

Postprint: Multi-level Combination Synchronization of Eight Chaotic Systems Based on Adaptive Control

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Date: 2018-11-29T00:00:00+00:00

Abstract

To address the issues of single structural pattern and few system variables in traditional chaotic systems, a chaotic circuit system is designed. This circuit exhibits relatively complex dynamical behaviors, high sensitivity, and strong anti-interference capability. With the variation of circuit parameters, different dynamical behaviors of the system are analyzed and discussed in detail by combining adaptive stability criteria and chaotic motion theory; furthermore, the corresponding circuit is built according to the circuit diagram, and the dynamical behaviors of the circuit are observed using an oscilloscope, with its behaviors being consistent with the MATLAB simulation results, further demonstrating the feasibility and flexibility of the circuit. On this basis, the paper focuses on proposing and investigating multi-level combination synchronization based on this system, achieving multi-level combination synchronization among eight systems by constructing different controllers. Simulation results show that this synchronization scheme achieves excellent performance in terms of convergence speed and accuracy.

Full Text

Multi-stage Combination Synchronization of Eight Chaotic Systems Based on Adaptive Control

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Abstract

Traditional chaotic systems often suffer from simplistic structural patterns and limited system variables. To address these limitations, this paper proposes a

novel chaotic circuit system that exhibits complex dynamical behaviors, high sensitivity, and strong anti-interference capability. By varying the circuit parameters, we thoroughly analyze the system's diverse dynamical behaviors using adaptive stability criteria and chaos theory. Furthermore, we construct the corresponding physical circuit and observe its dynamical behaviors via oscilloscope, with experimental results matching MATLAB simulations and thereby validating the circuit's feasibility and flexibility. Building upon this foundation, we propose and investigate a multi-stage combination synchronization scheme based on the proposed system. Through the design of different controllers, we achieve multi-stage combination synchronization among eight chaotic systems. Simulation results demonstrate that the proposed synchronization scheme offers excellent convergence speed and precision.

Keywords: chaotic circuit; period; bifurcation; transient chaos; combination synchronization; system error; controller

0 Introduction

The control and application of chaotic systems have become a significant research direction in nonlinear science, particularly with the development of nonlinear control theory. Chaos-based secure communication, in particular, has garnered widespread attention in practical applications. In nonlinear dynamical systems, nonlinear functions induce chaotic behavior, and different nonlinear functions produce distinct dynamical characteristics and features. Common nonlinear functions include quadratic, cubic, piecewise linear, and signum functions, which appear in systems such as the Genesio system [1], Sprott system [2], Hénon system [3], Arneodo-Couillet system [4], Chua system [5], and Elwakil system [6]. However, many chaotic system design approaches are based on classical systems like the Lorenz system [7], Rössler system [8], and Chen system [9], which generally have simple structures and few variables. This limitation motivates the need to develop easily implementable circuits from a practical perspective.

The application of chaos synchronization to secure communication has become a hotly researched and competitive area in recent years [10, 11]. The primary objective of secure communication is to prevent transmitted signals from being intercepted while enabling effective signal recovery at the receiver. Chaotic signals possess special properties—including pseudo-randomness, high sensitivity to initial conditions and parameters, broadband spectrum, and long-term unpredictability—that meet the special requirements for secure communication signals. Consequently, chaos synchronization [12-14] holds promising potential for secure communication applications.

Specifically, chaos synchronization refers to the convergence of a response system's state toward that of a drive system. Current synchronization types include complete synchronization [15], phase synchronization [16], projective synchro-

nization [17], generalized synchronization [18, 19], lag synchronization [20], and partial synchronization [21]. However, most research focuses on single-drive, single-response synchronization schemes, with limited investigation into multi-drive, multi-response systems and combination synchronization among multiple systems [22–24]. This gap necessitates greater attention to multi-system research.

This paper designs a chaotic circuit system with complex trajectories, high sensitivity, and strong anti-interference capability. We focus on developing a multi-stage combination synchronization scheme for this system, achieving synchronization among eight systems through different controller designs. Experimental verification demonstrates the system's complex dynamical characteristics and confirms the accuracy and feasibility of the proposed synchronization scheme.

1 System Model and Analysis

1.1 Circuit Model

The chaotic system is represented by the following third-order equations:

$$\begin{cases} \dot{x} = ay \\ \dot{y} = -x + bxy + y + z \\ \dot{z} = -cy \end{cases}$$

where a , b , and c are system parameters, and x , y , z are state variables. When $a = 1$, $b = 1$, $c = 1$, the Lyapunov exponents of system (1) are shown in [Figure 1: see original paper], revealing $LE_1 > 0$, $LE_2 < 0$, and $LE_3 < 0$, indicating chaotic behavior. With initial conditions $(x(0), y(0), z(0)) = (1, 1, 1)$, the chaotic attractor of system (1) is displayed in [Figure 2: see original paper].

1.2 Influence of System Parameters

This section employs bifurcation diagrams, Lyapunov exponents, and other methods to examine how parameters affect the system's dynamical behavior.

1.2.1 Influence of Parameter a With parameter $a \in [0, 5]$, $b = 1$, $c = 1$, and initial conditions $(x(0), y(0), z(0)) = (1, 1, 1)$, the Lyapunov exponent diagram and corresponding bifurcation diagram are shown in [Figure 3: see original paper] and [Figure 4: see original paper]. Analysis reveals that when $a \in [0, 0.7]$, the maximum Lyapunov exponent $LE_1 > 0$, gradually decreasing toward zero. As shown in Figure 5: see original paper, when $a = 0.2$, system (1) exhibits chaotic phenomena. As a continues increasing in $[0.7, 5]$, the system re-enters chaos. By arbitrarily selecting values in $[0.7, 5]$, chaotic states can be observed, as illustrated in Figure 5: see original paper. To investigate the effect of initial

conditions, [Figure 5: see original paper] uses red curves for trajectories starting from $(0.8, 1, 1.4)$ and blue curves for those from $(1, 1, 1)$.

1.2.2 Influence of Parameter b With parameter $b \in [0, 5]$, $a = 1$, $c = 1$, and initial conditions $(x(0), y(0), z(0)) = (1, 1, 1)$, the Lyapunov exponent diagram and corresponding bifurcation diagram are shown in [Figure 6: see original paper] and [Figure 7: see original paper]. System (1) exhibits rich and complex dynamical behaviors as b varies. [Figure 6: see original paper] shows that as b increases, the maximum Lyapunov exponent LE_1 transitions from positive to zero, indicating a shift from chaotic to periodic behavior. When $b \in [0.1, 0.2]$ and $b \in [2.6, 2.9]$, $LE_1 < 0$, and period-doubling phenomena occur, as shown in Figure 8: see original paper. Chaotic behavior primarily concentrates in three intervals: $[0, 0.1] \cup [0.2, 2.6] \cup [2.9, 5]$, as depicted in Figure 8: see original paper. Here, red curves represent trajectories from initial condition $(1.5, 2.1, 1.3)$, while blue curves represent those from $(1, 1, 1)$.

1.2.3 Influence of Parameter c With parameter $c \in [0.7, 5]$, $a = 1$, $b = 1$, and initial conditions $(x(0), y(0), z(0)) = (1, 1, 1)$, the Lyapunov exponent diagram and corresponding bifurcation diagram are shown in [Figure 9: see original paper] and [Figure 10: see original paper]. Analysis indicates that the maximum Lyapunov exponent LE_1 transitions from zero to positive, revealing a transient phase at the starting point. As c increases in $[0.7, 1.4]$, system (1) enters and maintains chaos, as shown in Figure 11: see original paper. When $c \in [1.4, 1.9]$, $LE_1 = 0$, and the system undergoes period-doubling, as shown in Figure 11: see original paper. For $c \in [1.9, 5]$, LE_1 becomes positive again, and the system re-enters chaos. In [Figure 11: see original paper], red curves represent trajectories from $(0.5, 1, 2.2)$, while blue curves represent those from $(1, 1, 1)$.

2 Circuit Implementation and Validation

Using Multisim, we constructed the circuit diagram for system (1) to realize its dynamical characteristics and observed the phenomena via oscilloscope. As shown in [Figure 12: see original paper], the chaotic circuit consists of resistors, capacitors, operational amplifiers, and multipliers. Operational amplifiers with resistors and capacitors implement integration, while operational amplifiers with resistors perform scaling and addition operations. Multipliers enable multiplication between signals.

Based on the component selection and Multisim circuit diagram, we built the physical circuit and used an oscilloscope to obtain the corresponding chaotic attractors for system (1), as shown in [Figure 13: see original paper]. The oscilloscope results in [Figure 14: see original paper] clearly match the MATLAB simulation in [Figure 2: see original paper], validating the correctness and feasibility of chaotic system (1).

With $a = 1$, $b = 1$, $c = 1$, system (1) can be constructed as:

$$\begin{cases} \dot{x} = \frac{1}{R_5 C_1} y \\ \dot{y} = -\frac{1}{R_3 C_2} x + \frac{1}{R_{12} C_2} xy + \frac{1}{R_8 C_2} y + \frac{1}{R_{11} C_2} z \\ \dot{z} = -\frac{1}{R_6 C_3} y \end{cases}$$

Comparing (1) and (2) and equating corresponding coefficients yields the following component values: $R_1 = R_8 = R_{14} = 100 \text{ k}\Omega$; $R_2 = R_3 = R_4 = R_5 = R_6 = 10 \text{ k}\Omega$; $R_7 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = 10 \text{ k}\Omega$; and capacitance values $C_1 = C_2 = C_3 = 10 \text{ nF}$.

3 Multi-stage Combination Synchronization

Based on stability theory, consider multiple chaotic systems with controllers:

$$\dot{x}_i = f_i(x_i) + u_i, \quad i = 1, 2 \quad (3)$$

$$\dot{y}_i = g_i(y_i) + v_i, \quad i = 1, 2 \quad (4)$$

$$\dot{z}_i = h_i(z_i) + w_i, \quad i = 1, 2 \quad (5)$$

$$\dot{\psi}_i = \sigma_i(\psi_i) + \xi_i, \quad i = 1, 2 \quad (6)$$

where $x_i, y_i, z_i, \psi_i \in \mathbb{R}^n$ are state vectors of systems (3)-(6), f_i, g_i, h_i, σ_i are continuous functions, and u_i, v_i, w_i, ξ_i are controllers to be designed.

Definition 1. Define the synchronization errors between combined systems (3)(4) and (5)(6), between (5)(6) and (7)(8), between (7)(8) and (9)(10), and between (9)(10) and (3)(4) as:

$$\begin{aligned} e_1 &= k_1 x_1 + l_1 x_2 - k_2 y_1 - l_2 y_2 \\ e_2 &= l_1 x_1 - m_1 x_2 - l_2 y_1 + m_2 y_2 \\ e_3 &= m_1 x_1 - p_1 x_2 - m_2 y_1 + p_2 y_2 \\ e_4 &= p_1 x_1 + k_1 x_2 - p_2 y_1 - k_2 y_2 \end{aligned}$$

where k_i, l_i, m_i, p_i ($i = 1, 2$) are constant coefficients. If $\lim_{t \rightarrow \infty} e_i = 0$ ($i = 1, 2, 3, 4$), then combined systems (3)(4) and (5)(6) achieve synchronization, and similarly for the other combinations.

For concreteness, using system (1) as the basis, consider eight three-dimensional systems with controllers:

$$\dot{x}_{1j} = x_{1,j+1} + u_{1j}, \quad j = 1, 2 \quad (12)$$

$$\dot{x}_{2j} = -x_{1j} + x_{1j}x_{2j} + x_{2j} + x_{3j} + u_{2j}, \quad j = 1, 2 \quad (13)$$

$$\dot{x}_{3j} = -x_{2j} + u_{3j}, \quad j = 1, 2 \quad (14)$$

$$\dot{y}_{1j} = y_{1,j+1} + v_{1j}, \quad j = 1, 2 \quad (15)$$

$$\dot{y}_{2j} = -y_{1j} + y_{1j}y_{2j} + y_{2j} + y_{3j} + v_{2j}, \quad j = 1, 2 \quad (16)$$

$$\dot{y}_{3j} = -y_{2j} + v_{3j}, \quad j = 1, 2 \quad (17)$$

$$\dot{z}_{1j} = z_{1,j+1} + w_{1j}, \quad j = 1, 2 \quad (18)$$

$$\dot{z}_{2j} = -z_{1j} + z_{1j}z_{2j} + z_{2j} + z_{3j} + w_{2j}, \quad j = 1, 2 \quad (19)$$

$$\dot{z}_{3j} = -z_{2j} + w_{3j}, \quad j = 1, 2 \quad (20)$$

$$\dot{\psi}_{1j} = \psi_{1,j+1} + \xi_{1j}, \quad j = 1, 2 \quad (21)$$

$$\dot{\psi}_{2j} = -\psi_{1j} + \psi_{1j}\psi_{2j} + \psi_{2j} + \psi_{3j} + \xi_{2j}, \quad j = 1, 2 \quad (22)$$

where $u_{ij}, v_{ij}, w_{ij}, \xi_{ij}$ ($i = 1, 2, 3; j = 1, 2$) are controllers to be designed.

The multi-stage combination synchronization controllers are designed as:

$$U_1 = u_{11} + u_{12} - v_{11} - v_{12}$$

$$U_2 = u_{21} + u_{22} - v_{21} - v_{22}$$

$$U_3 = u_{31} + u_{32} - v_{31} - v_{32}$$

$$U_4 = v_{11} + v_{12} - w_{11} - w_{12}$$

$$U_5 = v_{21} + v_{22} - w_{21} - w_{22}$$

$$U_6 = v_{31} + v_{32} - w_{31} - w_{32}$$

$$U_7 = w_{11} + w_{12} - \xi_{11} - \xi_{12}$$

$$U_8 = w_{21} + w_{22} - \xi_{21} - \xi_{22}$$

$$U_9 = w_{31} + w_{32} - \xi_{31} - \xi_{32}$$

$$U_{10} = \xi_{11} + \xi_{12} - u_{11} - u_{12}$$

$$U_{11} = \xi_{21} + \xi_{22} - u_{21} - u_{22}$$

$$U_{12} = \xi_{31} + \xi_{32} - u_{31} - u_{32}$$

Theorem 1. If the combination controllers for system error (11) satisfy conditions (23)-(25), then systems (12)-(19) achieve multi-stage combination synchronization.

Proof. Construct the following Lyapunov function:

$$V(t) = \frac{1}{2} \sum_{i=1}^{12} e_i^2 \quad (26)$$

Differentiating with respect to time t yields:

$$\dot{V}(t) = \sum_{i=1}^{12} e_i \dot{e}_i = e_{11} \dot{e}_{11} + e_{12} \dot{e}_{12} + \dots + e_{42} \dot{e}_{42} \quad (27)$$

From systems (12)-(19), we obtain:

$$\begin{aligned} \dot{e}_{11} &= k_1 \dot{x}_{11} + l_1 \dot{x}_{21} - k_2 \dot{y}_{11} - l_2 \dot{y}_{21} \\ &= k_1(x_{12} + u_{11}) + l_1(-x_{11} + x_{11}x_{21} + x_{21} + x_{31} + u_{21}) \\ &\quad - k_2(y_{12} + v_{11}) - l_2(-y_{11} + y_{11}y_{21} + y_{21} + y_{31} + v_{21}) \end{aligned}$$

Similar expressions can be derived for $\dot{e}_{12}, \dot{e}_{13}, \dots, \dot{e}_{42}$. Substituting the controllers (20)-(22) and simplifying using conditions (23)-(25) yields:

$$\dot{V}(t) = -\alpha_1 e_{11}^2 - \beta_1 e_{12}^2 - \gamma_1 e_{13}^2 - \alpha_2 e_{21}^2 - \beta_2 e_{22}^2 - \gamma_2 e_{23}^2 - \alpha_3 e_{31}^2 - \beta_3 e_{32}^2 - \gamma_3 e_{33}^2 - \alpha_4 e_{41}^2 - \beta_4 e_{42}^2 - \gamma_4 e_{43}^2 \leq 0 \quad (34)$$

Since $\alpha_i, \beta_i, \gamma_i$ ($i = 1, 2, 3, 4$) are positive constants, $\dot{V}(t)$ is non-positive. According to Lyapunov stability theory, the system errors (11) converge asymptotically, i.e., $\lim_{t \rightarrow \infty} e_i = 0$. This completes the proof.

The error convergence diagram is shown in [Figure 14: see original paper].

4 Simulation Results

Simulation experiments verify the theoretical correctness. Assuming the parameters and initial conditions are given as specified, the initial error values calculated from (9) are $(e_{11}(0), e_{12}(0), e_{13}(0), e_{14}(0)) = (1, -1, 1, -1)$ and $(e_{21}(0), e_{22}(0), e_{23}(0), e_{24}(0)) = (3, -1, 0, -2)$. The simulation results confirm the effectiveness of the proposed synchronization scheme.

5 Conclusion

This paper designs a novel chaotic circuit and analyzes its complex dynamical behaviors through adaptive stability criteria. The influence of system parameters on dynamical behavior is discussed in detail, revealing excellent chaotic characteristics. Furthermore, we propose and rigorously prove a new synchronization scheme—multi-stage combination synchronization—achieving synchronization among eight systems through different controller designs and error analysis. Experimental results demonstrate the feasibility of the proposed approach.

References

- [1] Luo Runzi, Zeng Yanhui. The control and synchronization of fractional-order Genesio-Tesi system [J]. *Nonlinear Dynamics*, 2017, 88(3): 1-11.
- [2] Lai Qiang, Chen Shiming. Generating multiple chaotic attractors from spott B system [J]. *International Journal of Bifurcation and Chaos*, 2016, 26(11): 1650177.
- [3] Song Bingbing, Ding Qun. Comparisons of typical discrete logistic map and henon map [J]. *Intelligent Data Analysis and Its Applications*, 2014, 297: 267-275.
- [4] Vaidyanathan S. Output regulation of Arneodo-Coulet chaotic system [C]//*Proc of International Conference on Computer Science & Information Technology*. 2011: 98-107.
- [5] Vaidyanathan S, Rasappan S. Global chaos synchronization of n-scroll Chua circuit and Lur' e system using backstepping control design with recursive feedback [J]. *Arabian Journal for Science and Engineering*, 2014, 39(4): 3351-3364.
- [6] Elwakil A. Fractional-order circuits and systems: an emerging interdisciplinary research area [J]. *IEEE Circuits and Systems Magazine*, 2010, 10(4): 40-50.
- [7] Li Chunbiao, Julienclinton S. Coexisting hidden attractors in a 4D simplified lorenz system [J]. *International Journal of Bifurcation and Chaos*, 2014, 24(3): 739-748.
- [8] Frasca M, Gambuzza L V, Buscarino A, et al. Memristor based adaptive coupling for synchronization of two Rössler systems [J]. *Advances in Neural Networks: Computational and Theoretical Issues*, 2015, 37: 39-46.
- [9] Zhang Fuchen, Liao Xiaofeng, Mu Chunlai, et al. On global boundedness of the Chen system [J]. *Discrete and Continuous Dynamical Systems-Series B*, 2017, 22(4): 1673-1681.
- [10] Abd M H, Tahir F R, Al-Suhail G, et al. An adaptive observer synchronization using chaotic time-delay system for secure communication [J]. *Nonlinear Dynamics*, 2017, 90(4): 2583-2598.
- [11] Zhao Jianfeng, Wang Shuying, Chang Yingxiang, et al. A novel image encryption scheme based on an improper fractional-order chaotic system [J]. *Nonlinear Dynamics*, 2015, 80(4): 1721-1729.
- [12] Wang Sha, Yu Yongguang, Wen Guoguang. Hybrid projective synchronization of time-delayed fractional order chaotic systems [J]. *Nonlinear Analysis: Hybrid Systems*, 2014, 11: 129-138.
- [13] 李震波, 唐驾时. 参数扰动下的混沌同步控制及其保密通信方案 [J]. *控制理论与应用*, 2014, 31(5): 592-600. (Li Zhengbo, Tang Jiashi. Chaotic synchronization

control under parametric perturbation and its secure communication scheme [J]. Control Theory and Application, 2014, 31(5): 592-600.)

[14] 潘光, 魏静. 一种分数阶混沌系统同步的自适应滑模控制器设计 [J]. 物理学报, 2015, 64(4): 040501-040507. (Pan Guang, Wei Jing. Design of an adaptive sliding mode controller for synchronization of fractional order chaotic systems [J]. Acta Physica Sinica, 2015, 64(4): 040501-040507.)

[15] Pecora Louism, Carrol Thomas L. Synchronization in chaotic systems [J]. Chaos, 2015, 25: 097611-1-097611-11.

[16] Wu Xiaofeng, Cai Jianping, Wang Muhong. Robust synchronization of chaotic horizontal platform systems with phase difference [J]. Journal of Sound and Vibration, 2007, 305(3): 481-491.

[17] Du Hongyue, Zeng Qianghuang, Wang Changhong. Modified function projective synchronization of chaotic system [J]. Chaos, Solitons, Fractals, 2009, 42(4): 2399-2404.

[18] Wang Shibing, Wang Xingyuan, Han Bo. Complex Generalized Synchronization and Parameter Identification of Nonidentical Nonlinear Complex Systems [J]. International Journal of Modern Physics B, 2016, 27(30): 1350195.

[19] 韦相, 赵军产. 参数未知耦合时滞不同复杂网络的广义同步 [J]. 控制理论与应用, 2016, 33(6): 825-831. (Wei Xiang, Zhao Junchan. Generalized synchronization of complex networks with unknown coupling parameters and coupled delays [J]. Control Theory and Application, 2016, 33(6): 825-831.)

[20] Xia Yonghui, Yang Zijing, Han Maoan. Lag synchronization of unknown chaotic delayed Yang-Yang-Type fuzzy neural networks with noise perturbation based on adaptive control and parameter identification [J]. IEEE Trans on Neural Networks, 2009, 20(7): 1165-1180.

[21] Wu Wei, Chen Tianping. Partial synchronization linearly and symmetrically coupled ordinary differential systems [J]. Physica D: Nonlinear Phenomena, 2009, 238(4): 355-364.

[22] 张檬, 韩敏. 基于滑模控制法的不确定复杂网络间有限时间组合同步 [J]. 控制与决策, 2017, 32(8): 1553-1536. (Zhang Meng, Han Min. Finite time combination synchronization of uncertain complex networks based on sliding mode control [J]. Control and Decision, 2017, 32(8): 1553-1536.)

[23] Luo Runzi, Wang Yinglan, Deng Shucheng. Combination synchronization of three classic chaotic systems using active backs stepping design [J]. Chaos, 2011, 21: 043114-1-043114-6.

[24] Sun Junwei, Shen Yi, Wang Xiaoping, et al. Finite-time combination-combination synchronization of four different chaotic systems with unknown parameters via sliding mode control [J]. Nonlinear Dynamics, 2014, 76(1): 383-397.

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