Image Denoising Based on Wavelet Thresholding and Total Variation Model: Postprint

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Abstract

To address the issues of incomplete denoising and image edge blurring caused by wavelet threshold functions, a denoising method combining adaptive wavelet thresholding with total variation model is proposed. By exploiting the time-frequency characteristics of wavelet transform, the noisy image is decomposed to obtain wavelet coefficients across different dimensions. The low-frequency wavelet coefficients are denoised using the total variation model, while for high-frequency coefficients, different optimal thresholds are selected according to the decomposition scale, thereby overcoming the limitations of a unified threshold and enhancing the algorithm' s adaptability. Theoretical analysis and simulation results demonstrate that the proposed method integrates the denoising advantages of both wavelet transform and total variation model, preserves image edges and details more completely while effectively removing noise, and achieves higher structural similarity index and peak signal-to-noise ratio.

Full Text

Preamble

Method for Image Denoising Based on Wavelet Transform and Total Variational Model

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Abstract: Traditional wavelet threshold algorithms for image noise reduction suffer from incomplete denoising and edge blurring. To address these drawbacks, this paper proposes a denoising method that combines adaptive wavelet thresholding with a total variation model. Leveraging the time-frequency characteristics of wavelet transform, the noisy image is decomposed to obtain wavelet coefficients at various scales. The low-frequency coefficients are denoised using the total variation model, while for high-frequency coefficients, different optimal thresholds are selected based on decomposition scale, overcoming the limitations of universal thresholds and enhancing algorithm adaptability. Theoretical analysis and simulation results demonstrate that the proposed method combines the advantages of both wavelet transform and total variation models, effectively removing noise while preserving image edges and details more completely, achieving higher structural similarity and peak signal-to-noise ratios.

Keywords: image denoising; adaptive thresholding; wavelet transform; total variation model

0 Introduction

Images inevitably suffer from noise interference during acquisition, transmission, and conversion processes, which adversely affects image recognition, analysis, and understanding [1]. Therefore, denoising is necessary to enable more accurate subsequent analysis and information extraction. Current image denoising algorithms primarily fall into three categories: filtering-based methods, multi-scale transform-based methods, and partial differential equation (PDE)-based methods [2].

Filtering-based approaches include median filtering [3], Kalman filtering [4], and Wiener filtering [5], which primarily achieve denoising by removing high-frequency components. However, this approach also leads to loss of structural information. Multi-scale transforms, represented by wavelet transform, exhibit excellent time-frequency analysis characteristics [6] and are widely used in image denoising. Wavelet threshold denoising is the main approach within this category [7,8], but it suffers from over-killing of wavelet coefficients, resulting in reconstruction errors and edge blurring.

To address these issues, researchers have proposed adaptive threshold denoising methods, though these still have limitations in preserving image details. For instance, methods based on neighboring wavelet coefficients [9] investigate the influence of neighborhood coefficients on threshold selection and improve universal thresholds but neglect subband effects. Adaptive filtering using mask filters [10] suffers from high computational complexity in optimization. Methods that adjust thresholds using sampling point length [11] produce edge blurring and detail loss due to inaccurate threshold estimation and image size dependencies.

PDE-based image denoising methods have gained widespread application due to their excellent edge-preserving properties [12-14]. Rudin et al. proposed the total variation (TV) model [13], which formulates image denoising as an energy functional minimization problem, interpreting the denoising process from a novel perspective. While effectively protecting edge and texture information during denoising, TV models suffer from staircase effects under high noise conditions [15], affecting denoising performance.

To overcome these limitations, this paper proposes an image denoising method combining adaptive wavelet thresholding with a total variation model. The method leverages the time-frequency analysis capability of wavelet denoising and the detail-preserving advantages of total variation models. The noisy image is first decomposed via wavelet transform into low-frequency and high-frequency regions, which are processed separately. The low-frequency region employs the total variation model, while the high-frequency region uses an adaptive wavelet threshold that varies with decomposition scale and subband, overcoming universal threshold limitations and improving adaptability. Simulation experiments on images with different noise levels and texture distributions validate the proposed method' s effectiveness.

1 Total Variation Model and Wavelet Threshold Denoising

1.1 Total Variation Model

The total variation model was first introduced to image denoising by Rudin et al. [13]. TV-based image denoising represents image energy as a functional and removes noise by minimizing the system energy under constraints, transforming the denoising problem into an energy minimization problem.

Let $f_0(x, y)$ denote the noisy image, Ω the image domain, $f(x, y)$ the original clean image, and $n(x, y)$ Gaussian noise with variance σ^2. The total variation of f is defined as:

$$TV(f) = \int_\Omega |\nabla f| dxdy$$

Image contamination by noise is a forward problem. Denoising is an inverse problem that may yield non-unique or unstable solutions. Regularization methods are typically employed to constrain the solution space. Therefore, TV-based image denoising can be formulated as the following minimization problem:

$$\min E(f) = \frac{1}{2} \int_\Omega (f_0 - f)^2 dxdy + \lambda \int_\Omega |\nabla f| dxdy$$

where $\frac{1}{2} \int_\Omega (f_0 - f)^2 dxdy$ is the data fidelity term representing the approximation between the restored and noisy images, primarily preserving original image characteristics and reducing distortion. $\lambda \int_\Omega |\nabla f| dxdy$ is the regularization term that effectively reflects edge information in the image.

The derived Euler-Lagrange equation is:

$$\lambda \left(\frac{\nabla f}{|\nabla f|} \right) - (f_0 - f) = 0$$

From Equation (3), the diffusion coefficient is $1/|\nabla f|$. At image edges where $|\nabla f|$ is large, the diffusion coefficient is small, resulting in weak diffusion that preserves edges. In smooth regions where $|\nabla f|$ is small, the diffusion coefficient is large, enabling strong diffusion for noise removal.

1.2 Wavelet Threshold Denoising

Based on the different singularities of signals and noise in the wavelet domain [16], useful signals manifest as low-frequency components while noise appears as high-frequency components. After wavelet transformation, signal wavelet coefficients are larger than noise wavelet coefficients. By determining an appropriate threshold, coefficients with absolute values smaller than the threshold are set to zero while larger coefficients are preserved or shrunk, yielding estimated wavelet coefficients for signal reconstruction. The denoising process is illustrated in [Figure 1: see original paper].

Wavelet threshold denoising typically employs hard or soft thresholding. Hard thresholding compares the absolute value of wavelet coefficients against a threshold T, setting coefficients smaller than T to zero while leaving larger coefficients unchanged. Let $w_{i,j}$ denote the wavelet coefficients and $\hat{w}_{i,j}$ the thresholded result:

$$\hat{w}_{i,j} = \begin{cases} w_{i,j}, & |w_{i,j}| \geq T \\ 0, & |w_{i,j}| < T \end{cases}$$

Soft thresholding differs by subtracting the threshold from coefficients whose absolute values exceed T:

$$\hat{w}_{i,j} = \begin{cases} \mathrm{sign}(w_{i,j})(|w_{i,j}| - T), & |w_{i,j}| \geq T \\ 0, & |w_{i,j}| < T \end{cases}$$

While these methods are easy to implement and quickly produce estimated coefficients, they have limitations. Hard thresholding is discontinuous at $|w_{i,j}| = T$, causing oscillations in the reconstructed signal and over-killing of wavelet coefficients. Soft thresholding yields continuous $\hat{w}_{i,j}$ but introduces a constant bias between $\hat{w}_{i,j}$ and $w_{i,j}$ when $|w_{i,j}| \geq T$, affecting the approximation between reconstructed and true signals.

2 Adaptive Wavelet Threshold and Total Variation Combined Denoising Method

2.1 Adaptive Wavelet Threshold

To address the over-killing phenomenon and edge blurring caused by universal threshold functions, this paper proposes an adaptive wavelet threshold that considers neighborhood coefficient correlations. Within small neighborhoods, wavelet coefficients exhibit correlation, so thresholding should consider neighboring coefficients to preserve image details. Therefore, adjacent coefficients are introduced into the threshold function [17].

Let $d_{j,k}$ represent the wavelet coefficients at scale j and position k, and $\hat{d}_{j,k}$ the thresholded result. The threshold function is:

$$\hat{d}_{j,k} = \begin{cases} d_{j,k} - \alpha \frac{\sigma^2}{S_{j,k}^2} d_{j,k}, & |d_{j,k}| \geq T \\ 0, & \text{otherwise} \end{cases}$$

where $S_{j,k}^2$ is an energy estimate for each high-frequency subband, R is the selected neighborhood window size, and m, n are indices for the subband coefficient matrix.

Wavelet coefficients exhibit approximate exponential decay with increasing decomposition scale, meaning noise energy gradually weakens during signal decomposition. The horizontal and vertical high-frequency subbands contain larger coefficient values and energy than the diagonal subband, indicating less noise in horizontal and vertical subbands. Using a universal threshold estimated from the diagonal subband removes some image information from vertical and horizontal subbands, causing blurring. Based on this characteristic, we adjust the threshold by varying parameter α in Equation (6). According to the exponential decay property, we set $\alpha = 2^{-i}$ where i is the decomposition level. Adjusting α selects appropriate threshold functions for different scales, ensuring overall continuity of $\hat{d}_{j,k}$ to avoid signal oscillations and improve denoising performance.

The parameter λ balances the diffusion and fidelity terms. Larger λ values increase fidelity term influence, making denoised images closer to the original. This paper determines λ based on peak signal-to-noise ratio (PSNR) values after denoising, selecting the value that yields maximum PSNR. The gradient descent method solves Equation (10) for TV-denoised low-frequency subimages, while Equation (6) processes high-frequency subimages with the adaptive wavelet threshold.

2.2 Combined Denoising Method

Since wavelet thresholding causes edge blurring while total variation preserves edges, combining these methods can achieve noise removal while retaining edge information. Let $\{V_j\}$ constitute a multiresolution analysis on $L^2(R)$ with scaling function $\phi(x)$ and wavelet function $\psi(x)$. According to Mallat' s pyramid algorithm, the fast decomposition of 2D discrete wavelet transform can be expressed as:

$$\begin{cases} c_{i+1}(m,n) = \sum_l \sum_k h(l-2m)h(k-2n)c_i(l,k) \\ d^1_{i+1}(m,n) = \sum_l \sum_k g(l-2m)h(k-2n)c_i(l,k) \\ d^2_{i+1}(m,n) = \sum_l \sum_k h(l-2m)g(k-2n)c_i(l,k) \\ d^3_{i+1}(m,n) = \sum_l \sum_k g(l-2m)g(k-2n)c_i(l,k) \end{cases}$$

where h and g are high-pass and low-pass filters respectively, $c_{i,m,n}$ is the approximation of the original image, and $d^1_{i,m,n}$, $d^2_{i,m,n}$, $d^3_{i,m,n}$ are high-frequency coefficients in vertical, horizontal, and diagonal directions.

Since the low-frequency subband $c_{i,m,n}$ is an approximation of the original image, it is expanded to the same dimensions as the original image. The TV fidelity term becomes:

$$E_{NEW} = \frac{1}{2}\int_\Omega (f_0 - c_{i,m,n})^2 dxdy + \lambda \int_\Omega |\nabla f| dxdy$$

with the corresponding Euler-Lagrange equation:

$$\lambda \text{div}\left(\frac{\nabla f}{|\nabla f|}\right) + (c_{i,m,n} - f) = 0$$

The implementation flowchart is shown in [Figure 2: see original paper]. The specific steps are:

a) **Wavelet Transform**: Select the sym5 wavelet basis for I-level decomposition of the image to obtain low-frequency subimage LL_i, horizontal high-frequency subimage HL_i, vertical high-frequency subimage LH_i, and diagonal high-frequency subimage HH_i.

b) **Total Variation Processing**: Denoise low-frequency subimage LL_i using the TV model to obtain processed low-frequency subimage LL'_i.

c) **Thresholding**: Apply thresholding to wavelet coefficients of three directional high-frequency subimages using Equation (6) to obtain processed high-frequency subimages HL'_i, LH'_i, HH'_i.

d) **Image Reconstruction**: Perform wavelet inverse transform and reconstruction on the TV-processed low-frequency part LL'_i and thresholded high-frequency parts HL'_i, LH'_i, HH'_i to obtain the final denoised image.

3 Experimental Results and Analysis

To validate the denoising performance of the proposed method, simulation experiments compare it with ROF, non-local means (NLM) filtering, and adaptive wavelet thresholding methods. Experiments are conducted on three images with different details: Barbara, Peppers, and Lena. Additive white Gaussian noise with mean 0 and standard deviations $\sigma = 15, 20, 25, 30$ is added to each image. The sym5 wavelet basis is used for all experiments.

3.1 Denoising Results Comparison

To observe denoising effects at different noise levels, denoising results for Barbara, Peppers, and Lena images with noise standard deviations of 15, 20, 25, and 30 are presented using ROF, NLM, adaptive wavelet thresholding, and the proposed method, as shown in through .

To better compare structure preservation, Sobel edge detection is applied to denoising results for Barbara, Peppers, and Lena images contaminated with Gaussian noise (mean 0, $\sigma = 30$). The edge detection results are shown in [Figure 3: see original paper] through [Figure 5: see original paper].

As shown in through , as noise variance increases, the PSNR values of all four methods gradually decrease. For all images—whether texture-rich (Barbara, Peppers) or edge-rich (Lena)—the proposed method achieves the highest PSNR and lowest MSE, indicating superior denoising performance. The ROF method over-smooths detail information (e.g., Barbara' s scarf, pants, and tablecloth; Lena's hat) and darkens background brightness. NLM filtering produces blurred edges and distortion (e.g., edges in Peppers and Lena, and Barbara' s head and arms). Adaptive wavelet thresholding yields incomplete denoising with edge blurring (e.g., Lena' s hair, image backgrounds, and Barbara' s right tablecloth). In contrast, the proposed method produces naturally smooth images with well-preserved edges and details. Edge detection results in [Figure 3: see original paper] through [Figure 5: see original paper] show that the proposed method extracts more complete edge information.

3.2 Evaluation Criteria

For objective evaluation, peak signal-to-noise ratio (PSNR), mean square error (MSE), and structural similarity (SSIM) are used as metrics:

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{255^2}{\mathrm{MSE}}\right)$$

$$\mathrm{MSE} = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}[f(i,j) - g(i,j)]^2$$

$$\mathrm{SSIM} = \frac{(2\mu_f\mu_g + c_1)(2\sigma_{fg} + c_2)}{(\mu_f^2 + \mu_g^2 + c_1)(\sigma_f^2 + \sigma_g^2 + c_2)}$$

where $f(i,j)$ is the noisy image, $g(i,j)$ the denoised image, M and N are image dimensions, $\mu_f, \mu_g, \sigma_f^2, \sigma_g^2, \sigma_{fg}$ are means, variances, and covariance, and c_1, c_2 are constants.

PSNR reflects the difference between original and denoised images, with larger values indicating better approximation. MSE measures the squared error expectation, with smaller values indicating better denoising. SSIM directly evaluates structural similarity, with larger values indicating better visual quality.

With noise standard deviations set to 15, 20, 25, and 30, PSNR, MSE, and SSIM values are computed for all methods on Barbara, Peppers, and Lena images. The results are presented in through .

As shown in , PSNR decreases with increasing σ for all methods. The proposed method consistently achieves the highest PSNR. shows it also produces the lowest MSE. reveals that when $\sigma = 15$, SSIM values are similar across methods. However, when $\sigma = 30$, the proposed method' s SSIM decreases by only 0.128 compared to 0.234 average decrease for other methods, demonstrating superior visual quality.

4 Conclusion

To address incomplete denoising and edge blurring in wavelet threshold methods, this paper proposes an image denoising method combining adaptive wavelet thresholding with a total variation model. The method leverages the advantage of wavelet thresholding in determining optimal thresholds based on noise conditions at different scales and directions in high-frequency components, overcoming universal threshold limitations and enhancing adaptability. Simultaneously, it utilizes the TV model' s effectiveness in low-frequency denoising to better preserve original edge information.

Experimental results on images with various Gaussian noise variances demonstrate that the proposed method effectively removes noise while achieving lower MSE and higher PSNR, with better preservation of texture details.

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Note: Figure translations are in progress. See original paper for figures.