Modified Newtonian Dynamics with Inverse Dissipation Potential as an Alternative to Dark Matter and Dark Energy

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Abstract

This paper introduces the inverse dissipation potential into the Newtonian Dynamic equation and studies the motion equations of the objects in the isolated gravitational system. It is found that at large scales it can derive the dynamical equation of cosmic expansion similar to the ΛCDM model and yield the flat rotation curves for spiral galaxy. Different from the usual dark matter models, the derived flat rotation curves are the result of time accumulation rather than the direct action of mechanics. And the Tully-Fisher relationship is also discussed, it is found that the basic constant a0 in the MOND model and the form of the function have a clear corresponding physical significance in the model of this paper.

Full Text

Preamble

Modified Newtonian Dynamics with Inverse Dissipation Potential as an Alternative to Dark Matter and Dark Energy
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Abstract

This paper introduces an inverse dissipation potential into Newton’ s dynamic equation and studies the equations of motion for objects in isolated gravitational systems. It is found that at large scales, this can derive a cosmic expansion dynamic equation similar to the ΛCDM model and produce flat rotation curves for spiral galaxies. Unlike conventional dark matter models, the flat rotation curves derived here result from temporal accumulation rather than direct mechanical action. The Tully-Fisher relationship is also discussed, and it is found that the fundamental constant a and the form of the function   in the MOND model have clear corresponding physical significance in the model presented in this paper.

Keywords: dark matter, dark energy, Newtonian dynamic equation, inverse dissipation, MOND

1. Introduction

Dark matter and dark energy are two major problems in today's cosmology [1-4]. These two "invisible"substances produce completely distinct effects: dark matter generates attractive gravity, while dark energy produces repulsive anti-gravity. Various theoretical models based on general relativity have been proposed to explain these phenomena. Broadly speaking, there are two modification approaches. By modifying the left-hand side of the Einstein field equations, the spacetime geometry itself is altered, yielding theories such as f(R) gravity [5,6], brane-world gravity [7], and MOND [8,9]. The other approach modifies the right-hand side of the Einstein field equations by introducing additional fields or matter within spacetime, as exemplified by ΛCDM [10], Quintessence [11], and phantom models [12,13].

However, none of the current theories can perfectly resolve all issues. In particular, since searches for dark matter particles and direct measurements of dark energy have thus far found nothing [14,15], this prompts us to consider that modifying the gravity equation might be a worthwhile option to explore. As can be seen in Refs. [16-21], many scholars have conducted extensive work in this area.

Among the various models that seek to replace dark matter, the MOND model proposed by M. Milgrom is a widely discussed one. This model presents an empirical formula that introduces a fundamental constant a to explain certain observational phenomena [22-28]; for example, it can successfully explain the Tully-Fisher relationship [24,25]. However, the model still faces challenges in explaining other observational data, such as mass discrepancies in galaxy clusters [29], and the theory itself violates the fundamental law of momentum conservation [30].

In short, the MOND theory was not perfect at the time of its inception. Therefore, based on the original MOND theory, Bekenstein et al., T.G. Zlosnik et al., and M. Milgrom proposed relativistic MOND theories—namely Tensor-Vector-Scalar Theory (TeVeS) [20,26], Einstein-Aether theory [31], and subsequently Bimetric MOND Theory [32,33], respectively. Although these theories greatly enrich the content of MOND, problems persist [34,35].

The partial success of MOND theory and its extensions vaguely implies that it may stem from a more fundamental theoretical framework. Hence, this paper, based on non-relativistic Newtonian dynamics, employs the theoretical method of modifying inertia [36] by introducing a dissipative potential into Newton' s dynamic equation to discuss its influence on the motion of objects at large scales, and attempts to further reveal the deep physical significance of the fundamental constant a in MOND theory.

2. Newtonian Dynamic Equation with Dissipative Potential Energy

2.1 Modified Equation and Lagrangian

In the non-relativistic framework, the Newtonian dynamic equation is

$$m\mathbf{a} = \mathbf{F} \tag{1}$$

where m is the mass of the object and $\mathbf{a}$ is the kinematic acceleration.

There are typically two ways to modify the Newtonian dynamic equation: the first is to modify the right-hand side to establish a modified inertia theory, and the second is to modify the left-hand side to establish a modified gravity theory. In the relativistic case, these two modification approaches are equivalent. However, in the non-relativistic case, applying the modified inertia theory, Eq. (1) can be assumed to take the following modified form:

$$m\mathbf{a} = \mathbf{F} - \lambda\mathbf{v} \tag{2}$$

where λ is a constant and $\mathbf{v}$ is the velocity of the moving object.

For an isolated gravitational system Σ consisting of only two objects, where an object with mass m moves within a 2D plane in the gravitational field of a quasi-stationary object with mass M ($M \gg m$), we can obtain the Lagrangian for system Σ based on Eq. (2):

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GMm}{r} - \lambda r\dot{r}\dot{\theta} \tag{3}$$

where (r, θ) is the polar coordinate system of the 2D plane and G is the gravitational constant.

Since a velocity-dependent dissipation function is introduced in Eq. (2), the corresponding dissipation potential is

$$\Phi = \frac{1}{2}\lambda r^2\dot{\theta}^2 \tag{4}$$

According to the Hamiltonian principle for a dissipative system, the equation of motion for the object with mass m is

$$\begin{cases} m\ddot{r} - mr\dot{\theta}^2 + \frac{GMm}{r^2} + \lambda r\dot{\theta}^2 = 0 \\ mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} - \lambda r\dot{r} = 0 \end{cases} \tag{5}$$

2.2 Cosmological Expansion

When the object with mass m moves along the radial direction of the polar coordinate system, namely $\theta = 0$, the second equation of Eq. (5) can be rewritten as

$$\dot{r} = \frac{GM}{\lambda r} \tag{6}$$

As is known, the cosmological equations can be derived from Newtonian mechanics [37]. For a sphere with radius r, if the physical coordinate is $r = a(t)R$ (where R is the comoving coordinate of cosmic expansion and $a(t)$ is the cosmic expansion factor), the density is $\rho = \rho_0 a^{-3}$, and the mass of material inside the sphere is $M = \frac{4\pi}{3}\rho r^3$, then from Eq. (6) we obtain

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_0 a^{-1} + \frac{2GM}{\lambda a R^2} \tag{7}$$

Substituting Hubble' s law into Eq. (7), we obtain

$$H^2 = \frac{8\pi G}{3}\rho_0 + \frac{2GM}{\lambda a^2 R^2} \tag{8}$$

Comparing Eq. (8) with the time-time component of the field equation in the ΛCDM model [10], we find

$$\Lambda = \frac{2GM}{\lambda a^2 R^2} \tag{9}$$

Equation (9) shows that the Newtonian dynamic equation with inverse dissipation potential can derive dynamic equations similar to the expansive cosmological ΛCDM model at large scales.

2.3 Flat Rotation Curves

Now we discuss the second case. For a given initial condition at $t = 0$, an object with mass m orbits a quasi-stationary object with mass M in its gravitational field, i.e., $\dot{\theta} = \text{constant}$. In a dissipative system, both r and $\dot{\theta}$ will change with time. Let us assume the second equation in Eq. (5) can be rewritten as

$$mr^2\dot{\theta} = \text{constant} \tag{10}$$

Considering the initial condition, when the gravitational field is very weak, namely

$$\frac{GM}{r^2} \ll \lambda \dot{r} \tag{11}$$

then substituting Eq. (11) into Eq. (10) yields

$$\dot{\theta} = \frac{\lambda}{mr} \tag{12}$$

Solving Eq. (12), the radius r evolves as

$$r(t) = r_0 e^{\frac{\lambda}{m}t} \tag{13}$$

From the first equation in Eq. (5), we obtain

$$\ddot{r} - r\dot{\theta}^2 + \frac{GM}{r^2} + \frac{\lambda}{m} r\dot{\theta}^2 = 0 \tag{14}$$

Therefore

$$V^2 = \frac{GM}{r} \tag{15}$$

Since the rotational tangential velocity is $V = r\dot{\theta}$, and using Eq. (12), we find

$$V = \sqrt{\frac{GM}{r_0}} = V_0 \tag{16}$$

It can be seen from Eq. (13) and Eq. (16) that the radius increases exponentially with time, while the rotational tangential velocity of the object remains constant, which coincides with the asymptotically flat rotational velocity properties typically exhibited by some spiral galaxies [38,39].

3. MOND Model

As is known, M. Milgrom introduced a fundamental constant a_0 into MOND theory, which modifies the relationship between actual acceleration and Newtonian acceleration in Newtonian mechanics as follows [8,9]:

$$a = \mu\left(\frac{a}{a_0}\right) a_N \tag{17}$$

where $a_N = GM/r^2$ is the Newtonian acceleration. When $a/a_0 \gg 1$, $\mu(a/a_0) \sim 1$; when $a/a_0 \ll 1$, $\mu(a/a_0) \sim a/a_0$.

From Eq. (12), it is shown that when the gravitational field in the initial condition is very weak, the tangential velocity of the object with mass m remains essentially unchanged with time, while the distance r can still increase. Suppose the intensity of the weak gravitational field initially corresponds to g_0, that is

$$\lambda = mg_0 \tag{18}$$

Substituting Eq. (18) into Eq. (16), we obtain

$$V^4 = GMg_0 \tag{19}$$

Rewriting Eq. (19) yields

$$M = \frac{V^4}{Gg_0} \propto \left(\frac{M}{L}\right) L \tag{20}$$

where M/L is the mass-to-light ratio.

We can see that Eq. (20) is precisely the Tully-Fisher relationship. Comparing Eq. (20) with Eq. (17), we find that for the MOND theory, $g_0 = a_0$.

The centripetal/actual acceleration of the object obtained from Eq. (13) is

$$a = \frac{V^2}{r} = \frac{GM}{r^2} \tag{21}$$

The Newtonian acceleration is

$$a_N = \frac{GM}{r^2} \tag{22}$$

Dividing Eq. (22) by Eq. (21) gives

$$\frac{a_N}{a} = \frac{r}{r_0} \tag{23}$$

Hence, comparing Eq. (23) with Eq. (17), it is shown that the model in this paper can derive the form of the empirical function μ introduced in the MOND theory.

4. Conclusion

This paper studies an isolated gravitational system by introducing an inverse dissipation potential into Newton' s dynamic equation. It is found that the modified dynamic equation can simultaneously derive a cosmic expansion dynamic equation similar to the ΛCDM model and the asymptotically flat rotational velocity properties typically exhibited by spiral galaxies—effects generally attributed to dark energy and dark matter, respectively. This suggests that the dark energy effect and the dark matter effect may be two aspects of the same physical reality following the introduction of the inverse dissipation potential.

Unlike conventional dark matter models, this type of asymptotically flat rotational velocity property of spiral galaxies results from long-term temporal accumulation rather than direct mechanical action. Moreover, in the derived cosmic expansion dynamic equation, an equivalent time-varying cosmological constant appears, which is consistent with the Quintessence and phantom models.

Finally, similar to the MOND model, the modified Newton' s dynamic equation also yields the Tully-Fisher relationship. Furthermore, the fundamental constant a_0 introduced in the MOND model and the form of the function μ exhibit clear corresponding physical significance in the model presented in this paper. However, the discussion in this paper is not yet a relativistic extension of Newtonian gravity, which requires further study.

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