and μ_2 are known real constants. For simplicity, we define $\tau(t) = \tau_1(t) + \tau_2(t)$, $\tau = \tau_1 + \tau_2$, and $u = u_1 + u_2$.

Assumption 1: The time-varying delays $\tau_i(t)$ $(i = 1, 2)$ denote additive time-varying delays and satisfy the above conditions.

Assumption 2: Each neuron activation function $f_i(\cdot)$ in (1) is bounded and satisfies:

$$L_i^- \leq \frac{f_i(\sigma_1) - f_i(\sigma_2)}{\sigma_1 - \sigma_2} \leq L_i^+, \quad \forall \sigma_1, \sigma_2 \in \mathbb{R}, \sigma_1 \neq \sigma_2, \quad i = 1, 2, \dots, n \quad (3)$$

where L_i^- and L_i^+ are known real scalars that can be positive, negative, or zero. This means the obtained activation function can be non-monotonic and more general than typical sigmoid functions and Lipschitz conditions.

Under Assumption 2, system (1) has an equilibrium point x^*. Through the transformation $y(\cdot) = x(\cdot) - x^*$, $g(y(\cdot)) = f(y(\cdot) + x^*) - f(x^*)$, system (1) can be transformed into:

$$\dot{y}(t) = -Dy(t) + A(t)g(y(t)) + B(t)g(y(t - \tau_1(t) - \tau_2(t))) \quad (6)$$

Assumption 3: Each activation function $g_i(\cdot)$ in (6) is continuous and bounded, satisfying $g_i(0) = 0$ and:

$$L_i^- \leq \frac{g_i(\sigma_1) - g_i(\sigma_2)}{\sigma_1 - \sigma_2} \leq L_i^+, \quad \forall \sigma_1, \sigma_2 \in \mathbb{R}, \sigma_1 \neq \sigma_2, \quad i = 1, 2, \dots, n \quad (7)$$

When an external disturbance appears in (6), it can be written as:

$$\begin{cases} \dot{y}(t) = -Dy(t) + A(t)g(y(t)) + B(t)g(y(t - \tau_1(t) - \tau_2(t))) + w(t) \\ z(t) = g(y(t)) \\ y(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \quad (8)$$

where $w(t) \in L_2[0,\infty)$ represents the disturbance input that cannot be fully measured and is assumed to satisfy $\int_0^\infty w^T(t)w(t)dt < \infty$, meaning it is a finite-energy function. The system can be expressed in the following form:

$$\begin{cases} \dot{y}(t) = -Dy(t) + Ag(y(t)) + Bg(y(t-\tau(t))) + Cp(t) + w(t) \\ p(t) = F(t)q(t) \\ q(t) = \varepsilon_a y(t) + \varepsilon_b y(t-\tau(t)) + \varepsilon_d g(y(t)) + \varepsilon_d g(y(t-\tau(t))) \\ y(t) = \varphi(t), \quad t \in [-\tau, 0] \\ z(t) = g(y(t)) \end{cases} \tag{9}$$

where C, ε_a, ε_b, and ε_d are known constant matrices; $F(t)$ is an unknown real-time-varying matrix satisfying $F^T(t)F(t) \leq I$ for all $t > 0$.

Definition 1: The uncertain neural network (8) is said to be strictly (Q,S,R)-γ-dissipative if, under zero initial conditions, the following inequality holds for any $t_f \geq 0$:

$$\int_0^{t_f} f(\omega,z,w)dt \geq \gamma \int_0^{t_f} w^T(t)w(t)dt \quad (10)$$

where $f(\omega,z,w)$ is the energy supply function defined as $f(\omega,z,w) = z^T(t)Qz(t) + 2z^T(t)Sw(t) + w^T(t)Rw(t)$, with Q, S, and R being real-valued matrices and Q, R symmetric. Therefore, (10) can be written as the dissipativity performance index for uncertain neural network (8):

$$\int_0^{t_f} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R-\gamma I \end{bmatrix} \begin{bmatrix} z(t) \\ w(t) \end{bmatrix} dt \leq 0 \quad (11)$$

Lemma 1 [18]: For any constant matrix $W \in \mathbb{R}^{n\times n}$ with $W > 0$, the following inequality holds:

$$-\int_a^b \dot{y}^T(s)W\dot{y}(s)ds \leq -\frac{1}{b-a}\left(\int_a^b \dot{y}(s)ds\right)^T W \left(\int_a^b \dot{y}(s)ds\right)$$

Lemma 2 [19]: Let $f_1, f_2, \dots, f_m : D \to \mathbb{R}$ have positive values on an open subset D of $\mathbb{R}^n$. Then the reciprocally convex combination of f_i over D satisfies:

$$\min_{\{\alpha_i|\alpha_i>0,\sum_i \alpha_i=1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{ij}(t)} \sum_{i\neq j} g_{ij}(t)$$

subject to:

$$\{g_{ij}:\mathbb{R}^m\to\mathbb{R}, g_{ji}(t)=g_{ij}(t), \begin{bmatrix} f_i(t) & g_{ij}(t) \\ g_{ij}(t) & f_j(t)\end{bmatrix}\geq 0\}$$

Lemma 3: For any constant matrix $W\in\mathbb{R}^{n\times n}$ with $W>0$, and vectors $x,y,z\in\mathbb{R}^n$, the following inequality holds:

$$-\int_{t-\tau(t)}^{t}\dot{y}^T(s)W\dot{y}(s)ds\leq\xi^T(t)\left[\frac{\tau(t)}{\tau}W_1+\frac{\tau-\tau(t)}{\tau}W_2\right]\xi(t)$$

where W_1, W_2, and W_3 are defined matrices, and $\xi(t)$ is an augmented state vector.

Proof: When $0\leq\tau(t)\leq\tau$, we have $\frac{\tau(t)}{\tau}+\frac{\tau-\tau(t)}{\tau}=1$. Therefore, for any constant matrix $W>0$, the inequality holds. Through simple rearrangement, (13) is obtained, completing the proof.

---

2 Stability Analysis

In this section, a more general activation function model is established based on the LKF and LMI techniques proposed in [19]. For simplicity, we define matrices and vectors as block matrices. The notation used in this section is defined as follows:

Theorem 1: If Assumptions 1 and 2 hold, then for any positive scalars τ_1, τ_2, and μ, the uncertain neural network (6) is globally asymptotically stable if there exist positive-definite matrices $P_i\in\mathbb{R}^{n\times n}$ $(i=1,2,\dots,12)$, positive diagonal matrices $Q_i\in\mathbb{R}^{n\times n}$ $(i=1,2,3)$, arbitrary matrices $M_k\in\mathbb{R}^{n\times n}$ $(k=1,2,\dots,12)$, and a positive scalar ε such that the following LMI holds:

$$\Psi=\begin{bmatrix}\Psi_{11} & \Psi_{12} & \cdots & \Psi_{1,12}\\ * & \Psi_{22} & \cdots & \Psi_{2,12}\\ \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & \Psi_{12,12}\end{bmatrix}<0 \quad (14)$$

Proof: Consider the Lyapunov-Krasovskii functional with double integral terms:

$$V(t)=V_1(t)+V_2(t)+V_3(t) \quad (15)$$

where

$$V_1(t)=y^T(t)Py(t)+\sum_{i=1}^{2}\int_{t-\tau_i(t)}^{t}y^T(s)Q_iy(s)ds+\int_{t-\tau(t)}^{t}y^T(s)Q_3y(s)ds$$

$$V_2(t) = \sum_{i=1}^{2} \int_{-\tau_i}^{0} \int_{t+\theta}^{t} \dot{y}^T(s) R_i \dot{y}(s) ds d\theta + \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{y}^T(s) R_3 \dot{y}(s) ds d\theta$$

$$V_3(t) = \sum_{i=1}^{2} \int_{-\tau_i}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{y}^T(s) W_i \dot{y}(s) ds d\lambda d\theta$$

The time derivative of $V(t)$ along the trajectory of system (6) is:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \quad (16)$$

Case 1: When $0 \leq \tau_1(t) \leq \tau_1$ and $0 \leq \tau_2(t) \leq \tau_2$. For Case 1, we have the following conditions. The time derivative of $V(t)$ is:

$$\dot{V}(t) \leq \xi^T(t) \left[\Omega + \Psi + \Phi\right] \xi(t) \quad (17)$$

where $\xi(t)$ is the augmented state vector. According to Lemma 3, we have:

$$-\int_{t-\tau(t)}^{t} \dot{y}^T(s) W \dot{y}(s) ds \leq \xi^T(t) \left[\frac{\tau(t)}{\tau} W_1 + \frac{\tau - \tau(t)}{\tau} W_2\right] \xi(t) \quad (18)$$

Therefore, for any diagonal matrices M_i $(i = 1, 2, \dots, 6)$ with appropriate dimensions, according to Lemma 1, we obtain:

$$\dot{V}(t) \leq \xi^T(t) \Psi \xi(t) \quad (19)$$

Thus, there exists a positive parameter ε satisfying:

$$\dot{V}(t) \leq -\kappa \|y(t)\|^2 \quad (20)$$

for a sufficiently small $\kappa > 0$. Moreover, for any matrices S_1, S_2, the following equality holds:

$$2\xi^T(t) S_1 \left[-Dy(t) + Ag(y(t)) + Bg(y(t - \tau(t)))\right] = 0 \quad (21)$$

Combining the above results, we obtain $\dot{V}(t) < 0$.

Case 2: When $\tau_1(t) > \tau_1$ or $\tau_2(t) > \tau_2$. The discussion for Case 2 is similar to Case 1. By introducing parameter σ in the neuron activation function and considering two subintervals according to (7), we can easily obtain:

$$\dot{V}(t) \leq \xi^T(t) \Psi \xi(t) < 0 \quad (22)$$

Therefore, based on (24), (26), and the LMI condition, we have $\dot{V}(t) < 0$, which implies that for a sufficiently small parameter $\kappa > 0$, $\dot{V}(t) \leq -\kappa \|y(t)\|^2$. This proves that the uncertain neural network (6) is globally asymptotically stable.

Corollary 1: If Assumptions 1, 2, and 3 hold, then for any positive scalars τ_1, τ_2, and μ, the uncertain neural network (8) is globally asymptotically stable and strictly (Q, S, R)-γ-dissipative if there exist positive-definite matrices $P_i \in \mathbb{R}^{n\times n}$ $(i = 1, 2, \dots, 12)$, positive diagonal matrices $Q_i \in \mathbb{R}^{n\times n}$ $(i = 1, 2, 3)$, arbitrary matrices $M_k \in \mathbb{R}^{n\times n}$ $(k = 1, 2, \dots, 12)$, and a positive scalar ε such that the following LMI holds:

$$\begin{bmatrix} \Psi & \Gamma \\ * & \Phi \end{bmatrix} < 0 \quad (23)$$

Proof: For dissipativity analysis, we use an LKF similar to (15) and define the dissipativity performance index for system (8) as:

$$J(\gamma, t_f) = \int_0^{t_f} \left[z^T(t)Qz(t) + 2z^T(t)Sw(t) + w^T(t)Rw(t) - \gamma w^T(t)w(t) \right] dt \quad (24)$$

Following the proof of Theorem 1 for Case 1 and Case 2, we obtain:

$$\dot{V}(t) - J(\gamma, t) \leq \xi^T(t) \left[\Psi + \Gamma \Phi^{-1} \Gamma^T \right] \xi(t) < 0 \quad (25)$$

Under zero initial conditions, this ensures that (10) holds, which means the uncertain neural network (8) is strictly (Q, S, R)-γ-dissipative, completing the proof.

For uncertain neural networks with each uncertain term in system (6), we can write:

$$\dot{y}(t) = -Dy(t) + Ag(y(t)) + Bg(y(t - \tau(t))) + Cp(t) \quad (31)$$

Corollary 2: If Assumptions 1 and 2 hold, then for any positive scalars τ_1, τ_2, and μ, the uncertain neural network (31) is globally asymptotically stable if there exist positive-definite matrices $P_i \in \mathbb{R}^{n\times n}$ $(i = 1, 2, \dots, 12)$, positive diagonal matrices $Q_i \in \mathbb{R}^{n\times n}$ $(i = 1, 2, 3)$, arbitrary matrices $M_k \in \mathbb{R}^{n\times n}$ $(k = 1, 2, \dots, 12)$, and a positive scalar ε such that LMI (14) holds.

3 Experimental Analysis

This section compares the results with those in [8,9,11,13,12].

Example 1: Consider the uncertain neural network (6) with the following parameters:

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & -0.5 \\ 0.1 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.4 \end{bmatrix}$$

Using the MATLAB LMI Toolbox, LMI (14) is found to be feasible. The maximum allowable bound can be obtained from Theorem 1. Consider activation functions $g_1(s) = 0.2(\tanh(s) + 1)$, $g_2(s) = 0.4(\tanh(s) + 1)$, with time-varying delay components $\tau_1(t) = 0.3+0.3\sin(t)$, $\tau_2(t) = 0.7+0.1\sin(t)$, and initial state $y(0) = [0.2, -0.2]^T$. The simulation results for different τ_1 and τ_2 are shown in Table 1.

Comparison between this paper and [11]

Example 2: Consider the uncertain neural network (8) with the following parameters:

$$D = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.2 \end{bmatrix}, \quad A = \begin{bmatrix} 0.5 & -0.5 \\ 0.1 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.4 \end{bmatrix}$$

For dissipativity performance, we select:

$$Q = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad S = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad R = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

with parameters $\tau_1 = 0.1$, $\tau_2 = 0.3$, $\delta = 0.3$, $u_1 = 0.2$, $u_2 = 0.1$. The optimal dissipativity performance level γ can be obtained from Corollary 1, as shown in Table 2.

Optimal dissipativity performance for different τ_2

Example 3: Consider the uncertain neural network (31) with the following parameters:

$$D = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad A = \begin{bmatrix} 0.2 & -0.5 \\ 0.1 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.4 \end{bmatrix}$$

Consider activation functions $g_1(s) = 0.2(\tanh(s)+1)$, $g_2(s) = 0.4(\tanh(s)+1)$, with time-varying delay components $\tau_1(t) = 0.4 + 0.4\sin(t)$, $\tau_2(t) = 1.835 + 0.1\sin(t)$, and initial state $y(0) = [0.5, -0.5]^T$. The global asymptotic stability simulation results are shown in Table 3.

Global asymptotic stability simulation results of example 3

4 Conclusion

This paper has extensively investigated the global asymptotic stability and dissipativity analysis for a class of uncertain continuous-time neural networks with time delays. By employing a more general activation function method, Lyapunov stability theory, and LMI techniques, new sufficient conditions were obtained in the form of linear matrix inequalities. Unlike previous results, a novel single integral inequality was adopted to handle the non-conservative derivative of the LKF. Finally, numerical simulations demonstrated the effectiveness of the research findings.

In practical engineering applications such as remote control and networked control systems, signals transmitted between two nodes can be dual-segmented. Systems considering two delay components are therefore significant. Traditional linear and optimal control precise mathematical models often ignore uncertainties, making it difficult for designed controllers to achieve expected performance. Consequently, the dissipativity theory studied in this paper provides an energy-related input/output description model for control system design and analysis, offering a more flexible theoretical foundation for modern control applications in robotics, active vibration damping, electromechanical systems, internal combustion engines, circuit theory, and other engineering directions. Future work will attempt to extend these results to neural networks with many discontinuous activations.

References

[1] Sainath T N, Weiss R J, Wilson K W, et al. Multichannel signal processing with deep neural networks for automatic speech recognition [J]. IEEE//ACM Trans on Audio, Speech, and Language Processing, 2017, 25 (5): 965-979.

[2] Song Hui, Dai Jiejie, Sheng Gehao, et al. GIS partial discharge pattern recognition via deep convolutional neural network under complex data source [J]. IEEE Trans on Dielectrics and Electrical Insulation, 2018, 25 (2): 702-709.

[3] Kalaam R N, Muyeen S M, Ahmed A D, et al. Optimisation of controller parameters for grid-tied photovoltaic system at faulty network using artificial neural network-based cuckoo search algorithm [J]. IET Renewable Power Generation, 2017, 11 (12): 1517-1526.

[4] Emary E, Zawbaa H M, Grosan C. Experienced gray wolf optimization through reinforcement learning and neural networks [J]. IEEE Trans on Neural Networks and Learning Systems, 2018, 29 (3): 681-694.

[5] Fu Caifen, Tan Wen. Decentralised load frequency control for power systems with communication delays via active disturbance rejection [J]. IET Generation,

Transmission & Distribution, 2018, 12 (6): 1397-1403.

[6] Le Van Hien, Le Huy V, Phat V N. Improved delay-dependent exponential stability of singular systems with mixed interval time-varying delays [J]. IET Control Theory & Applications, 2015, 9 (9): 1364-1372.

[7] Zhao Yu, Gao Huijun, Mou Shaoshuai. Asymptotic stability analysis of neural networks with successive time delay components [J]. Neurocomputing, 2008, 71: 2848-2856.

[8] Shao Hanyong, Han Qinglong. New delay-dependent stability criteria for neural networks with two additive time-varying delay components [J]. IEEE Trans on Neural Networks, 2011, 22 (5): 812-818.

[9] Zhang Chuanke, He Yong, Jiang L, et al. Delay-dependent stability criteria for generalized neural networks with two delay components [J]. IEEE Trans on Neural Networks, 2014, 25 (7): 1263-1276.

[10] Samidurai R, Manivannan R, Ahn C K, et al. New criteria for stability of generalized neural networks including Markov jump parameters and additive time delays [J]. IEEE Trans on Systems, Man, and Cybernetics: Systems, 2018, 48 (4): 485-499.

[11] Liu Yu, Lee S, Lee H. Robust delay-dependent stability criteria for uncertain neural networks with two additive time-varying delay components [J]. Neurocomputing, 2015, 151: 770-775.

[12] Ding Liming, He Yong, Liao Yiwei, et al. New result for generalized neural networks with additive time-varying delays using free-matrix-based integral inequality method [J]. Neurocomputing, 2017, 238: 205-211.

[13] Muthukumar P, Subramanian K. Stability criteria for Markovian jump neural networks with modedependent additive time-varying delays via quadratic convex combination [J]. Neurocomputing, 2016, 205: 75-83.

[14] Samidurai R, Rajavel S, Sriraman R, et al. Novel results on stability analysis of neutral-type neural networks with additive time-varying delay components and leakage delay [J]. International Journal Control Autom., 2017, 15: 1-13.

[15] Hill D, Moylan P. The stability of nonlinear dissipative systems [J]. IEEE Trans on Automatic Control, 1976, 21 (5): 708-711.

[16] Feng Zhiguang, Lam J. Stability and dissipativity analysis of distributed delay cellular neural networks [J]. IEEE Trans on Neural Networks, 2011, 22 (6): 976-981.

[17] Wu Zhengguang, Lam J, Su H, et al. Stability and dissipativity analysis of static neural networks with time delay [J]. IEEE Trans on Neural Networks and Learning Systems, 2012, 23 (2): 199-210.

[18] Gu K, Kharitonov V, Chen J. Stability of time-delay systems [M]. [S. l.]: Birkhauser, 2003.

[19] Wang Xin, She Kun, Zhong Shouming, et al. On extended dissipativity analysis for neural networks with time-varying delay and general activation functions [J]. Advances in Difference Equations, 2016, 79: 1-16.

Note: Figure translations are in progress. See original paper for figures.