E(x) Characteristic Analysis of Polling Systems with Station State Differentiation - Postprint

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Abstract

To improve the operational efficiency and resource utilization of polling control systems, a limited (K=2) polling control system based on station sleep-active states is proposed. The system' s mathematical model is studied using the probability generating function and embedded Markov chain method. Simulation results show that theoretical values closely match experimental values, indicating that the analysis method is correct and reasonable. Based on the limited (K=2) policy, the server serves stations according to their sleep-active states. The adoption of the limited (K=2) service strategy ensures that system fairness is not compromised, while distinguishing station sleep-active states avoids serving dormant stations without information packets. Compared with existing service strategies, system performance is significantly improved.

Full Text

Analysis of E(x) Characteristics of Polling System that Distinguishes Site Status

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Abstract

To improve the working efficiency and resource utilization of polling control systems, this paper proposes a limited (K=2) polling control system that distinguishes between busy and idle site states. The mathematical model of the system is studied using probability generating functions and the embedded Markov chain method. Simulation results demonstrate that theoretical values closely match experimental values, validating the correctness and rationality of the analytical method. Based on the limited (K=2) policy, the system serves stations according to their busy/idle status. The limited (K=2) service strategy ensures system fairness is not compromised, while distinguishing between busy and idle states avoids serving dormant stations without information packets. Compared with existing service strategies, system performance is significantly improved.

Keywords: K=2; busy/idle state; polling; efficiency

0 Introduction

In polling control systems, querying idle stations without information packets wastes system resources. While literature [7] attempts to avoid idle queries by allocating channels only to stations with transmission demands, the polling schedule remains independent of station status. For idle stations, the receiver must wait until the first time slot passes without receiving data before switching to the next sender. When stations remain idle, the receiver must query these idle stations during every round of channel allocation, resulting in wasted system resources.

Wireless sensor networks integrate sensor technology, embedded computing, and wireless communication, finding widespread applications in military, environmental monitoring, and healthcare domains [1]. Literature [2, 3] analyzes polling queueing systems in cognitive radio networks and product development management models, while literature [4] examines polling control systems in wireless networks, and literature [5] investigates their application in FPGA data acquisition systems.

Literature [6] addresses limited (K=N) service systems in polling, where different K values can distinguish service priorities—the larger the K value, the more information packets sent per service, and the higher the priority, thus achieving multi-priority control. However, literature [6] does not provide theoretical analysis for limited (K>1) service strategies. With the rapid development of modern network technology and strong demand for differentiated service priorities, the limited (K=1) service strategy can no longer meet practical needs. There is an urgent requirement for limited (K>1) polling control strategies that can both distinguish service priorities and maintain system fairness, while laying a theoretical foundation for subsequent research on limited service.

To address these issues, this paper proposes a limited (K=2) polling system model that distinguishes between busy and idle stations (Busy and Idle Polling Limited K=2, BIPL2) based on dynamic principles [8]. The system classifies stations as busy or idle according to whether they have information packets to send, and updates the polling schedule each round based on station buffer status, serving only busy stations with service demands. This reduces the average waiting time and improves system working efficiency. Using probability generating functions [9] and embedded Markov chains [10], the system model is thoroughly analyzed. Finally, simulation experiments verify the correctness and rationality of the theoretical analysis.

1.1 Model Principle Analysis

The BIPL2 system consists of N stations and one server, as shown in Figure 1 [Figure 1: see original paper]. The server first updates the status of each station, allowing it to serve only busy stations with transmission demands. This eliminates the need for the server to listen to and query idle stations, thereby reducing the average queue length and average waiting time. The service process adopts a limited (K=2) service strategy, where only two information packets are sent during each polling cycle.

Using the embedded Markov chain method, under system stability conditions [15], the Markov process is homogeneous, irreducible, aperiodic, and has a unique stationary distribution. The probability distribution in steady state is denoted by . At time t, the system state can be expressed as···

When the server begins serving a busy station i at time t, the relationship···

1.2 Variable Definition

Let v_i represent the service time required by the server to transmit information packets from station i according to the limited service strategy. Let η_{ij} denote the number of information packets arriving at station j during the service time of station i. The probability generating function of the system state variables at time t is defined as $G_i(z_1, z_2, ..., z_N)$.

Let $G_i(0, 0, ..., 0)$ represent the probability generating function of the system state variables when the number of information packets stored in all N terminal station buffers is 0 at time t.

1.3 Model Assumptions

a) Information packets arriving at each station follow independent and identically distributed Poisson processes, with probability generating function $A_i(z)$, mean λ_i, and variance $\sigma^2_{\lambda_i} = A''_i(1) + \lambda_i - \lambda_i^2$.

b) The service time for any information packet follows an independent and identical probability distribution, with probability generating function $B_i(z)$, mean β_i, and variance $\sigma^2_{\beta_i} = B''_i(1) + \beta_i - \beta_i^2$.

c) The time required for the server to switch between stations after completing service follows a probability distribution with generating function $R_i(z)$, mean γ_i, and variance $\sigma^2_{\gamma_i} = R''_i(1) + \gamma_i - \gamma_i^2$.

d) Each terminal station has sufficiently large buffer capacity to prevent data loss.

e) Cached information packets in the buffer are transmitted according to a First-Come-First-Served (FCFS) strategy.

1.4 Probability Generating Function

Assume that at time t_n, busy station i receives service from the server. When station i completes service of cached information packets according to the limited (K=2) rule, the server moves to serve station i+1, and station i+1 receives service at time t_{n+1}. Define the random variable $\xi_i(t_n)$ as the number of data information packets waiting in the buffer of station i at time t_n. The system state at time t_n can be expressed as $\{\xi_1(t_n), \xi_2(t_n), ..., \xi_N(t_n)\}$.

The polling system has a relatively fixed number of stations, and the system state at service start times is countable. This discrete-time countable state variable constitutes an embedded Markov chain. Under stable system conditions [15], the probability generating function is···

When the server begins serving busy station i at time t_n, the relationship is:

$$\begin{cases} \xi_j(t_{n+1}) = \xi_j(t_n) + \eta_{ij} & j \neq i \\ \xi_i(t_{n+1}) = \xi_i(t_n) + \eta_{ii} - 2 \end{cases}$$

The probability generating function of the system state is:

$$G_{i+1}(z_1, z_2, ..., z_N) = \lim_{t \to \infty} E\left[\prod_{j=1}^{N} z_j^{\xi_j(t_{n+1})}\right]$$

1.5 Average Queue Length

Define $g_i(j)$ as the average queue length of station j when station i is being served at time t_n. Define the first-order partial derivative characteristic of the system as:

$$g_i(j) = \left. \frac{\partial G_i(z_1, z_2, ..., z_N)}{\partial z_j} \right|_{z_1, z_2, ..., z_N \to 1}$$

Define the second-order partial derivative characteristic as:

$$g_i(j,k) = \left. \frac{\partial^2 G_i(z_1, z_2, ..., z_N)}{\partial z_j \partial z_k} \right|_{z_1, z_2, ..., z_N \to 1}$$

Based on equations (4)-(6), taking first and second order partial derivatives of equation (3) and simplifying yields:

For $i \neq j \neq k$:

$$g_i(j,k) = \frac{\lambda_j \lambda_k \theta}{1 - N\lambda\beta}\left[(1+\gamma\beta) + \frac{\lambda\gamma\beta}{2}\right]$$

For $i = j \neq k$:

$$g_i(i,k) = \frac{\lambda_i \lambda_k \theta}{1 - N\lambda\beta}\left[(1+\gamma\beta) + \frac{\lambda\gamma\beta}{2}\right] + \frac{2\lambda_k\beta}{1 - N\lambda\beta}$$

For $i = j = k$:

$$g_i(i,i) = \frac{\lambda_i^2 \theta}{1 - N\lambda\beta}\left[(1+\gamma\beta) + \frac{\lambda\gamma\beta}{2}\right] + \frac{4\lambda_i\beta}{1 - N\lambda\beta}$$

where $\theta = \frac{N\gamma}{1-N\lambda\beta}$.

1.6 Average Waiting Time

Define the average waiting time as the average time interval from when an information packet arrives at a station until its transmission service begins.

The average waiting time $E[w_i]$ for station i is derived as:

$$E[w_i] = \frac{g_i(i)}{\lambda_i} - \beta$$

Substituting the expressions for $g_i(i)$ yields:

$$E[w_i] = \frac{\lambda_i \theta}{1 - N\lambda\beta}\left[(1+\gamma\beta) + \frac{\lambda\gamma\beta}{2}\right] + \frac{2\beta}{1 - N\lambda\beta} - \beta$$

1.7 Average Polling Cycle

Define the system average polling cycle as the average time between two consecutive queries to the same station, derived from first-order derivatives:

$$T_c = \frac{N\gamma}{1 - N\lambda\beta}$$

1.8 System Throughput

Define system throughput as the number of information packets the system can transmit per unit time:

$$\text{Throughput} = \frac{N\lambda\beta}{T_c} = \frac{N\lambda\beta(1-N\lambda\beta)}{N\gamma}$$

2.1 Experimental Simulation

Based on the established BIPL2 system model, numerical calculations and simulation experiments were conducted under stable system conditions. Theoretical values for the BIPL2 system were calculated using equations (13)-(16).

The simulation was completed on the MATLAB2014a platform. Poisson distribution sequences with mean λ were generated to simulate the number of arriving information packets at stations. The simulated communication process was ideal, with zero packet loss and retransmission rates. The normalized time axis was divided into time slots.

Simulation Parameters: a) The number of information packets entering any station's buffer during a unit time slot follows a Poisson distribution. b) Symmetric control system: information packets arriving at each station follow identical probability distributions ($\lambda_i = \lambda$, $\beta_i = \beta$, $\gamma_i = \gamma$). c) System stability condition: $N\lambda\beta < 1$. d) Simulation parameters are labeled below each figure.

2.2 Results Analysis

Figures 2-5 demonstrate that the theoretical analysis method reasonably describes the BIPL2 polling control system, with good agreement between theoretical calculations and computer simulation results.

Figure 2 [Figure 2: see original paper] illustrates the relationship between throughput and station information packet arrival rate, showing that system throughput increases linearly with arrival rate. However, as shown in Figure 3 [Figure 3: see original paper], increased arrival rate also causes average waiting time to increase. Therefore, when considering how to improve system throughput, average waiting time should be considered as a constraint.

Figures 4 [Figure 4: see original paper] and 5 [Figure 5: see original paper] depict the relationship between average queue length, average polling cycle, and system load. Both metrics increase with system load, with small errors between theoretical and experimental values, consistent with theoretical derivations.

Figure 6 [Figure 6: see original paper] shows that the BIPL2 system achieves significant improvement in average waiting time compared to the simple limited

(K=1) system. As system arrival rate increases, the average waiting time for K=1 increases dramatically, while BIPL2' s average waiting time changes slowly and stabilizes. When the number of stations N increases from 10 to 20, K=1' s average waiting time increases rapidly with large fluctuations, whereas BIPL2 increases slowly with minimal fluctuations.

Figure 7 [Figure 7: see original paper] compares the average waiting time of single gated service and exhaustive service with BIPL2 under the same experimental conditions. The BIPL2 system' s average waiting time is significantly smaller than both single gated and exhaustive services. While the three classic polling service strategies (exhaustive, gated, and limited) have sequentially increasing average waiting times under identical conditions, BIPL2' s average waiting time is substantially lower because it avoids querying idle stations without information packets, thereby reducing system average waiting time and improving efficiency.

Figure 8 [Figure 8: see original paper] compares the throughput of BIPL2 and limited (K=1) systems, showing that under the same average waiting time conditions, the BIPL2 system achieves greater throughput.

The busy and idle limited (K=1) polling system model (BIPL1) was derived for comparison. Using average waiting time as the comparison metric, Figure 9 [Figure 9: see original paper] shows that as information packet arrival rate increases, the BIPL2 system achieves good service priority differentiation compared to the BIPL1 system, with significantly lower average waiting time. As stated in the paper, the limited (K=2) service strategy can send two information packets per service, increasing the number of packets sent per service compared to limited (K=1). Therefore, the BIPL2 system maintains system fairness while reducing average waiting time.

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3 Conclusion

This paper proposes the BIPL2 system, which, based on the limited (K=2) service strategy and leveraging station busy/idle status, serves only busy stations with information packets. This reduces query energy consumption for idle stations and improves system working efficiency and resource utilization. The system mathematical model was established using probability generating functions and embedded Markov chains, with precise analytical solutions for key system performance parameters.

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Note: Figure translations are in progress. See original paper for figures.