Modified Newtonian Dynamics Equation with Inverse Dissipation Potential: A Possible Model for Explaining Dark Matter and Dark Energy

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Abstract

This paper introduces an inverse dissipation potential into the Newtonian dynamical equations and investigates the equations of motion for objects within isolated gravitational systems. It is found that on large scales, this approach can derive cosmological expansion dynamical equations analogous to those of the standard model, as well as the asymptotically flat rotation velocity property of disk galaxies. Unlike conventional dark matter models, this asymptotically flat rotation velocity property emerges as a result of temporal accumulation rather than direct mechanical action. The Tully-Fisher relation is also discussed, revealing that the fundamental acceleration constant a_0 in the MOND model and the form of the μ-function possess clear corresponding physical interpretations within the framework presented herein.

Full Text

Modified Newtonian Dynamics with Inverse Dissipation Potential: A Possible Model for Explaining Dark Matter and Dark Energy

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Abstract

In this paper, we introduce an inverse dissipation potential into the Newtonian dynamics equation and investigate the equations of motion for objects in isolated gravitational systems. We find that at large scales, this approach can derive cosmological expansion dynamics similar to the ΛCDM model and reproduce the asymptotically flat rotation velocity profiles of disk galaxies. Unlike conventional dark matter models, this asymptotically flat rotation velocity emerges as a result of temporal accumulation rather than direct mechanical action. We also discuss the Tully-Fisher relation and find that the natural constant a and the form of the   function in MOND have clear physical correspondences in our model.

Keywords: Dark matter; dark energy; Newtonian dynamic equation; inverse dissipation potential; MOND

Introduction

Dark matter and dark energy represent two major puzzles in modern cosmology [1-4]. These "invisible" forms of matter exhibit two distinct effects: dark matter produces gravitational attraction, while dark energy generates repulsive gravity. To explain these phenomena, various theoretical models have been proposed within the framework of general relativity, falling into two main categories. The first approach modifies the left-hand side of Einstein' s field equations—that is, the spacetime geometry itself—including theories such as f(R) gravity [5-6], brane-world gravity [7], and MOND [8-9]. The second approach modifies the right-hand side of Einstein' s field equations by introducing additional fields or matter into spacetime, including models such as ΛCDM [10], Quintessence [11], and phantom [12-13]. However, none of these theories have yet provided a perfect solution to all problems. In particular, experimental searches for dark matter particles and measurements of dark energy have so far yielded no definitive results [14-15], suggesting that modifying the gravitational equations themselves may be a worthwhile avenue to explore, a direction in which several researchers have already made significant efforts [16-21].

Among the various alternatives to dark matter, the MOND model proposed by M. Milgrom has been widely discussed. This model introduces a fundamental natural constant a and provides an empirical formula that successfully explains several observed phenomena at galactic scales [22-28], such as the Tully-Fisher relation [24-25]. However, the model still faces challenges in explaining other observational data and suffers from theoretical issues, including its inability to account for mass discrepancies in galaxy clusters [29] and its violation of momentum conservation, a fundamental physical law [30]. Consequently, while the original MOND theory was incomplete, Bekenstein and colleagues developed a relativistic extension known as Tensor-Vector-Scalar Theory (TeVeS) [20,26], T. G. Zlosnik et al. proposed Einstein-Aether theory [31], and M. Milgrom subsequently introduced Bimetric MOND Theory [32-33]. These extensions have substantially enriched the MOND framework, though they introduce other problems [34-35].

Nevertheless, the partial successes of MOND and its extensions suggest that they may originate from a more fundamental theoretical framework. This paper attempts to build upon non-relativistic Newtonian dynamics using the modified inertia approach [36], specifically by introducing a dissipation potential into the Newtonian dynamics equation. We investigate its effects on object motion at large scales and seek to reveal deeper physical origins for the natural constant a in MOND theory.

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Newtonian Dynamics with Dissipation Potential

In the non-relativistic regime, the Newtonian dynamics equation is:

$$m\mathbf{a} = \mathbf{F} \tag{1}$$

where m is the object mass and $\mathbf{a}$ is the acceleration. Following MOND' s philosophy, there are two primary approaches to modification: the first modifies the right-hand side of the equation by establishing a modified inertia theory, while the second modifies the left-hand side to create a modified gravity theory. In the relativistic case, inertia and gravity theories are equivalent. This work considers a modified inertia theory in the non-relativistic context, transforming Eq. (1) into:

$$m\mathbf{a} + \lambda(\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{F} \tag{2}$$

where λ is a constant and $\mathbf{v}$ is the object velocity.

For an isolated gravitational system Σ consisting of only two objects, where an object of mass m moves in the gravitational field of a quasi-stationary object of mass M ($M \gg m$) in the equatorial plane, the Lagrangian for system Σ based on Eq. (2) becomes:

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GMm}{r} \tag{3}$$

where (r, θ) are equatorial plane coordinates.

Equation (2) introduces a velocity-dependent dissipation function, with corresponding dissipation potential:

$$\Phi = -\frac{1}{2}\lambda v^2 \tag{4}$$

Applying Hamilton' s principle for dissipative systems yields the equation of motion for the object of mass m:

$$\begin{cases} m\ddot{r} - mr\dot{\theta}^2 + \frac{GMm}{r^2} + \lambda\dot{r} = 0 \\ mr\ddot{\theta} + 2m\dot{r}\dot{\theta} + \lambda r\dot{\theta} = 0 \end{cases} \tag{5}$$

Cosmological Implications

It is known that cosmological equations can be derived from Newtonian mechanics [37]. Taking the physical coordinate $\mathbf{r} = a(t)\mathbf{x}$ (where $\mathbf{x}$ is the comoving coordinate for cosmic expansion and $a(t)$ is the cosmic scale factor) and considering a sphere of radius r containing mass M, Eq. (6) yields:

$$\ddot{a} = -\frac{4\pi G}{3}\rho a - \frac{\lambda}{m}\dot{a} \tag{7}$$

where ρ is the matter density. According to Hubble' s law $\dot{a} = Ha$, Eq. (7) becomes:

$$\ddot{a} = -\frac{4\pi G}{3}\rho a - \frac{\lambda}{m}Ha \tag{8}$$

Comparing with the time-time component of the field equations in the ΛCDM model [10], we obtain:

$$\frac{\lambda}{m} = \frac{\Lambda}{3H} \tag{9}$$

Equation (9) demonstrates that the Newtonian dynamics equation with inverse dissipation potential can produce cosmological expansion dynamics similar to those in the ΛCDM model at large scales.

Galactic Rotation Curves

Now consider an object of mass m undergoing circular motion ($\dot{r} = 0$) in the gravitational field of a quasi-stationary object of mass M under initial conditions. In dissipative systems, both r and θ vary with time. Setting $\dot{\theta} = \omega$, the second equation in (5) becomes:

$$\dot{\omega} + \frac{2\dot{r}}{r}\omega + \frac{\lambda}{m}\omega = 0 \tag{10}$$

Under initial conditions where the gravitational field is weak ($r \to \infty$), we have $\dot{r} \gg r\dot{\theta}$. Substituting Eq. (12) into Eq. (11) gives:

$$\ddot{r} + \frac{\lambda}{m}\dot{r} = 0 \tag{13}$$

Solving Eq. (13) yields:

$$r(t) = r_0 e^{-\frac{\lambda}{m}t} \tag{14}$$

From the first equation in (5), we obtain:

$$\dot{\theta} = \sqrt{\frac{GM}{r^3}} \tag{15}$$

Combining Eqs. (14) and (15) gives:

$$V_\theta = r\dot{\theta} = V_0 e^{\frac{\lambda}{2m}t} \tag{16}$$

where V_θ is the rotational velocity and V_0 is the initial rotational velocity. Equations (14) and (16) show that while the radius r increases exponentially with time, the rotational velocity of the object of mass m remains constant. This behavior is consistent with the asymptotically flat rotation velocity profiles observed in many disk galaxies [38-39].

Connection to MOND Theory

In M. Milgrom' s MOND theory, a natural constant a_0 is introduced, modifying the relationship between actual acceleration and Newtonian acceleration as follows [8-9]:

$$\mathbf{a}\mu\left(\frac{a}{a_0}\right) = \mathbf{a}_N \tag{17}$$

From Eq. (12), we see that in a weak gravitational field under initial conditions, the linear velocity of the object of mass m remains essentially constant while the distance r can continue to increase. Let the weak gravitational field strength be g_0, then:

$$g_0 = \frac{GM}{r^2} \tag{18}$$

Substituting Eq. (18) into Eq. (16) yields:

$$V^4 = GMa_0 \tag{19}$$

From Eq. (19), we can derive:

$$L \propto M^4 \tag{20}$$

where L/M is the mass-to-light ratio. Equation (20) represents the Tully-Fisher relation. Comparing Eq. (19) with MOND theory reveals that $g_0 = a_0$.

Furthermore, from Eq. (14), the actual acceleration of the object is:

$$a = \ddot{r} = \left(\frac{\lambda}{m}\right)^2 r \tag{21}$$

while the Newtonian acceleration is:

$$a_N = \frac{GM}{r^2} \tag{22}$$

Combining Eqs. (21) and (22) gives:

$$\mu\left(\frac{a}{a_0}\right) = \frac{a_0}{a} \tag{23}$$

Comparing Eqs. (17) and (23) demonstrates that our model can derive the empirical function form introduced in MOND theory.

Discussion and Conclusion

This paper attempts to introduce an inverse dissipation potential into the Newtonian dynamics equation and study an isolated gravitational system. We find that the modified dynamics equation can simultaneously produce cosmological expansion dynamics similar to the ΛCDM model and asymptotically flat rotation velocity profiles for disk galaxies—phenomena typically attributed to dark energy and dark matter, respectively. This suggests that with the introduction of the inverse dissipation potential, dark energy and dark matter effects represent two aspects of the same physical essence.

Unlike conventional dark matter models, the asymptotically flat rotation velocity profiles of disk galaxies emerge as a result of long-term temporal accumulation rather than direct mechanical action. Furthermore, the derived cosmological expansion equation exhibits an effective time-varying cosmological constant, consistent with Quintessence and phantom models.

Finally, like the MOND model, this modified Newtonian dynamics equation successfully reproduces the Tully-Fisher relation, revealing that the natural constant a_0 and the function form introduced in MOND have clear physical interpretations in our model. However, the present discussion remains a non-relativistic modification of Newtonian gravity, and further extension to a relativistic framework is needed in future work.

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Note: Figure translations are in progress. See original paper for figures.