A Robust Quantum Watermarking Algorithm Based on Key-Controlled Least Significant Bit Modification Technique Postprint

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Abstract

More effective protection of copyright for quantum images represents an important research topic in quantum watermarking technology. Based on logarithmic polar coordinate representation of quantum images, a novel quantum watermarking algorithm is proposed. According to the values of a shared key set between communicating parties, the sender selects one bit from the high four bits of pixel grayscale values in the quantum carrier image as a control bit; subsequently, based on the value of this selected control bit, the sender embeds watermark information into either the least significant bit or the second least significant bit of the quantum carrier image. This key-based controlled least significant bit modification technique enhances both the transparency and robustness of quantum watermarked images. Experimental simulations and performance analysis conducted in MATLAB further demonstrate that the proposed algorithm exhibits favorable performance in terms of transparency, robustness, and embedding capacity.

Full Text

Preamble

Robust Quantum Watermarking Algorithm Based on Key to Implement Controlled Least Significant Qubit Modification Technique

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Abstract: How to better protect the copyright of quantum images is an important research subject in quantum watermarking technology. Based on the representation of quantum log-polar images, this paper proposes a novel quantum watermarking algorithm. According to the value of the key shared by both communicating parties, the sender selects one of the high four qubits of the pixel gray value of the quantum carrier image as the controlled qubit. The watermarking information is then embedded into the least significant qubit or the peripheral least significant qubit of the quantum carrier image based on the value of the selected controlled qubit. Utilizing the key to implement the controlled least significant qubit modification technique improves the transparency and robustness of the quantum watermark image. Experimental simulation and performance analysis based on MATLAB show that the new algorithm has good performance in terms of transparency, robustness, and embedding capacity.

Keywords: copyright protection; quantum watermarking technology; quantum log-polar images; least significant qubit modification technique; transparency; robustness; embedding capacity

0 Introduction

As a novel computing model, quantum computing can leverage the special properties of quantum mechanics, such as superposition and entanglement, to store, process, and transmit information. In recent years, with the rapid development of quantum computing and the widespread application of digital images, various quantum image representation models have been proposed to meet the demand for storing digital images using quantum states. These include Qubit Lattice [2], Entangled Image [3], Real Ket [4], FRQI [5], NEQR [6], QUALPI [7], and NCQI [8]. Among them, QUALPI is a quantum image representation model based on log-polar coordinates that exhibits rotational and scaling invariance [9]. The three transformations of QUALPI quantum images—symmetric transformation, rotation transformation, and scaling transformation—can be performed flexibly and rapidly.

The proposal of multiple quantum image representation models has enabled broad application prospects for quantum images in communication networks. Based on the characteristics of different quantum image representation models, quantum image steganography techniques [10–12] and quantum watermarking techniques [13–16] have been developed accordingly. Compared with quantum image steganography, which focuses on the concealment of secret information in quantum images, quantum watermarking technology plays a crucial role in copyright protection of quantum images. Quantum watermarking technology embeds identification information (i.e., quantum watermarks) into quantum carriers (including text, images, video, etc.) without affecting the utility value of the quantum carrier or allowing unauthorized third parties to easily detect the presence of the quantum watermark. Through this hidden watermark information, copyright protection can be achieved.

The main research achievements in quantum watermarking technology to date are as follows. In 2013, Zhang et al. [13] proposed a general quantum watermarking algorithm that could identify the true watermark embedder based on the extracted watermark, and any non-copyright owner could not remove the embedded watermark. In the same year, Zhang et al. [14] also proposed a quantum watermarking algorithm based on Quantum Fourier Transform (QFT). This algorithm embeds quantum watermark image information into the Fourier coefficients of the quantum carrier image, utilizing the properties of Fourier transform to ensure that the watermarked quantum carrier image can resist unavoidable noise. In 2016, applying the classical Least Significant Bit (LSB [17]) modification technique, Sang et al. [15] first introduced the concept of quantum Least Significant Qubit (LSQb) modification, then proposed a quantum watermarking algorithm based on the quantum color image representation model (NCQI), making the algorithm easy to implement and capable of hiding large amounts of information.

Transparency, robustness, and embedding capacity are three important metrics for evaluating quantum watermarking algorithm performance, with robustness being the key evaluation criterion. Any proposed quantum watermarking algorithm is only secure if it can withstand various attacks from unauthorized third parties. The misuse of a weakly robust quantum watermarking algorithm may cause losses to legitimate copyright owners. If unauthorized third parties attempt to delete or destroy watermarks in an image, it will cause severe distortion to the carrier image, and this malicious attack also prevents the unauthorized party from obtaining watermark information. However, if unauthorized third parties employ attacks such as scanning and copying, geometric transformation, and noise pollution, these cause minimal impact on the carrier image while still allowing the unauthorized party to obtain partial watermark information. These unintentional attacks are commonly used by unauthorized third parties. The three quantum watermarking algorithms described above focus on analyzing the impact of scanning/copying and noise pollution attacks but neglect the effect of geometric transformation attacks on watermarked quantum carrier images. Therefore, proposing a robust quantum watermarking algorithm that can effectively resist geometric transformation attacks has important practical significance.

To address this need, Qu et al. [16] proposed a robust quantum watermarking algorithm based on QUALPI quantum images in 2017. This algorithm embeds quantum watermark images into quantum carrier images using LSQb modification techniques, then leverages the rotational and scaling invariance properties of QUALPI quantum images to enable watermarked quantum carrier images to effectively resist common geometric transformation attacks such as rotation, flipping, and scaling, demonstrating good robustness. Although this algorithm utilizes the operational simplicity of LSQb modification techniques, when unauthorized third parties obtain the watermarked quantum carrier image and read out the least significant bit plane, they still have a high probability of discovering the watermark information, creating a security vulnerability. In 1984,

Bennett et al. [18] proposed the first quantum key distribution protocol, BB84, which enables legitimate communicators to share a random key while preventing unauthorized third parties from obtaining any information about the key. Therefore, this paper proposes a robust quantum watermarking algorithm based on a key-controlled least significant bit modification technique. Even if unauthorized third parties obtain the watermarked quantum carrier image, they cannot recover the watermark information without knowing the key, thereby further improving the algorithm' s robustness.

1 Quantum Log-Polar Image Representation Model

Assume the sampling resolutions for the logarithmic radius and angular direction of a log-polar digital image are 2^m and 2^n, respectively. Then, based on the Quantum Log-Polar Image representation model (QUALPI), the digital image can be represented as

$$|F\rangle = \frac{1}{2^{(m+n)/2}} \sum_{\rho=0}^{2^m-1} \sum_{\theta=0}^{2^n-1} |G(\rho,\theta)\rangle \otimes |\rho\rangle \otimes |\theta\rangle$$

where $|G(\rho,\theta)\rangle$ represents the gray value of the -th pixel. If the gray range of the digital image is 2^q, the pixel' s gray value can be represented by a binary sequence

$$|G(\rho,\theta)\rangle = |c_{q-1}c_{q-2}\cdots c_0\rangle, \quad c_i \in \{0,1\}, i = 0,1,\ldots,q-1$$

Correspondingly, the logarithmic radius and angular direction can also be encoded as

$$|\rho\rangle = |\rho_{m-1}\rho_{m-2}\cdots\rho_0\rangle, \quad |\theta\rangle = |\theta_{n-1}\theta_{n-2}\cdots\theta_0\rangle$$

Equation (1) shows that the entire QUALPI quantum image is stored in a normalized and equiprobable quantum superposition state, where each basis state represents a pixel. The basis state consists of the tensor product of three quantum bit sequences, with all pixel information—including gray value, logarithmic radius, and angular direction—stored in the basis state.

[Figure 1: see original paper] is an example of a QUALPI quantum image with size 4$×$4 and gray range of 256. Using the QUALPI model, it can be represented as

$$|F\rangle = \frac{1}{4}(|00000000\rangle \otimes |00\rangle \otimes |00\rangle + |01111111\rangle \otimes |01\rangle \otimes |00\rangle + |00000001\rangle \otimes |10\rangle \otimes |00\rangle + |00101001\rangle \otimes |11\rangle \otimes |$$

2 Quantum Watermarking Algorithm

Using a key-based controlled Least Significant Qubit (LSQb) modification technique, this paper proposes a novel quantum watermarking algorithm. The new algorithm consists of two parts: the embedding process and the extraction process of the quantum watermark image. The flowchart is shown in [Figure 2: see original paper].

2.1 Embedding Process of Quantum Watermark Image

The embedding process of the quantum watermark image is described as follows:

a) Assume the digital carrier image is a gray-scale image of size 2^m × 2^n, and the digital watermark image is a binary image of size 2^m × 2^n. Prepare the quantum carrier image |C and quantum watermark image |W based on the QUALPI model, expressed as equations (5) and (6) respectively.

b) The sender and receiver share a key K:

$$K = (k_0, k_1, \ldots, k_{2^{m+n}-1}), \quad k_t \in \{0,1\}, t = 0, 1, \ldots, 2^{m+n} - 1$$

where t corresponds to the t-th pixel of the quantum carrier image |C .

Based on the value of key bits k_{2t}k_{2t+1}, the sender selects one bit from the high four bits of the gray value |C_{ji} of the j-th pixel of the quantum carrier image as the control bit, with the following rules:

- If k_{2t}k_{2t+1} = 00, select |c_{7ji} as the control bit
- If k_{2t}k_{2t+1} = 01, select |c_{6ji} as the control bit
- If k_{2t}k_{2t+1} = 10, select |c_{5ji} as the control bit
- If k_{2t}k_{2t+1} = 11, select |c_{4ji} as the control bit

Based on the selected control bit value, the sender embeds the gray value |w_{0ji} of the t-th pixel of the quantum watermark image into either the least significant bit |c_{0ji} or the peripheral least significant bit |c_{1ji} :

(a) If the control bit is |c_{7ji} , |c_{6ji} , |c_{5ji} , or |c_{4ji} , then embed |w_{0ji} into the least significant bit |c_{0ji} . The specific unitary operation is:

Define a unitary transformation U_{ji}^S:

$$U_{ji}^S = I \otimes I \otimes \cdots \otimes I \otimes \sigma_x \otimes I \otimes \cdots \otimes I$$

where *σx is the Pauli-X matrix. When the control bit value differs from /w*{0ji} , flip the value of |c_{0ji} . The unitary transformation U_{ji}^S acts on the quantum carrier image |C as:

$$U_{ji}^{S}|C\rangle = \frac{1}{2^{(m+n)/2}} \sum_{\rho=0}^{2^m-1} \sum_{\theta=0}^{2^n-1} |c_{7ji}c_{6ji}c_{5ji}c_{4ji}c_{3ji}c_{2ji}c_{1ji}c_{0ji} \oplus w_{0ji}\rangle \otimes |\rho\rangle \otimes |\theta\rangle$$

(b) If the control bit is |c_{3ji} or |c_{2ji} , then embed |w_{0ji} into the peripheral least significant bit |c_{1ji} . Define two unitary transformations V_{ji}^S and V_{ji}^D:

When the control bit value matches |w_{0ji} , the unitary transformation V_{ji}^S acts on the quantum carrier image |C ; when they differ, the unitary transformation V_{ji}^D acts on |C :

Through unitary transformation V_{ji}^S or V_{ji}^D, |w_{0ji} can be embedded into |c_{1ji} .

This key-based controlled LSQb modification embedding method can be implemented by the quantum circuit designed in [Figure 3: see original paper]. In [Figure 3: see original paper], the symbols "•" , " " , and " " represent 1-control, 0-control, and CNOT gates respectively, and the constant input qubit is an auxiliary qubit. For convenience, the quantum circuit block diagram of the key-based controlled LSQb modification technique omits auxiliary inputs and irrelevant outputs, as shown in [Figure 4: see original paper].

c) By executing step b) 2^{m+n} times, all information of the quantum watermark image |W is embedded into the quantum carrier image |C , obtaining the watermarked quantum carrier image |CW :

$$|CW\rangle = \frac{1}{2^{(m+n)/2}} \sum_{\rho=0}^{2^m-1} \sum_{\theta=0}^{2^n-1} |c_{7ji}c_{6ji}c_{5ji}c_{4ji}c_{3ji}c_{2ji}c'_{1ji}c'_{0ji}\rangle \otimes |\rho\rangle \otimes |\theta\rangle$$

To improve the implementability of the new algorithm, this paper designs an efficient quantum circuit for watermark image embedding, as shown in [Figure 5: see original paper]. This key-based controlled LSQb modification embedding method ensures that the watermarked quantum carrier image has no obvious visual difference from the original quantum carrier image |C , while also providing the new algorithm with good robustness.

Thus, the embedding process of the quantum watermark image is completed.

2.2 Extraction Process of Quantum Watermark Image

The extraction process of the quantum watermark image is described as follows:

a) In Hilbert space, the watermarked quantum carrier image |CW is a complex vector that needs to be decomposed into the tensor product form of gray value information and pixel position information. Assuming the watermarked quantum carrier image |CW has vector Q, then Q can be decomposed as:

$$Q = \left(\frac{1}{2^{(m+n)/2}} \sum_{\rho=0}^{2^m-1} \sum_{\theta=0}^{2^n-1} |a_{2^{m+n}-1} \cdots a_0\rangle \otimes |\rho_{m-1} \cdots \rho_0\rangle \otimes |\theta_{n-1} \cdots \theta_0\rangle \right)$$

where the symbol " "denotes the tensor product. The first and second parts of the tensor product are the gray value information and corresponding pixel position information of the watermarked quantum carrier image |CW , respectively.

b) To extract watermark information, convert the gray value information a_0, ···, a_{2^{m+n}-1} into corresponding binary codes b_0, ···, b_{2^{m+n}-1}, where b_t corresponds to the gray value of the t-th pixel of the watermarked quantum carrier image |CW :

$$b_t = b_{7ji} b_{6ji} b_{5ji} b_{4ji} b_{3ji} b_{2ji} b_{1ji} b_{0ji}$$

c) Based on the key value k_{2t}k_{2t+1} and the pre-established rules between communicating parties, the receiver determines the control bit from the high four bits of gray value b_t. If the control bit is |c_{7ji} , |c_{6ji} , |c_{5ji} , or |c_{4ji} , then the least significant bit b_{0ji} of gray value b_t is the watermark information; otherwise, the peripheral least significant bit b_{1ji} is the watermark information. This key-based controlled LSQb extraction method can be implemented by the quantum circuit designed in [Figure 6: see original paper]. The quantum circuit block diagram of the key-based controlled LSQb extraction technique, which omits irrelevant outputs, is shown in [Figure 7: see original paper].

This paper also designs an efficient quantum circuit for watermark image extraction, as shown in [Figure 8: see original paper]. In [Figure 8: see original paper], the output qubits corresponding to the 2^{m+n} input qubits are the gray value information of each pixel of the quantum watermark image |W .

From the above steps, it can be seen that the extraction process of the quantum watermark image is essentially the inverse of the embedding process.

3 Experimental Simulation Results and Performance Analysis

The performance of quantum watermarking algorithms is primarily evaluated by three important metrics: transparency, robustness, and embedding capacity. Transparency requires that the quantum watermark embedded in the quantum carrier be imperceptible and not affect the normal use of the quantum carrier. Robustness means that after experiencing various malicious or unintentional attacks, the quantum watermark can still maintain partial integrity and be accurately identified. Embedding capacity calculates the amount of quantum watermark information embedded in the quantum carrier. To better analyze the performance of the proposed quantum watermarking algorithm in terms

of transparency, robustness, and embedding capacity, this section presents experimental data obtained from the new algorithm in the MATLAB R2012a environment on a classical computer. The carrier gray-scale images used in experiments are Girl, Woman, Lena, and Vegetables, while the watermark binary images are Thumbs-up, Recycling, Ribbon, and Butterfly, as shown in [Figure 9: see original paper] and [Figure 10: see original paper], respectively, with image sizes of 256$\times$256.

3.1 Transparency

The fidelity between the original quantum carrier image and the watermarked quantum carrier image is the most common method for evaluating the transparency of quantum watermarking algorithms. In classical digital image processing, Peak Signal-to-Noise Ratio (PSNR) is the most widely used method for image quality assessment. Assuming I is the original carrier image and J is the watermarked carrier image, equations (22) and (23) give the calculations for Mean Square Error (MSE) and PSNR, respectively:

$$MSE = \frac{1}{mn}\sum_{\rho=0}^{m-1}\sum_{\theta=0}^{n-1}[I(\rho,\theta) - J(\rho,\theta)]^2$$

$$PSNR = 20\log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right)$$

where I(,) and J(,) represent the gray values of the original carrier image I and watermarked carrier image J at the (,)-th pixel, respectively; MAX_I is the maximum gray value of the original carrier image I.

The watermark binary images Thumbs-up, Recycling, Ribbon, and Butterfly are embedded into the carrier gray-scale images Girl, Woman, Lena, and Vegetables using the key-based controlled least significant bit modification technique, obtaining watermarked carrier images as shown in [Figure 11: see original paper]. In [Figure 11: see original paper], the human eye cannot effectively distinguish between the original carrier images and the watermarked carrier images. Additionally, the PSNR values between the original carrier images and their corresponding watermarked carrier images in [Figure 11: see original paper] are calculated, with results shown in . In , all PSNR values exceed the image quality standard of 38 dB, indicating that the watermarked carrier images have minimal distortion. Therefore, the new algorithm demonstrates good transparency.

3.2 Robustness

Robustness is the key to evaluating quantum watermarking algorithms. The quantum watermark embedded in the quantum carrier should not only be difficult for unauthorized third parties to detect but also remain relatively unaffected when the watermarked quantum carrier is attacked.

Attack methods by unauthorized third parties are divided into malicious attacks and unintentional attacks. Malicious attacks (such as data tampering and lossy compression) have a high probability of deleting or even destroying the quantum watermark embedded in the quantum carrier, preventing both communicating parties and unauthorized third parties from obtaining the quantum watermark. Therefore, quantum watermarking algorithms typically focus on analyzing the impact of unintentional attacks by unauthorized third parties. Common unintentional attacks include scanning and copying, geometric transformation, and noise pollution.

To ensure the proposed quantum watermarking algorithm demonstrates good robustness, this section analyzes the new algorithm' s robustness against three attack types: scanning and copying, geometric transformation, and noise pollution.

First, analyze the impact of scanning and copying attacks on the new algorithm. When unauthorized third parties scan and copy the watermarked quantum carrier image, they can obtain the least significant bit plane and the peripheral least significant bit plane of the image' s pixel information. Attempts to extract watermark information from these two bit planes are futile because the quantum watermark image is embedded into the quantum carrier image using the key-based controlled least significant bit modification technique, and the key shared between communicating parties is absolutely secure. Therefore, unauthorized third parties cannot know the specific locations where the quantum watermark image is embedded into the quantum carrier image. The key shared between communicating parties enhances the robustness of the new algorithm.

Next, analyze the impact of geometric transformation attacks on the new algorithm based on the characteristics of the Quantum Log-Polar Image representation model (QUALPI). QUALPI quantum images possess rotational and scaling invariance, enabling the three transformations of QUALPI quantum images—symmetric transformation, rotation transformation, and scaling transformation—to be performed flexibly and rapidly. When unauthorized third parties apply geometric transformation attacks to watermarked quantum carrier images, the receiver can still recover the original watermarked quantum carrier image through unitary transformations and extract the quantum watermark image.

To demonstrate this argument, the watermarked carrier images from the four examples in [Figure 11: see original paper] are subjected to geometric transformations including rotation, horizontal axis flipping, vertical axis flipping, and scaling. The simulation results are shown in [Figure 12: see original paper]. In [Figure 12: see original paper], the watermarked carrier images attacked by geometric transformations can still be recovered. Using the key-based controlled LSQb extraction technique to extract watermark images from the watermarked carrier images still yields high image quality. The use of the QUALPI model to represent carrier images and watermark images also improves the robustness of the new algorithm.

Finally, analyze the impact of noise pollution attacks on the new algorithm using Bit Error Rate (BER), another commonly used metric in classical digital image processing. When watermarked carrier images are transmitted in communication networks, channel noise causes output images to have errors, making them somewhat different from the original watermarked carrier images. Since the human eye cannot distinguish between these two images, BER is used to measure the impact of noise pollution during image transmission. BER is defined as the reciprocal of PSNR:

$$BER = \frac{1}{PSNR}$$

Based on the PSNR values given in , the calculated BER values are shown in . In , the BER values of the new algorithm are relatively small, indicating that noise pollution has minimal impact on the new algorithm.

3.3 Embedding Capacity

The embedding capacity of quantum watermarking algorithms can be measured by embedding rate and modification rate. According to the definition of embedding rate (embedding rate = number of embedded watermark bits / number of all pixels in carrier image), the new algorithm' s embedding rate is 1. Additionally, according to the definition of modification rate (modification rate = number of bits modified per pixel in carrier image / number of embedded watermark bits), assuming the 0-value bits and 1-value bits of the watermark binary image are uniformly distributed, the least significant bit or peripheral least significant bit of the carrier image' s unit pixel has a 50% probability of being modified, giving the new algorithm a modification rate of 0.5.

4 Conclusion

Utilizing the rotational and scaling invariance characteristics of the Quantum Log-Polar Image representation model (QUALPI) and the operational simplicity of the key-based controlled least significant bit modification technique, this paper proposes a novel robust quantum watermarking algorithm. Compared with previous quantum watermarking algorithms, the new algorithm leverages the features of QUALPI quantum images to ensure that when watermarked quantum carrier images are attacked by geometric transformations such as rotation, flipping, and scaling, the embedded quantum watermark images can still be extracted, guaranteeing the robustness of watermarked quantum carrier images. Furthermore, the key-based controlled least significant bit modification embedding method prevents unauthorized third parties from recovering quantum watermark images even when they perform scanning and copying attacks on watermarked quantum carrier images without key information, thereby further protecting the copyright of quantum images.

Experimental simulations based on MATLAB, combined with PSNR and BER from classical digital image processing, demonstrate the transparency and robustness of the new algorithm. The simulation results and experimental data show that the new algorithm has good transparency and robustness. Analysis of the embedding capacity indicates that the new algorithm has an embedding rate of 1, which is a considerable rate.

Another innovation of this paper is the design of quantum circuits for both the key-based controlled least significant bit modification technique and the key-based controlled least significant bit extraction technique. On one hand, this ensures the new algorithm strictly follows quantum mechanics principles during implementation and is feasible under current physical experimental conditions; on the other hand, it makes the new algorithm highly practical.

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Note: Figure translations are in progress. See original paper for figures.