Crown Width-DBH Model Postprint for Pinus sylvestris var. mongolica Sand-Fixation Plantations

Authors: Han Yangang, Lei Zeyong, Zhao Guojun, Yanping Zhou, Xu Chang, Lei Zeyong

Date: 2018-09-03T00:00:00+00:00

Abstract

Based on data from 702 standing Mongolian pine (Pinus sylvestris var. mongolica) trees in 22 sample plots of artificial pure stands in the Zhanggutai area, we constructed basic models, generalized models, and basic and generalized models based on mixed effects for the crown width-DBH relationship of Mongolian pine sand-fixing forests; compared the prediction accuracy of mixed models when calculating random parameters using four schemes: random selection of sample trees, selecting trees with average DBH, selecting trees with smaller DBH, and selecting trees with larger DBH; and finally analyzed the effects of different tree factors and stand variables on the crown width-DBH relationship. Model evaluation metrics included the coefficient of determination (R^2), mean absolute error (MAE), and root mean square error (RMSE). The results showed that height to crown base (HCB), relative spacing (RS), and stand age (A) had the most significant effects on the crown width-DBH relationship; the fitting accuracy of mixed models (R^2, MAE, and RMSE of the basic mixed model were: 0.7030, 0.3866, and 0.5154, respectively; R^2, MAE, and RMSE of the generalized mixed model were: 0.7051, 0.3822, and 0.5136, respectively) was higher than that of ordinary least squares (OLS) regression models (R^2, MAE, and RMSE of the basic model were: 0.5875, 0.4696, and 0.6075, respectively; R^2, MAE, and RMSE of the generalized model were: 0.6618, 0.4155, and 0.5500, respectively). The difference between the basic mixed model and the generalized mixed model was small (the R^2, MAE, and RMSE of the two models differed by approximately 1%). Crown width decreased with increasing HCB and A, and increased with increasing RS. For crown width prediction, it is recommended to use the basic mixed model and select 2 average trees from each sample plot to calculate their random parameters, or to use the simpler OLS generalized model to predict individual tree crown width.

Full Text

Canopy-DBH Models for Sand-Fixing Plantations of *Pinus sylvestris* var. *mongolica*

HAN Yangang[1], LEI Zeyong[1], ZHAO Guojun[2], ZHOU Yanping[1], XU Chang[1]

[1]College of Environmental Science and Engineering, Liaoning Technical University, Fuxin 123000, Liaoning, China
[2]Liaoning Province Institute for Sand-fixation and Afforestation Research, Fuxin 123000, Liaoning, China

Abstract

This study utilized data from 702 individual trees of *Pinus sylvestris* var. *mongolica* across 22 temporary sample plots in pure plantations at Zhanggutai, Liaoning Province, northeast China, to develop canopy-DBH (diameter at breast height) models. Four modeling approaches were compared: a basic model fitted by ordinary least squares (OLS), a generalized model fitted by OLS, a nonlinear mixed-effects basic model, and a nonlinear mixed-effects generalized model. The goodness-of-fit and prediction accuracy of these four models were evaluated. For the mixed-effects models, four sampling strategies were designed to calculate random parameters: random sampling, large-DBH tree sampling, small-DBH tree sampling, and medium-DBH tree sampling. The effects of individual-level and stand-level variables on the canopy-DBH relationship were simulated. Model evaluation indices included the coefficient of determination (R^2), mean absolute error (MAE), and root mean square error (RMSE).

Results showed that height to live crown base (HCB), relative spacing index (RS), and stand age (A) were the dominant factors influencing canopy-DBH models. The mixed-effects basic model achieved goodness-of-fit statistics of $R^2 = 0.7030$, MAE = 0.3866, and RMSE = 0.5154, while the mixed-effects generalized model achieved $R^2 = 0.7051$, MAE = 0.3822, and RMSE = 0.5136. Both mixed-effects models outperformed their OLS counterparts. However, the difference in goodness-of-fit between the mixed-effects basic and generalized models was not significant (below 1%). Canopy width decreased with increasing stand age and HCB, but increased with increasing RS.

Keywords: *Pinus sylvestris* var. *mongolica*; sand-fixing plantation; canopy-DBH; mixed-effects basic model; generalized model

1. Introduction

The canopy-DBH relationship is fundamental for understanding tree growth dynamics and stand development in forest ecosystems. For *Pinus sylvestris* var. *mongolica* plantations established for sand fixation, accurate canopy width prediction is essential for assessing crown competition, light interception, and overall stand productivity. Previous studies have demonstrated that mixed-effects models provide superior predictive performance compared to traditional OLS approaches by accounting for hierarchical data structures and between-plot variability [?, ?].

Mixed-effects modeling has become increasingly important in forestry research for characterizing individual tree attributes. The methodology effectively partitions variation between fixed population-level parameters and random plot-level effects, providing more robust predictions across different stand conditions [?]. For canopy-DBH relationships, both tree-level factors (e.g., HCB) and stand-level variables (e.g., RS, age) significantly influence model performance [?].

2. Materials and Methods

2.1 Data Collection The study data comprised 702 trees from 22 temporary sample plots in *Pinus sylvestris* var. *mongolica* plantations at Zhanggutai, Liaoning Province. The dataset included measurements of canopy width (CW), DBH, HCB, relative spacing index (RS), stand age (A), and other stand characteristics. The plots represented a range of stand densities and age classes typical of sand-fixing plantations in the region.

2.2 Model Specification Ten candidate model forms were evaluated for the canopy-DBH relationship (Table 1). These included linear, polynomial, power, exponential, logarithmic, and several nonlinear growth functions (Mitscherlich, Gompertz, Logistic). The power function (Model 3) was ultimately selected as the base model based on preliminary analysis.

Table 1. Candidate models for canopy-DBH relationship

Model	Form
M1	$CW = \beta_0 + \beta_1 DBH + \varepsilon$
M2	$CW = \beta_0 + \beta_1 DBH + \beta_2 DBH^2 + \varepsilon$
M3	$CW = \beta_0 DBH^{\beta_1} + \varepsilon$
M4	$CW = \beta_0 \beta_1^{DBH} + \varepsilon$
M5	$CW = \beta_0 \exp(\beta_1 DBH) + \varepsilon$
M6	$CW = \beta_0 + \beta_1 \ln(DBH) + \varepsilon$
M7	$CW = \exp(\beta_0 + \beta_1 DBH) + \varepsilon$
M8	$CW = 1/(\beta_0 + \beta_1 DBH) + \varepsilon$
M9	$CW = [DBH/(\beta_0 + \beta_1 DBH)]^2 + \varepsilon$
M10	$CW = \beta_0[1 - \exp(-\beta_1 DBH)] + \varepsilon$

where β_0, β_1, and β_2 are model parameters, and ε is the error term.

Table 2. Basic information of the investigated forest stands of *Pinus sylvestris* var. *mongolica*

Parameter	Value
Number of plots	22
Number of trees (n)	501 (modeling)
Number of trees (n)	201 (validation)
Stand age (A)	16.09 years (range)
Mean DBH	10.04 cm
Mean canopy width	13.61 m
Stand density	22.29 trees/ha
Mean HCB	14.28 m
Mean RS	33.34

2.3 Statistical Methods The analysis employed both OLS and nonlinear mixed-effects modeling approaches. The general mixed-effects model structure was:

$$y_i = f(\beta, u_i, x_i) + \varepsilon_i$$

where y_i is the response vector for plot i, β represents fixed effects parameters, u_i are random effects, x_i denotes covariates, and ε_i is the residual error. Random effects were assumed normally distributed: $u_i \sim N(0, D)$, and residuals: $\varepsilon_i \sim N(0, R_i)$.

For model selection, we used Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and log-likelihood values. Prediction accuracy was assessed using R², MAE, and RMSE.

Four sampling strategies were implemented to estimate random parameters: 1. Random sampling (MPA) 2. Large-DBH tree sampling (MPS) 3. Small-DBH tree sampling 4. Medium-DBH tree sampling

3. Results

3.1 Model Fitting and Selection Among the 12 candidate models evaluated, the power function (Model 3) and modified power function (Model 10) demonstrated superior performance. Model 3, expressed as:

$$CW_{ij} = \beta_0 DBH_{ij}^{\beta_1} + \varepsilon_{ij}$$

where CW_{ij} and DBH_{ij} are canopy width and DBH for tree j in plot i, respectively, provided the best balance of simplicity and predictive power.

Table 4. Fitting results and statistical indices of different canopy-DBH models

Model				AIC	BIC	-2LL	R²	MAE	RMSE
OLS (7)	0.5043	0.7150	-	928.31	940.96	922.31	0.5875	0.4696	0.6075
OLS (8)	0.3427	-0.0077	0.8882	834.78	860.07	822.78	0.6618	0.4155	0.5500
MPA (9)	0.3095	0.1310	-0.0012	828.55	845.42	820.55	0.7030	0.3866	0.5154
MPS (10)	0.3234	0.1219	-0.0013	792.97	806.97	786.97	0.7051	0.3822	0.5136

The mixed-effects models (MPA and MPS) substantially outperformed OLS models, with R² improvements of 12.65% and reductions in MAE (11.51%) and RMSE (9.46%). The difference between MPA and MPS was not statistically significant (F = 36.34, P < 0.001).

3.2 Parameter Estimation Random effects were significant for the intercept parameter ( ) across plots. The final mixed-effects model structure incorporated HCB, RS, and stand age as covariates:

$$CW_{ij} = (\beta_0 + u_i) DBH_{ij}^{\beta_1} + \varepsilon_{ij}$$

for the basic model, and

$$CW_{ij} = (\beta_0 + u_i + \beta_1 HCB_{ij} + \beta_2 RS_i + \beta_3 A_i) DBH_{ij}^{\beta_4} + \varepsilon_{ij}$$

for the generalized model.

The likelihood ratio test confirmed the superiority of the full mixed-effects generalized model (LRT = 27.58, P < 0.0001). Compared to the basic mixed-effects model, the generalized version increased R² by 0.30% and reduced MAE and RMSE by 1.14% and 0.35%, respectively.

3.3 Sampling Strategy Evaluation [Figure 1: see original paper] shows the residual distribution of canopy mixed-effects models. [Figure 2: see original paper] compares observed versus predicted canopy-DBH relationships across sample plots. [Figure 3: see original paper] illustrates the relationship between prediction accuracy and sampling intensity.

The MPS sampling strategy (large-DBH trees) provided the most accurate parameter estimation, reducing MAE and RMSE by 28.1% and 20.9% compared to random sampling for two sample plots. For three sample plots, the improvement was even more pronounced, with reductions of 31.5% and 24.3%, respectively.

3.4 Simulation of Factor Effects [Figure 4: see original paper] presents canopy-DBH simulation curves under different stand conditions. Canopy width decreased nonlinearly with increasing stand age and HCB, but increased with RS. The mixed-effects models captured these relationships more accurately than OLS models, particularly at the extremes of the DBH distribution.

4. Discussion

The incorporation of random effects significantly improved model performance by accounting for plot-to-plot variability in stand structure and site conditions. HCB emerged as a critical individual-level predictor, reflecting its direct control on crown dimensions. As HCB increases, available crown length decreases, resulting in narrower canopies [**?**].

RS, a measure of competition intensity, positively influenced canopy width, consistent with ecological theory that trees in less crowded conditions develop larger crowns [**?**]. Stand age effects were nonlinear, with canopy expansion slowing in older stands due to increased competition and physiological aging [**?**].

The marginal improvement of the generalized model over the basic mixed-effects model suggests that while stand-level variables are important, the random effects structure captures most of the between-plot variation. This aligns with findings from previous studies on conifer species [**?**, **?**].

Sampling strategy significantly affected parameter estimation accuracy. Targeting large-DBH trees (MPS) proved most efficient, as these trees represent the realized potential of site conditions and competition history. This approach reduced sample size requirements by approximately 30% while maintaining equivalent prediction accuracy.

5. Conclusions

Nonlinear mixed-effects models provide superior predictive accuracy for canopy-DBH relationships in *Pinus sylvestris* var. *mongolica* plantations compared to traditional OLS methods. The power function form with random intercepts offers an optimal balance of parsimony and precision. Key factors influencing canopy width include HCB, RS, and stand age. For operational implementation, sampling approximately 2-3 large-DBH trees per plot provides efficient random parameter estimation. These models support improved silvicultural decision-making in sand-fixing plantations by enabling accurate crown dimension predictions across diverse stand conditions.

References

[1] Yang Y, Huang S. Allometric modelling of crown width for white spruce by fixed- and mixed-effects models[J]. Forestry Chronicle, 2017, 93(2): 138-147.
[2] Fu LY, Sun H. Individual crown diameter prediction for *Cunninghamia lanceolata* forests based on mixed effects models[J]. Scientia Silvae Sinicae,

2013, 49(8): 65-74.

[3] Liu J, Xu XY, Ding AQ, et al. Application of digitized measurement in upwind project area of desert shrubs[J]. Arid Zone Research, 2017, 34(5): 1070-1076.

[4] Zhao WJ, Liu XD, Jin M, et al. Analysis on community structure of *Picea crassifolia* forests in the Qilian Mountains[J]. Arid Zone Research, 2012, 29(4): 615-620.

[5] Pretzsch H, Biber P, Uhle E, et al. Crown size and growing space requirement of common tree species in urban centres, parks, and forests[J]. Urban Forestry and Urban Greening, 2015, 14(3): 466-479.

[6] Bechtold WA. Crown-diameter prediction models for 87 species of stand-grown trees in the Eastern United States[J]. Southern Journal of Applied Forestry, 2003, 27(4): 269-278.

[7] Gill SJ, Biging GS, Murphy EC. Modelling conifer tree crown radius and estimating canopy cover[J]. Forest Ecology and Management, 2000, 126(3): 405-416.

[8] Sharma RP, Vacek Z, Vacek S. Individual tree crown width models for Norway spruce and European beech in Czech Republic[J]. Forest Ecology and Management, 2016, 366: 208-220.

[9] Sharma RP, Bílek L, Vacek Z, et al. Modelling crown width-diameter relationship for Scots pine in central Europe[J]. Trees, 2017: 1-15.

[10] Thorpe HC, Astrup R, Trowbridge A, et al. Competition and tree crowns: A neighborhood analysis of three boreal tree species[J]. Forest Ecology and Management, 2010, 259(8): 1586-1596.

[11] Crecente-Campo F, Álvarez-González JG, Castedo-Dorado F, et al. Development of crown profile models for *Pinus pinaster* Ait. and *Pinus sylvestris* L. in northwestern Spain[J]. Forestry, 2013, 86(4): 481-491.

[12] Pinheiro JC, Bates DM. Mixed-Effects Models in S and S-PLUS[M]. New York: Springer-Verlag, 2000: 271-395.

[13] Zang H, Lei XD, Zhang HR, et al. Nonlinear mixed-effects height-diameter model of *Pinus koraiensis*[J]. Journal of Beijing Forestry University, 2016, 38(6): 8-16.

[14] Jiang LC, Li FR. Application of Mixed Effects Model in Forestry Modeling[M]. Beijing: Science Press, 2014: 1-20.

[15] Fu LY, Sun H, Sharma RP, et al. Nonlinear mixed-effects crown width models for individual trees of Chinese fir (*Cunninghamia lanceolata*) in South-Central China[J]. Forest Ecology and Management, 2013, 302(6): 210-220.

[16] Fu LY, Sharma RP, Hao KJ, et al. A generalized interregional nonlinear mixed-effects crown width model for Prince Rupprecht larch in Northern China[J]. Forest Ecology and Management, 2017, 389: 364-373.

[17] Zhu JJ, Fan ZP, Zeng DH, et al. Comparison of stand structure and growth between artificial and natural forests of *Pinus sylvestris* var. *mongolica* on sandy land[J]. Journal of Forest Research, 2003, 14(2): 103-111.

[18] Zhu JJ, Zeng DH, Kang HZ, et al. Decline of *Pinus sylvestris* var. *mongolica* plantations on sandy land[M]. Beijing: China Forestry Publishing House, 2005: 10-40.

[19] Lei ZY, Zhou YP, Zhao GJ, et al. Effects of competition on tree height growth of sand-fixation plantations of *Pinus sylvestris* var. *mongolica* in Northwest Liaoning Province[J]. Arid Zone Research, 2018, 35(1): 961-966.

[20] Jiao SR. Report on the causes of the early decline of *Pinus sylvestris* var. *mongolica* shelterbelt and its preventive and control measures in Zhanggutai of Liaoning Province[J]. Scientia Silvae Sinicae, 2001, 37(2): 131-138.

[21] Xu H, Sun YJ, Wang XJ, et al. Linear mixed-effects models to describe individual tree crown width for China-fir in Fujian Province, Southeast China[J]. PLoS One, 2015, 10(4): e0122257.

[22] Fang Z, Bailey RL. Nonlinear mixed effects modeling for slash pine dominant height growth following intensive silvicultural treatments[J]. Forest Science, 2001, 47(3): 287-300.

[23] Huang S, Douglas PW, Yang Y, et al. Assessing the impacts of species composition, top height and density on individual tree height prediction of quaking aspen in boreal mixed woods[J]. Forest Ecology and Management, 2009, 258(7): 1235-1247.

[24] Temesgen HT, Monleon VJM, Hann DW. Analysis and comparison of nonlinear tree height prediction strategies for Douglas-fir forests[J]. Canadian Journal of Forest Research, 2008, 38(38): 553-565.

[25] Sharma RP, Breidenbach J. Modelling height-diameter relationships for Norway spruce, Scots pine, and downy birch using Norwegian national forest inventory data[J]. Forest Science and Technology, 2015, 11(1): 44-53.

[26] Gómez-García E, Fonseca TF, Crecente-Campo F, et al. Height-diameter models for maritime pine in Portugal: A comparison of basic, generalized and mixed-effects models[J]. Forest-Biogeosciences and Forestry, 2015, 8: 72-78.

[27] Power H, LeMay V, Berniger F, et al. Differences in crown characteristics between black (*Picea mariana*) and white spruce (*Picea glauca*)[J]. Canadian Journal of Forest Research, 2012, 42(9): 1733-1743.

[28] Zhao D, Michael K, Bruce B. Development and applications of the relative spacing model for loblolly pine plantations[J]. Forest Ecology and Management, 2010, 259(10): 1922-1929.

[29] Dong C, Wu BG, Han YY, et al. An individual crown width model for Chinese fir plantation based on modified function[J]. Journal of Northeast Forestry University, 2015, 43(5): 49-53.

[30] Ishii H, McDowell N. Age-related development of crown structure in coastal Douglas-fir trees[J]. Forest Ecology and Management, 2002, 169(3): 257-270.

[31] Uzoh FCC, Oliver WW. Individual tree diameter increment model for managed even-aged stands of ponderosa pine throughout the western United States using a multilevel linear mixed effects model[J]. Forest Ecology and Management, 2008, 256(3): 438-445.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.