

## Postprint of Critical Road Segment Identification in Urban Traffic Networks Based on Directed Weighted Complex Networks

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### Abstract

Although critical road segments in urban traffic networks are few in number, they exert substantial influence on the overall flow capacity of the entire network. To address the limitations of current complex network-based identification methods for critical road segments in urban traffic networks, which neglect real-world influencing factors and the directional characteristics of road segments, this paper proposes a critical road segment identification methodology based on directed weighted complex networks. The first stage employs complex network theory to model the urban traffic network as a directed weighted complex network. The second stage utilizes the LinkRank algorithm to rank edge importance within the complex network, thereby identifying critical edges—that is, critical road segments in the urban traffic network. The third stage applies a modified Susceptible-Infective (SI) model to assess the impact of these critical road segments. The practicality and effectiveness of the proposed method are validated through analysis of the urban traffic network in Haining City, Zhejiang Province.

### Full Text

#### Preamble

#### Critical Road Sections Identification of Urban Traffic Road Network Based on Weighted and Directed Complex Networks

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**Abstract:** Although critical road sections constitute only a small portion of urban traffic networks, they exert substantial influence on overall network cir-

ulation. Addressing the limitations of existing complex network-based identification methods—which often neglect realistic influencing factors and the directional nature of road sections—this paper proposes a novel method for identifying critical road sections using directed weighted complex networks. The approach comprises three stages: First, complex network theory is employed to construct a directed weighted complex network model of the urban traffic road network. Second, the LinkRank algorithm is applied to rank edge importance within this network, thereby identifying critical edges that correspond to key road sections. Third, a variant susceptible-infective (SI) model is utilized to evaluate the impact of these critical sections. A case study analyzing the urban traffic network of Haining City, Zhejiang Province, demonstrates the practical utility and effectiveness of the proposed method.

**Keywords:** urban traffic road network; critical road sections; weighted and directed complex networks; LinkRank algorithm; variant SI model

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## 0 Introduction

Since the emergence of small-world characteristics [1] and scale-free properties [2], complex network theory and its applications have advanced rapidly. The identification of critical edges in complex networks has attracted considerable research attention. In urban traffic networks, critical edges—though few in number—significantly affect overall network circulation. When these key sections fail due to incidents such as traffic congestion, accidents, or construction, the entire network’s flow capacity can deteriorate severely, potentially leading to large-scale paralysis. Therefore, identifying critical road sections is essential for prioritizing daily traffic control and management efforts, as well as providing theoretical foundations and technical support for urban traffic planning and network redesign.

Numerous scholars have investigated critical edge identification methods. Ball et al. [3] proposed assessing edge importance by measuring changes in network shortest paths after edge removal. Girvan and Newman [4] introduced the concept of edge betweenness centrality to quantify edge importance, where higher betweenness indicates greater significance in resource transmission and network control. Reference [5] employed entropy weighting to evaluate three edge importance metrics—betweenness, spanning tree reduction rate, and average distance growth rate—combining them into a comprehensive weight for edge ranking. These methods have been applied across various domains: Fang et al. [6] identified critical lines in power grids using a maximum-flow-based complex network approach; reference [7] proposed a network cohesion-based method for critical line identification; and references [8, 9] identified critical edges in military and naval cooperative anti-missile networks by considering edge betweenness and endpoint support. Some researchers have also applied complex network theory to traffic networks. Chinthavali [10] developed an improved centrality index

based on information entropy to identify critical nodes (i.e., road sections) using dual topology structures. Zhang et al. [11, 12] proposed an automatic road network selection method based on dual-graph node importance evaluation, incorporating m-order neighbor concepts, node degree, betweenness, inter-node distances, and contributions from nodes and their 1- to m-order neighbors. Additional studies have similarly employed dual modeling with complex network metrics such as degree, k-core decomposition, betweenness, and closeness to identify critical road sections [13, 14].

However, existing complex network approaches for identifying critical urban traffic road sections exhibit two primary limitations: (a) they consider only structural factors like degree and betweenness while neglecting realistic factors affecting traffic flow, including road width, flatness, traffic volume, and urbanization density; and (b) they ignore the directional nature of urban road sections, as real-world traffic flows are often unidirectional or bidirectional with specific orientations.

To address these gaps, this paper proposes a critical road section identification method based on directed weighted complex networks. The approach first constructs a directed weighted complex network model that incorporates realistic factors such as traffic volume, road width, flatness, and urbanization density while preserving directionality. Second, the LinkRank algorithm ranks road section importance within this model to identify critical sections. Finally, a variant SI model evaluates the accuracy and effectiveness of the identification results.

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## 1 Urban Traffic Road Network Modeling Based on Complex Networks

Urban traffic networks are typically mapped to complex networks using either the primal method [16] or the dual method [17]. The primal method maps intersections to nodes and road sections to edges (Figure 1a), while the dual method maps intersections to edges and road sections to nodes (Figure 1c). This study employs the primal method with additional considerations for direction and weight:

- a) Intersections and road sections are mapped to nodes ( $N_i$ ) and undirected edges ( $E_{ij}$ ), respectively (Figure 1a).
- b) Traffic directionality is mapped to edge directions: bidirectional roads become bidirectional edges, while unidirectional roads become directed edges.
- c) Realistic factors affecting traffic flow are mapped to edge weights ( $W_{ij}$ ) through comprehensive evaluation (Figure 1b).

Table 1 presents the comprehensive evaluation framework for factors influencing

road section circulation. The composite evaluation value  $W_{ij}$  for a directed road section from intersection  $i$  to  $j$  is calculated as:

$$W_{ij} = \frac{1}{n} \sum_{k=1}^n f_k w_k$$

where  $f_k$  represents the score for factor  $k$ , and  $w_k$  denotes its relative weight determined through expert scoring. Traffic volume values represent averages during morning and evening peak hours (7:00–9:00 and 17:00–19:00).

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## 2 Road Section Importance Ranking Using LinkRank Algorithm

For critical edge identification in complex networks, Kim et al. [18] proposed the LinkRank algorithm, which effectively ranks edge importance in both directed and undirected networks. LinkRank extends Google's PageRank algorithm [19], whose formulation is:

$$\hat{T} = \alpha G + \beta \mathbf{1} \mathbf{1}^T$$

where  $\hat{T}$  represents the fixed row vector of matrix  $G$  (the PageRank vector), with each element  $\hat{T}_i$  indicating the probability that a random walk resides at node  $i$  in the stationary state. Initially, all  $\hat{T}_i$  values are set to  $1/n$ , where  $n$  is the number of nodes. The Google matrix  $G$  is computed as:

$$G_{ij} = \frac{A_{ij}}{A_{out,i}} (1 - \alpha) + \alpha \delta_{ij}$$

where  $A_{ij}$  denotes the adjacency matrix element;  $A_{out,i}$  represents the out-degree of node  $i$ ;  $(1 - \alpha)$  is the probability of randomly jumping to any page;  $\alpha = 0.85$  for dangling nodes and 0 otherwise; and  $\delta_{ij}$  is the probability of following a link (typically 0.85).

Based on PageRank, the LinkRank formula is:

$$L_{ij} = G_{ij} \hat{T}_j$$

where  $L_{ij}$  represents LinkRank matrix elements,  $\hat{T}_j$  are PageRank vector elements, and  $G_{ij}$  are Google matrix elements.

The importance ranking process is illustrated in Figure 2 [Figure 2: see original paper] and proceeds as follows:

- 1) **Obtain weighted adjacency matrix  $W$** : Represent the directed weighted complex network using a weighted adjacency matrix.
- 2) **Compute Google matrix  $G$** : Transform the weighted adjacency matrix using equation (3), where  $A_{ij}$  now represents weighted adjacency matrix elements and  $A_{out,i}$  denotes the sum of weights from node  $i$  to its neighbors.

- 3) **Initialize PageRank vector  $\hat{T}$** : Set each  $\hat{T}_i$  to  $1/n$ .
- 4) **Iterate**: Iteratively compute  $\hat{T}$  using equation (2). Calculate the difference vector  $\Delta$  between iterations; continue if any  $\Delta$  element  $> 0.0000001$ .
- 5) **Obtain final PageRank vector  $\hat{T}_L$** : Terminate iteration when all  $\Delta$  elements  $< 0.0000001$ .
- 6) **Compute LinkRank matrix  $L$** : Multiply the final PageRank vector  $\hat{T}_L$  by Google matrix  $G$  using equation (4).
- 7) **Generate ranking**: For unidirectional edges between nodes  $i$  and  $j$ , use the corresponding LinkRank value as the importance measure. For bidirectional edges, average  $L_{ij}$  and  $L_{ji}$  to obtain the importance value for road section  $E_{ij}$ . Sort all sections by importance to obtain the final ranking.

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### 3 Impact Assessment of Critical Road Sections Using Variant SI Model

The variant SI model [20] extends the standard SI model [21] by assuming each node has identical infectivity  $A$ , meaning each node generates  $A$  contacts per time step (where  $A$  is a constant, typically the network's average degree). From a structural perspective, the infection process is illustrated in Figure 3 [Figure 3: see original paper]. When infected nodes (black nodes in Figure 4 [Figure 4: see original paper]) appear, they contact neighbors: if  $A$  exceeds the infected node's degree, it contacts all neighbors; if  $A$  is smaller, it randomly selects  $A$  neighbors. Susceptible nodes (white nodes in Figure 3) become infected with probability  $\beta$  upon contact. This process continues iteratively until no susceptible nodes remain.

Urban road sections exist in either normal or disrupted states. Normal sections facilitate smooth traffic flow without affecting adjacent sections, while disrupted sections impede traffic and probabilistically affect neighboring sections. This dichotomy aligns perfectly with the variant SI model's node states, making it suitable for evaluating critical section impacts.

For assessment, the urban traffic network is remodeled using the dual method, mapping road sections to nodes and intersections to edges. The infection probability  $\beta_{ij}$  for each road section node is defined as:

where  $\beta_{ij}$  is the infection probability of directed section  $E_{ij}$ ,  $W_{ij}$  is its composite weight, and  $w_k$  represents factor weights.

The evaluation procedure involves:

- a) Selecting network nodes representing road sections ranked in the top 10%,

- between 10%-50%, and bottom 50% for virus propagation simulation.
- b) After the variant SI infection process, measuring each node' s comprehensive influence through its infection spread rate within the network.
  - c) Validating the identification method by confirming consistency between LinkRank rankings and infection results.

While the LinkRank algorithm effectively incorporates realistic factors and directionality, the dual-method remodeling for SI evaluation neglects directionality and converts realistic factors into uniform infection probabilities without considering neighbor-specific conditions. However, the variant SI model' s epidemic spreading effect appropriately captures the principle that more critical sections exert greater influence on neighboring and network-wide traffic, with influence manifested through infection rate magnitude. Thus, the evaluation approach remains valid.

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## 4 Application Analysis for Urban Traffic Network Critical Section Identification

With rapid economic development and increasing vehicle ownership, Haining City in Zhejiang Province faces severe traffic congestion, particularly during holidays or when incidents occur. Identifying critical sections is therefore crucial. Using Google Maps, we obtained the actual road network of Haining' s urban area (Figure 4 [Figure 4: see original paper]), selecting 54 major intersections and 86 road sections.

### 4.1 Complex Network Modeling of Urban Traffic Network

For Haining' s traffic network, we applied the primal method to map intersections and road sections to nodes and edges, respectively. Traffic directions were mapped to edge directions, and field surveys determined factor scores for each section. Composite importance values were calculated using equation (1) and mapped to edge weights. Expert scoring determined the relative weights for width, flatness, traffic volume, and urbanization density as  $w_1=1$ ,  $w_2=1$ ,  $w_3=2$ , and  $w_4=1$ , respectively. The network model was constructed using Gephi (Figure 5 [Figure 5: see original paper]).

### 4.2 Road Section Importance Ranking

The LinkRank algorithm ranked all road sections by importance. The ranking process was implemented in Java, with critical sections identified in Figure 6 [Figure 6: see original paper] and detailed importance values provided in Table 2 . Due to space limitations, only the top eight (1-8), middle four (41-44), and bottom four (83-86) sections are listed.

The top 10% of sections were designated as critical: E\_(21)(22), E\_(20)(21), E\_(33)(34), E\_(34)(41), E\_(19)(20), E\_(22)(29), E\_(27)(32), and

E\_(35)(36).

### 4.3 Impact Assessment of Critical Sections

The variant SI model evaluated critical section impacts. The network was remodeled using the dual method, mapping road sections to nodes and intersections to edges. Java implementations simulated infections on:

- Top 10% sections (E\_(21)(22), E\_(20)(21))
- 10%-50% sections (E\_(14)(15), E\_(40)(41))
- Bottom 50% sections (E\_(23)(36), E\_(36)(43))

Results in Figure 7 [Figure 7: see original paper] show that sections E\_(21)(22), E\_(20)(21), E\_(33)(34), E\_(34)(41), E\_(19)(20), E\_(22)(29), E\_(27)(32), and E\_(35)(36) exhibit progressively slower infection rates. Critically, the top 10% sections (E\_(21)(22), E\_(20)(21)) spread infection significantly faster than 10%-50% sections (E\_(14)(15), E\_(40)(41)), which in turn spread much faster than bottom 50% sections (E\_(23)(36), E\_(36)(43)). This confirms that LinkRank importance rankings align with actual influence, validating the identification method's accuracy and effectiveness.

From a practical perspective, the identified critical sections are located in the city center with narrow roads, high urbanization density, and persistent congestion during peak hours, further confirming the method's validity.

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## 5 Conclusion

Identifying critical road sections is vital for improving urban traffic network circulation. This study constructs a directed weighted complex network model incorporating realistic factors (road width, flatness, traffic volume, urbanization density) and directionality. The LinkRank algorithm ranks section importance within this model, considering both weights and directionality. Critical sections are then evaluated using a variant SI model. The Haining City case study demonstrates the method's effectiveness.

The selected factors are representative and universally applicable, making the model and identification method suitable for cities with diverse traffic characteristics. Future research should address two limitations: (1) incorporating intersection-related influencing factors, and (2) accounting for the dynamic nature of traffic factors rather than static values.

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