Blind Source Separation Algorithm Based on Givens Transformation and Second-Order Oscillation W-C-PSO Optimization (Postprint)

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Abstract

To address the issues of intelligent algorithms easily falling into local optima and exhibiting slow convergence when implementing blind source separation, a blind source separation algorithm based on Givens transformation and second-order oscillating particle swarm optimization is proposed. The algorithm first establishes a functional relationship between the inertia weight and learning factor parameters, enabling them to jointly regulate the iterative process and thereby enhance the algorithm' s overall integrity and global search capability. Subsequently, a second-order oscillation component is introduced to increase population diversity, making the algorithm less susceptible to local optima. Additionally, Givens transformation is employed to represent the separation matrix using rotation angles, which reduces computational complexity. Simulation results demonstrate that the algorithm can effectively achieve blind separation of both mechanical vibration signals and speech signals, offering faster convergence speed and superior separation performance compared to other algorithms.

Full Text

Preamble

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Abstract: Intelligent algorithms for blind source separation (BSS) often suffer from slow convergence and susceptibility to local optima. To address these issues, this paper proposes a BSS algorithm based on Givens transformation and second-order oscillatory particle swarm optimization (PSO). The algorithm constructs a functional relationship between inertia weight and learning factors to jointly regulate iteration, thereby enhancing overall algorithmic integrity and global search capability. A second-order oscillation term is introduced to increase population diversity and prevent premature convergence to local optima. Additionally, Givens transformation is employed to represent the separation matrix using rotation angles, reducing computational complexity. Simulation results demonstrate that the proposed algorithm effectively achieves blind separation of mechanical vibration signals and speech signals, offering faster convergence speed and superior separation performance compared to alternative methods.

Keywords: blind source separation; particle swarm optimization; second-order oscillatory particle swarm; Givens transformation

0 Introduction

Blind source separation (BSS) refers to the process of extracting unknown source signals from their mixtures when both the sources and transmission channels are unknown. It has important applications in parameter estimation, various signal analyses, and fault detection [1,2]. Traditional BSS algorithms have been well studied, including FastICA, natural gradient methods, and equivariant adaptive algorithms. These methods improve convergence speed through adaptive variable step sizes [3] and enhance steady-state error by adding momentum terms [4]. However, the selection of nonlinear functions based on source statistical characteristics involves many influencing factors, which contradicts the fundamental premise that source signal information is unknown.

Recently, more researchers have applied intelligent algorithms to BSS to address these challenges [5,6]. Reference [7] proposed the basic Particle Swarm Optimization (PSO) algorithm, which offers faster convergence, stronger global search capability, and a simpler iterative process compared to other intelligent algorithms such as ant colony and bee colony algorithms. Reference [8] introduced a PSO algorithm with adaptive inertia weight adjustment, which, although solving the nonlinear function selection problem, did not significantly improve convergence and remained prone to local optima. Reference [9] employed an adaptive genetic mutation mechanism to increase population diversity and improve separation performance, but the algorithm' s complexity increased substantially due to the need to calculate relationships between mutation probability and offspring.

To overcome these limitations, this paper proposes a blind source separation algorithm based on Givens transformation and second-order oscillatory W-C-PSO optimization. This method uses PSO to optimize the objective function by nonlinearly combining inertia weight and learning factors to jointly regulate particle iteration, thereby improving algorithmic integrity and global convergence capability. Second, a second-order oscillation mechanism is introduced to enhance particle diversity, making it easier for particles to escape local optima. Additionally, Givens transformation represents the separation matrix using rotation angles, reducing computational complexity and improving convergence speed. Experimental results on vibration and speech signals demonstrate that the proposed algorithm offers better convergence speed and global search capability compared to other algorithms, effectively solving the local optima problem.

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1 Blind Source Separation Model

The BSS model can be described as follows: Assuming n source signals and m sensors, the relationship between observed signals and source signals is expressed as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$$

where $\mathbf{x}(t) = [x_1, x_2, ..., x_m]^T$ is the m-dimensional observed data vector, $\mathbf{s}(t) = [s_1, s_2, ..., s_n]^T$ is the n-dimensional source signal vector, and $\mathbf{A}$ is the $m \times n$ mixing matrix. Background noise is not considered in this paper.

When both channel properties and source signals are unknown, the optimal separation matrix $\mathbf{W}(t)$ is obtained solely from observed data $\mathbf{x}(t)$ using appropriate learning algorithms, such that the algorithm output:

$$\mathbf{y}(t) = \mathbf{W}(t)\mathbf{x}(t)$$

represents the best estimate of the source signals.

Thus, the BSS process is transformed into an optimization problem where the separation matrix $\mathbf{W}$ is sought under a certain independence criterion to achieve mutual independence among components. This paper employs negentropy [10] as the basis for establishing the objective function to measure the independence of separated signals:

$$J(\mathbf{y}) = \sum_{i=1}^{n} k_i(y_i)$$

where the kurtosis $k_i(y_i) = E\{y_i^4\} - 3(E\{y_i^2\})^2$. Larger kurtosis values indicate higher statistical independence among separated signals, meaning better separation quality.

2.1 Basic Particle Swarm Algorithm

The PSO algorithm calculates fitness values based on the objective function, enabling particles to continuously update their velocities and positions to find the global optimum. The velocity and position update formulas are:

$$v_{i,j}(t+1) = v_{i,j}(t) + c_1 r_1 [p_{i,j} - x_{i,j}(t)] + c_2 r_2 [p_{g,j} - x_{i,j}(t)]$$

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1)$$

where c_1 and c_2 are the cognitive and social learning factors respectively; r_1 and r_2 are random numbers distributed in $[0,1]$; D is the dimension of the particle search space; $\mathbf{x}_i$ is the position of the i-th particle; $\mathbf{v}_i$ is the velocity of the i-th particle; $\mathbf{p}_i$ is the individual best position; and $\mathbf{p}_g$ is the global best position of the current population.

2.2 Improved Particle Swarm Algorithm

In traditional PSO, learning factors c_1 and c_2 are constants. Reference [11] introduced inertia weight ω into PSO to effectively balance convergence speed and search capability:

$$v_{i,j}(t+1) = \omega v_{i,j}(t) + c_1 r_1 [p_{i,j} - x_{i,j}(t)] + c_2 r_2 [p_{g,j} - x_{i,j}(t)]$$

The inertia weight typically uses a linear decreasing form:

$$\omega(t) = \omega_{\max} - \frac{(\omega_{\max} - \omega_{\min})t}{N_{\max}}$$

where t is the iteration number, $N_{\max}$ is the maximum number of iterations, $\omega_{\max}$ is the maximum inertia weight, and $\omega_{\min}$ is the minimum inertia weight.

While inertia weight improves search capability, equation (4) shows that learning factors c_1 and c_2 regulate step sizes toward the particle' s own best direction and global best position, respectively. Adjusting learning factors can prevent local optima and improve search performance. However, previous improvements treated these parameters independently, reducing PSO' s integrity and hindering global search. Therefore, this paper proposes nonlinear combination of these parameters to enhance overall performance:

$$\begin{cases} c_1(t) = \gamma + \eta + \varepsilon\omega(t) \\ c_2(t) = \gamma - \varepsilon\omega(t) \end{cases}$$

where γ, η, and ε are constant coefficients. This is the proposed W-C-PSO algorithm.

The inertia weight uses an exponential decreasing form:

$$\omega(t) = \omega_{\min} + (\omega_{\max} - \omega_{\min}) \exp\left(-\frac{20t}{T_{\max}}\right)$$

In PSO algorithms, population diversity decreases during search, hindering further exploration. Therefore, this paper introduces second-order oscillation into W-C-PSO, enabling particles to move toward better positions in the next iteration and enhancing diversity. When the search range expands, second-order oscillation helps particles escape local optima and improves global search capability.

The improved velocity iteration with second-order oscillation is:

$$\begin{aligned} v_{i,j}(t+1) = \omega(t)v_{i,j}(t) + c_1(t)r_1\{p_{i,j} - [(1+\xi_1)x_{i,j}(t) - \xi_1 x_{i,j}(t-1)]\} \\ + c_2(t)r_2\{p_{g,j} - [(1+\xi_2)x_{i,j}(t) - \xi_2 x_{i,j}(t-1)]\} \end{aligned}$$

where ξ_1 and ξ_2 are random numbers. During early iterations ($t < N_{\max}/2$), selecting $\xi_1, \xi_2 < 1$ enhances global search capability; during later iterations ($t \geq N_{\max}/2$), selecting $\xi_1, \xi_2 \geq 1$ accelerates convergence.

3.1 Givens Transformation

Equation (3) shows that processing an n-order matrix requires solving n^2 unknown parameters, involving substantial computation. To address this, QR decomposition is applied to the mixing matrix. Let the whitening matrix be $\mathbf{P}$. Using QR decomposition, $\mathbf{P} = \mathbf{QR}$, where $\mathbf{Q}$ is a unitary matrix and $\mathbf{R}$ is an upper triangular matrix. After whitening:

$$\mathbf{z}(t) = \mathbf{P}\mathbf{x}(t) = \mathbf{P}\mathbf{A}\mathbf{s}(t) = \mathbf{QR}\mathbf{s}(t)$$

Since source signals are uncorrelated, $E\{\mathbf{s}(t)\mathbf{s}^T(t)\} = \mathbf{I}$, making $\mathbf{R}$ a unitary diagonal matrix. Thus:

$$\mathbf{z}(t) = \mathbf{QR}\mathbf{s}(t)$$

As separated signals are proportional to source signals, $\mathbf{R}$ can be regarded as an estimate of the source signals (i.e., the separated signals), and $\mathbf{Q}$ can be considered as an estimate of the separation matrix, meaning the separation matrix is orthogonal.

Any n-order orthogonal matrix with determinant not equal to 1 can be transformed into a product of Givens rotation matrices:

$$\mathbf{G}_{ij}(\theta_{ij}) = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & \cos\theta_{ij} & \cdots & \sin\theta_{ij} & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -\sin\theta_{ij} & \cdots & \cos\theta_{ij} & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

The separation matrix is:

$$\mathbf{W}(t) = \mathbf{G}_1\mathbf{G}_2 \cdots \mathbf{G}_{\frac{n(n-1)}{2}}$$

After Givens transformation, each Givens matrix contains one parameter: $\mathbf{G}_1$ has $n-1$ parameters, $\mathbf{G}_2$ has $n-2$, and so on, with $\mathbf{G}_{n-1}$ having 1 parameter. Therefore, the number of parameters in $\mathbf{W}(t)$ is:

$$(n-1) + (n-2) + \cdots + 1 = \frac{n(n-1)}{2}$$

Without Givens transformation, $\mathbf{W}(t)$ would have n^2 parameters. The parameter reduction is:

$$n^2 - \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$$

This not only ensures the orthogonality of $\mathbf{W}(t)$ but also halves the number of unknown parameters, reducing computational complexity.

3.2 Blind Source Separation Based on Second-Order Oscillation W-C-PSO

The second-order oscillatory W-C-PSO algorithm uses particle positions to represent the orthogonal separation matrix. After Givens transformation, the parameter representation reduces particle search dimension from n^2 to $n(n-1)/2$, improving efficiency. The combination of inertia weight and learning factors enhances global search capability, while the second-order oscillation mechanism increases population diversity, preventing local optima. The algorithm effectively achieves blind separation of dynamic and speech signals. The procedure is shown in Figure 1 [Figure 1: see original paper].

Algorithm Steps: a) Perform centering and whitening on observed dynamic mixed signals $\mathbf{x}(t)$. b) Initialize learning factors c_1, c_2 and inertia weight ω, set $\omega_{\max}$ and $\omega_{\min}$, randomly initialize separation matrices as initial particle positions, and represent them using Givens-transformed parameters. c) Calculate fitness values using equation (3). d) Compare each particle' s fitness with its individual optimum $\mathbf{p}_i$ and global optimum $\mathbf{p}_g$, and update them. e) Adaptively update ω, c_1, and c_2 using equations (8) and (9). f) Introduce second-order oscillation and update particle velocities and positions using equations (10) and (5). g) Iterate until termination conditions are met, then output separation results; otherwise return to step c).

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4 Simulation Experiments

To verify the effectiveness of the proposed Givens transformation and second-order oscillatory W-C-PSO algorithm, blind separation experiments were conducted on mechanical vibration and speech signals. A time-varying mixing matrix $\mathbf{A}(t) = \mathbf{A}_0 + \alpha \cdot \mathrm{rand}(n)$ was used, where $\mathbf{A}_0$ is initialized as a random matrix with the same dimensions as $\mathbf{A}$, meaning the mixed signals are randomly and dynamically changing.

4.1 Vibration Signal Blind Separation Experiment

Simulated mechanical vibration signals were used as source signals. Two source signals were constructed based on the vibration signal model [13]:

$$\begin{cases} s_1(t) = 1.5\cos(2\pi \cdot 250t) + 2\sin(\pi \cdot t) \\ s_2(t) = \sin(2\pi \cdot 150t \cdot [1 + \sin(20\pi t)]) \end{cases}$$

Algorithm parameters: population size = 20, $\omega_{\max} = 0.9$, $\omega_{\min} = 0.4$, $\gamma = 0.5$, $\eta = 1$, $\varepsilon = 0.5$, maximum iterations = 300. Results are shown in Figures 2 [Figure 2: see original paper] to 4 [Figure 4: see original paper].

Comparing source signals (Figure 2) with separated signals (Figure 4), the time-domain waveforms are essentially identical, and frequency spectra show maximum peaks at the same frequencies, demonstrating effective blind separation of mechanical vibration signals.

4.2 Speech Signal Blind Separation Experiment

To verify practical effectiveness, blind source separation was performed on two speech signals with the same parameters as Section 4.1. Results are shown in Figures 5 [Figure 5: see original paper] to 8 [Figure 8: see original paper].

Comparing Figures 5 and 7, waveforms confirm successful separation. Based on audible sound quality, amplitude ambiguity is negligible. Figure 8 [Figure 8:

see original paper] shows the block scatter plot of separated signals, where the two straight lines indicate high similarity between source and separated signals.

4.3 Algorithm Comparison

1) Convergence and Search Capability The Performance Index (PI) curve [13] is used as a metric:

$$\mathrm{PI}(k) = \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{|G_{ij}(k)|}{\max_j |G_{ij}(k)|} - 1\right) + \sum_{j=1}^{n}\left(\sum_{i=1}^{n}\frac{|G_{ij}(k)|}{\max_i |G_{ij}(k)|} - 1\right)$$

where k is iteration number and $G_{ij}(k)$ is the i, j-th element of the global matrix at k-th iteration.

Figure 9 [Figure 9: see original paper] compares convergence and global search capability among basic PSO, W-C-PSO, and the proposed algorithm. Basic PSO converges prematurely to local optima. W-C-PSO improves global search but reduces convergence speed. The proposed algorithm, using Givens transformation and second-order oscillation, improves both convergence speed and global search capability, easily escaping local optima.

Figure 10 [Figure 10: see original paper] compares with other improved PSO algorithms. Reference [8]' s adaptive inertia weight PSO has poor global search and easily falls into local optima. Reference [9]' s genetic mutation improves global search but has slower convergence due to higher complexity. The proposed algorithm offers better convergence speed and global search capability with less susceptibility to local optima.

2) Separation Performance The similarity coefficient evaluates separation performance:

$$\xi_{ij} = \left|\frac{\sum_{t=1}^{M} y_i(t)s_j(t)}{\sqrt{\sum_{t=1}^{M} y_i^2(t)\sum_{t=1}^{M} s_j^2(t)}}\right|$$

When $s_i(t)$ and $y_j(t)$ are independent, $\xi_{ij} = 0$; when $y_j(t) = c \cdot s_i(t)$ (c is constant), $\xi_{ij} = 1$. The closer $|\xi_{ij}|$ is to 1, the better the separation.

Simulations compared traditional EASI, reference [8]' s adaptive inertia weight PSO, reference [9]' s genetic mutation algorithm, and the proposed algorithm. EASI used $f(y) = y^3$. Average similarity coefficients using mechanical vibration signals are shown in Table 1 .

Table 1 shows that EASI and algorithms from references [8] and [9] have relatively large minimum correlation coefficients, indicating poor dynamic signal separation. The proposed algorithm achieves a maximum correlation coefficient of 0.9999 (closer to 1) and minimum of 0.0009, demonstrating superior separation performance and providing new insights for dynamic signal processing.

3) Running Time Table 2 presents algorithm running times. While second-order oscillation slows convergence compared to W-C-PSO, Figure 9 shows W-C-PSO' s performance is suboptimal. The proposed algorithm uses Givens matrix representation, solving only $n(n-1)/2$ parameters (Section 3.1), reducing running time and accelerating convergence.

5 Conclusion

This paper combines inertia weight and learning factors in traditional PSO to enhance algorithmic integrity and global search capability. A second-order oscillation mechanism increases particle diversity, preventing local optima. Givens transformation reduces complexity and improves convergence speed. Simulations on mechanical vibration and speech signals show superior performance in convergence, global search, and separation quality, providing a new foundation for intelligent algorithm improvement.

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Note: Figure translations are in progress. See original paper for figures.