Postprint: Spectral Efficiency Analysis of Massive MU-MISO Systems under Spatially Correlated Channels

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Abstract

Focusing on the impact of channel and power allocation on spectral efficiency in large-scale MU-MISO systems, this study investigates the spectral efficiency problem under spatially correlated channels. First, the steering matrix is determined through the path difference of transmitted signals to obtain the user transmit correlation matrix, and combined with large-scale shadow fading and a scattering environment with Gaussian distribution, a spatially correlated channel is established. Then, based on the spatially correlated channel, precoding is employed to reduce multi-user interference, and the impact of precoding on spectral efficiency is simulated. Finally, under the scenario of limited transmit power and each user' s received signal having a certain signal-to-interference ratio (SIR), a power allocation algorithm is proposed to maximize the total spectral efficiency of the system. Simulation results demonstrate that the spectral efficiency of RZF precoding in spatially correlated channels outperforms that of MRT precoding. Compared with equal power allocation, the proposed algorithm achieves significant improvement in spectral efficiency and possesses strong theoretical value and practical significance.

Full Text

Analysis of Spectrum Efficiency of Large-Scale MU-MISO Systems in Spatially Correlated Channels

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Abstract

This paper addresses the spectrum efficiency problem in large-scale MU-MISO systems under spatially correlated channels, where spectrum efficiency is affected by both channel characteristics and power allocation. First, a spatially correlated channel model is established by deriving the user transmit correlation matrix from the steering matrix determined by the wave path difference of transmitted signals, combined with large-scale shadow fading and a scattering environment with Gaussian distribution. Then, based on this spatially correlated channel, precoding is employed to reduce multi-user interference, and the impact of precoding on spectrum efficiency is simulated. Finally, a power allocation algorithm is proposed under scenarios with limited transmit power and constrained signal-to-interference ratio for each user' s received signal, aiming to maximize the total system spectrum efficiency. Simulation results demonstrate that RZF precoding achieves better spectrum efficiency than MRT precoding in spatially correlated channels. Compared with equal power allocation, the proposed algorithm significantly improves spectrum efficiency and holds strong theoretical and practical significance.

Keywords: multi-user; spatial correlation channel; power allocation

0 Introduction

With the rapid development of wireless services, the fourth-generation mobile communication system (4G) can no longer meet people' s demands for wireless transmission rates in terms of spectrum efficiency and power efficiency. Therefore, breakthroughs are needed in wireless transmission for the future fifth-generation mobile communication system (5G) to achieve the dual goals of higher spectrum efficiency and green wireless communication [1]. Multiple-input multiple-output (MIMO) technology [2] has attracted significant attention due to its great potential in improving spectrum efficiency and link reliability, making it one of the key research directions.

In MIMO wireless communication systems, based on the number of users, they can be divided into single-user MIMO (SU-MIMO) systems and multi-user MIMO (MU-MIMO) systems. Literature [3,4] studied the relationship between system spectrum efficiency and the number of transmit/receive antennas in SU-MIMO systems, concluding that spectrum efficiency grows linearly with the number of antennas. However, in multi-user MIMO systems, since multiple users share the same time-frequency resources to communicate with the base station, co-channel interference (CCI) exists among users. To reduce or eliminate interference between users, precoding techniques can be employed at the base station transmitter. Precoding techniques can be divided into linear precoding and non-linear precoding. In non-linear precoding, dirty paper coding (DPC) [5] can achieve the optimal precoding scheme for system spectrum efficiency, but its computational complexity is higher than that of linear precoding.

Linear precoding schemes include Zero Forcing (ZF) precoding, regularized zero forcing (RZF) precoding, block diagonalization (BD) precoding [8], minimum mean square error (MMSE) precoding [9,10], and signal to leakage and noise ratio (SLNR) precoding criterion [11].

Literature [12] analyzed the minimization of signal-to-interference-plus-noise ratio for users' received signals in large-scale MU-MIMO systems and derived the optimal expression for RZF precoding. Literature [13] derived the optimal parameter solution for RZF precoding, the optimal user number design for ZF precoding, and the optimal power allocation problems for both RZF and ZF when the number of antennas and users tends to infinity with a bounded ratio. Literature [14] studied the impact of base station transmit power and antenna number on MU-MISO system spectrum efficiency in channels with shadow fading, concluding that with fixed total base station transmit power, system spectrum efficiency approaches a constant value as the number of antennas increases. However, in practical MIMO systems, channel spatial correlation and scattering effects in the surrounding environment cause system spectrum efficiency performance to be lower than the ideal channel analysis [15]. Literature [16] studied the impact of spatially correlated channels on spectrum efficiency and energy efficiency in large-scale MIMO systems under time-division duplexing (TDD) mode, concluding that system spectrum efficiency in spatially correlated systems is lower than that in non-spatially-correlated systems. Based on the spatial correlation of MIMO channels, literature [17] established a spatially correlated channel model for large-scale MU-MIMO systems, assuming independent users, and studied the impact of spatial correlation on system spectrum efficiency under fixed user signal-to-noise ratio (SNR), concluding that spectrum efficiency approaches a constant value as the number of antennas increases in spatially correlated channels.

This paper considers the impact of precoding and power allocation on spectrum efficiency in large-scale MU-MISO systems under spatially correlated channels, and proposes a power allocation algorithm to maximize system spectrum efficiency under constraints of limited system power and required signal-to-interference ratio for users' received signals.

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1 System Model and Problem Description

The block diagram of the large-scale MU-MISO system is shown in [Figure 1: see original paper]. The system consists of a base station equipped with M antennas and N single-antenna users. The channel is assumed to be accurately estimated, and the signals transmitted from the base station to all users are represented as $X = \sum_{i=1}^{N} p_i x_i w_i$, where x_i denotes the data symbol transmitted to user i, and p_i represents the power allocated by the base station to user i. The transmit precoding matrix at the base station is denoted as $W = [w_1, w_2, \dots, w_N]$, where w_i represents the precoding vector for the signal transmitted to user i, with $w_i \in$

$\mathbb{C}^{M\times 1}$. The channel matrix can be expressed as $G = [g_1, g_2, \dots, g_N]$, where $g_i = [g_{i,1}, g_{i,2}, \cdots, g_{i,M}]^T \in \mathbb{C}^{M\times 1}$ represents the channel between the base station' s M antennas and user i. The noise matrix for all users is $Z = [z_1, z_2, \dots, z_N]^T$, where z_i is the additive white Gaussian noise for user i. The received signal vector for N users is $Y = [y_1, y_2, \dots, y_N]^T$, with y_i being the received signal of user i. The system received signal matrix is $Y = GWX + Z$.

Considering large-scale fading in signal propagation, the channel matrix can be expressed as $H = D^{1/2}G$, where $D = \text{diag}\{\beta_1, \beta_2, \dots, \beta_N\}$ represents the large-scale shadow fading matrix. Here, $\beta_i = \varphi\varepsilon_i d_i^{-\alpha}$ denotes the path loss and shadow fading from the base station to user i, where φ is a constant containing carrier frequency and antenna gain, α is the path loss exponent, d_i is the distance between the base station transmitter and user i, and ε_i follows a log-normal distribution $\varepsilon_i \sim 10\log\mathcal{N}(0, \sigma_f^2)$ representing shadow fading. The matrix $H = [h_1, h_2, \dots, h_N]^T$ represents the fading channel with spatial correlation from the base station to user i, where $h_i \in \mathbb{C}^{1\times M}$. The received signal for user i is expressed as $y_i = \beta_i h_i w_i p_i x_i + \sum_{j=1, j\neq i}^{N} \beta_i h_i w_j p_j x_j + z_i$.

1.1 Spatially Correlated Channel Modeling

Spatially correlated random MIMO channels are generated using the correlation matrix multiplication method [18] based on independent random MIMO channels. The correlation matrix can be expressed as $R_i = R_{r,i} \otimes R_{t,i}$, where R_i represents the spatial correlation matrix between the base station and user i, composed of the Kronecker product of the receive correlation matrix $R_{r,i}$ and the transmit correlation matrix $R_{t,i}$.

In large-scale MU-MISO systems, the base station transmitter has an array of M antennas, and the receiver consists of N single-antenna users. Assuming users are mutually independent (no correlation between users) and scatterers exist at the user receiver side, the channel with spatial correlation and scatterers [19] is expressed as $h_i = R_{r,i}^{1/2} h_{w,i} R_{t,i}^{1/2}$. Here, $h_{w,i} \sim \mathcal{CN}(0, I)$ represents the scattering environment at the user side and follows a complex Gaussian distribution with zero mean and unit variance identity matrix. $R_{t,i}$ is the transmit correlation matrix, and $R_{r,i}$ is the receive correlation matrix. Since users have single antennas, $R_{r,i} = 1$, thus $R_i = R_{t,i}$. From literature [20,21], the transmit correlation matrix can be expressed as $R_{t,i} = \int_{\theta_{\min}}^{\theta_{\max}} a(\theta)a^H(\theta)f(\theta)d\theta$, where $f(\theta)$ represents the power azimuth spectrum satisfying $\int_{\theta_{\min}}^{\theta_{\max}} f(\theta)d\theta = 1$.

Assuming the average transmit angle at user i' s transmitter is φ_i, let $\theta = \varphi_i + \delta$, where δ is the random angular deviation of the base station transmit angle caused by scatterers. δ follows a Gaussian distribution with zero mean and angular standard deviation σ_φ, i.e., $\delta \sim \mathcal{N}(0, \sigma_\varphi^2)$. $\theta_{\max}$ represents the maximum transmit angle after deviation from the average transmit angle, and $\theta_{\min}$ represents the minimum transmit angle after deviation. This paper considers a linear array at the base station transmitter, whose steering vector is expressed as $a(\theta) = [1, e^{j2\pi d \sin\theta/\lambda}, \ldots, e^{j2\pi(M-1)d\sin\theta/\lambda}]^T$, where d is the antenna spacing and λ is the wavelength of the downlink signal.

1.2 System Precoding

Due to co-channel interference in large-scale MU-MIMO systems, precoding techniques can be applied at the transmitter to reduce and eliminate interference. The SLNR precoding in literature [11] maximizes the signal-to-noise ratio of the target user for signal preprocessing, which can effectively eliminate inter-user interference. However, due to residual interference, system performance cannot be significantly improved and its algorithm complexity is high. MRT precoding is the simplest linear precoding, requiring only simple transformation of the channel matrix with low complexity. ZF precoding is the most classical algorithm that can effectively eliminate co-channel interference but does not consider noise effects. RZF precoding provides a trade-off between co-channel interference and noise effects compared to ZF precoding. This paper primarily considers MRT and RZF precoding to reduce inter-user interference.

Based on literature [14] and the spatially correlated channel, the MRT precoding for user i can be expressed as $w_i^{MRT} = h_i^H/\sqrt{\mathbb{E}\{\|h_i\|^2\}}$. Replacing MRT with the more complex regularized zero-forcing precoding (RZF), the RZF precoding matrix for user i [22] is expressed as $w_i^{RZF} = \frac{(\sum_{j=1}^N h_j^H h_j + \varphi I)^{-1} h_i^H}{\sqrt{\mathbb{E}\{\|(\sum_{j=1}^N h_j^H h_j + \varphi I)^{-1} h_i^H\|^2\}}}$, where φ is the regularization coefficient. The signal-to-interference-plus-noise ratio (SINR) for user i under different precoding schemes can be derived by combining the above expressions. The SINR for user i with MRT precoding is:

$$\mathrm{SINR}_i^{MRT} = \frac{p_i \beta_i^2 |h_i w_i|^2}{\sum_{j=1, j\neq i}^N p_j \beta_i^2 |h_i w_j|^2 + \sigma^2}$$

The SINR for user i with RZF precoding is:

$$\mathrm{SINR}_i^{RZF} = \frac{p_i \beta_i^2 |h_i w_i|^2}{\sum_{j=1, j\neq i}^N p_j \beta_i^2 |h_i w_j|^2 + \sigma^2}$$

The system spectrum efficiency is expressed as $R = \sum_{i=1}^N \mathbb{E}\{\log_2(1 + \mathrm{SINR}_i)\}$.

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2 System Power Allocation Algorithm

From the system spectrum efficiency expression in equation (13), we observe that spectrum efficiency depends on both power allocation and precoding selection. While precoding techniques can mitigate or eliminate inter-user interference, combining them with appropriate power allocation algorithms enables more efficient utilization of system spectrum efficiency. This paper investigates power allocation under power-constrained systems with MRT or RZF precoding, where each user must maintain a minimum signal-to-interference ratio.

The optimization objective is modeled using inequalities, establishing the following mathematical expression:

$$\begin{aligned}\max_{p_1,p_2,\ldots,p_N} \quad & \sum_{i=1}^{N} \log_2(1+\mathrm{SINR}_i) \\ \text{s.t.} \quad & \sum_{i=1}^{N} \mathrm{tr}(P_i) \le P_{\text{total}} \\ & \mathrm{SINR}_i \ge \eta_i, \quad i=1,2,\ldots,N\end{aligned}$$

where P_{total} represents the total system transmit power and η_i is the minimum required SINR for user i to maintain communication. By examining the system spectrum efficiency expression (13), we can simplify the formulation. The system spectrum efficiency can be expressed as:

$$R = \sum_{i=1}^{N} \log_2(1+\mathrm{SINR}_i) \ge \sum_{i=1}^{N} \log_2(\mathrm{SINR}_i) = \log_2 \prod_{i=1}^{N} \mathrm{SINR}_i$$

From equations (8) and (9), RZF precoding considers more factors and is more complex than MRT precoding. For generality, let w_i denote the precoding matrix for user i and h_i denote the matrix with spatial correlation and scattering for user i. By introducing a variable γ_i for each user' s SINR, the SINR for user i can be expressed as:

$$\mathrm{SINR}_i = \frac{p_i \beta_i^2 |h_i w_i|^2}{\sum_{j=1,j\ne i}^{N} p_j \beta_i^2 |h_i w_j|^2 + \sigma^2} \ge \gamma_i$$

The above expression can be transformed to:

$$\frac{p_i \beta_i^2 |h_i w_i|^2}{\gamma_i} \ge \sum_{j=1,j\ne i}^{N} p_j \beta_i^2 |h_i w_j|^2 + \sigma^2$$

Thus, the problem of maximizing system spectrum efficiency is converted to maximizing the product of minimum SINR values. The mathematical expression for the optimization objective becomes:

$$
\begin{aligned}
\max_{p_1,p_2,\dots,p_N} \quad & \prod_{i=1}^{N} \gamma_i \\
\text{s.t.} \quad & \sum_{i=1}^{N} p_i \le P_{\text{total}} \\
& \frac{p_i \beta_i^2 |h_i w_i|^2}{\gamma_i} \ge \sum_{j=1, j\ne i}^{N} p_j \beta_i^2 |h_i w_j|^2 + \sigma^2, \quad i = 1, 2, \dots, N
\end{aligned}
$$

The above objective expression is a geometric programming (GP) problem. According to literature [23,24], this can be transformed into a convex optimization problem. This paper solves equation (18) using convex optimization tools and the bisection algorithm from literature [25].

The algorithm steps are as follows:

a) Initialize parameters: set the minimum introduced variable $\gamma_{\text{lower}} = 0$, set the maximum value $\gamma_{\text{upper}} = \frac{P_{\text{total}}}{\sigma^2} \min_i \frac{\beta_i^2 |h_i w_i|^2}{\sum_{j=1}^{N} \beta_i^2 |h_i w_j|^2}$, set the minimum error tolerance ε, and compute the next best candidate value $\gamma_{\text{candidate}} = \frac{\gamma_{\text{upper}} + \gamma_{\text{lower}}}{2}$.
b) Substitute $\gamma_{\text{candidate}}$ into equation (18). If equation (18) is solvable, compute $\gamma_{\text{lower}} = \gamma_{\text{candidate}}$; otherwise, compute $\gamma_{\text{upper}} = \gamma_{\text{candidate}}$.
c) If $|\gamma_{\text{upper}} - \gamma_{\text{lower}}| > \varepsilon$, continue iteration; otherwise, stop.
d) If $|\gamma_{\text{upper}} - \gamma_{\text{lower}}| < \varepsilon$, the optimal power allocation $[p_1^{\text{opt}}, p_2^{\text{opt}}, \dots, p_N^{\text{opt}}]$ is obtained and iteration stops.

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3 Simulation Results and Analysis

This section verifies the effectiveness of the proposed scheme through simulations. In the simulations, each user has the same minimum SINR requirement $\eta_i = \eta$. Users are randomly distributed in a cell with radius R, and the base station' s average transmit angle is determined by the user' s location in the cell. The transmit correlation matrix is then determined using equations (5) and (6). This paper assumes no correlation between users, and the receive correlation matrix is represented by an identity matrix. The system simulation parameters are shown in .

Table 1 Simulation parameters - Log-normal shadow fading variance: $\sigma_{sh}^2 = 8$ dB - Path loss exponent: $\alpha = 3.8$ - Noise power spectral density: $N_0 = -94$ dBm/MHz - Power per base station antenna: $p_t = 100$ mW - Number of base station antennas: $M = 100$ - Signal bandwidth: $B = 20$ MHz - Cell radius: $R = 500$ m

[Figure 2: see original paper] simulates the effect of the number of base station antennas on system average spectrum efficiency performance under equal power allocation and spatially correlated channel conditions. The simulation uses 10 and 20 users in the system. From [Figure 2: see original paper], we can see that under the same precoding and number of users, system spectrum efficiency increases with the number of antennas. Under a fixed number of antennas, the spectrum efficiency of RZF precoding is higher than that of MRT precoding. When the number of antennas is small, RZF precoding outperforms ZF precoding, but as the number of antennas gradually increases, ZF precoding performance approaches that of RZF precoding.

[Figure 3: see original paper] simulates the relationship between the random angular deviation δ of the transmit angle and system spectrum efficiency. In the simulation, as the random angular deviation δ gradually increases, spectrum efficiency decreases. Comparing the curves for MRT and RZF precoding conditions, we observe that with the same random deviation angle and number of users, MRT precoding achieves lower spectrum efficiency than RZF precoding. Moreover, MRT precoding degrades faster than RZF precoding when the random deviation angle is below $10°$. As the random deviation angle continues to increase, RZF precoding shows more gradual changes compared to MRT precoding.

[Figure 4: see original paper] shows the cumulative distribution function (CDF) of spectrum efficiency for correlated and uncorrelated channels under different precoding schemes. Since user locations are uniformly distributed in the cell, we statistically compare the spectrum efficiency of correlated and uncorrelated channels. From [Figure 4: see original paper], we can see that under the same precoding condition, the CDF of spectrum efficiency in uncorrelated channels is better than that in correlated channels. In the same channel environment, RZF precoding achieves better spectrum efficiency than MRT precoding.

[Figure 5: see original paper] compares the spectrum efficiency of the proposed power allocation algorithm with equal power allocation under MRT and RZF precoding. From [Figure 5: see original paper], we can see that compared with equal power allocation, the proposed algorithm enables each user to achieve higher spectrum efficiency at the same CDF value. Furthermore, under the same power allocation, users with RZF precoding achieve better spectrum efficiency than those with MRT precoding.

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4 Conclusion

This paper investigates the spectrum efficiency of large-scale MU-MISO downlink systems based on channels with spatial correlation and large-scale flat fading. First, a channel model with large-scale shadow fading and spatial correlation is constructed. The spatially correlated channel with scatterers is established through correlation matrices, and an expression for solving the transmit spatial correlation matrix is derived. Then, expressions for RZF and MRT precoding are derived under the established correlated channel model. Finally, under limited system power and the constraint that each user’ s received signal must meet a certain signal-to-interference ratio, an optimization model for power allocation is established to maximize spectrum efficiency, and a power allocation algorithm is proposed to solve this model.

Compared with considering only ideal Rayleigh flat fading channel models, the channel model proposed in this paper is closer to practical channel environments. Simulation results demonstrate that in the constructed channel model, RZF precoding outperforms MRT precoding in terms of spectrum efficiency. Under the proposed channel model, the proposed scheme significantly improves spectrum efficiency compared with equal power allocation schemes.

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Note: Figure translations are in progress. See original paper for figures.