Intelligent Face Recognition Algorithm Based on Spectral Domain Feature Extraction and Linear Regression Classification (Postprint)

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Abstract

To address the problem of face image variations caused by illumination, expression changes, and occlusion in face recognition, an intelligent face recognition method combining spectral domain feature extraction with a linear regression classification algorithm is proposed. For the purpose of feature extraction, the Viola-Jones algorithm is first employed to extract the initial face region from the original image and convert it into a grayscale image of 120$×$120 pixels. Subsequently, a novel framework for computing Polar Fourier Transform (FFT) is proposed to obtain the primary amplitude spectrum features of the preprocessed face image; furthermore, 2D-DFT is performed on the preprocessed image and represented as 1D P-FFT. The feature values correspond to the maximum values in the amplitude of the 1D P-FFT, and the extracted feature values are utilized to construct symbolic objects representing face images. Finally, a fast and effective linear regression classification algorithm is employed to achieve classification. Various experiments were conducted on the AR and GT databases, achieving accuracies of 97.51% and 98.02%, respectively. Compared with some recently reported face recognition techniques, the proposed method demonstrates higher recognition accuracy.

Full Text

Preamble

Intelligent Face Recognition Algorithm Based on Spectral Feature Extraction and Linear Regression Classification

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Abstract: To address the problem of facial image variation caused by lighting, expression changes, and occlusion in face recognition, this paper proposes an intelligent face recognition method that combines spectral domain feature extraction with a linear regression classification algorithm. For feature extraction, the Viola-Jones algorithm is first used to extract the initial facial region from original images, which are then converted to grayscale images of 120$\times$120 pixels. A novel framework is proposed to compute the polar amplitude Fourier transform (FFT) to obtain the main amplitude spectral features of preprocessed facial images. The 2D-DFT is further performed on the preprocessed images and expressed as 1D P-FFT. The eigenvalues are the maximum values in the 1D P-FFT amplitude, and these extracted eigenvalues are used to construct symbolic objects representing facial images. Finally, classification is implemented using a fast and efficient linear regression classification algorithm.

Various experiments conducted on the AR and GT databases achieved accuracies of 97.51% and 98.02%, respectively. Compared with some recently reported face recognition techniques, the proposed method demonstrates higher recognition accuracy.

Keywords: face recognition; linear regression; fast Fourier transform; classification algorithm; spectral domain feature

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0 Introduction

Face recognition represents an active research area in computer vision, artificial intelligence, pattern recognition, and biometrics. Compared with other biometric modalities such as fingerprint, iris, and hand geometry, its most significant advantage lies in its ability to operate at a distance and in a non-intrusive manner. Over the past decades, numerous face recognition methods have been proposed and developed. However, the recognition accuracy of these methods is severely affected by illumination variation and occlusion. Extracting optimal features and developing effective classification methods constitute the two main steps in face recognition systems. Therefore, to improve system performance, finding a good feature extractor and an effective classifier is crucial. Additionally, reducing the dimensionality of the extracted feature space is necessary to enhance algorithm efficiency and robustness in the classification process.

In recent years, many appearance-based face recognition systems have been developed to reduce computational time for facial image classification. Some literature proposes appearance-based systems that directly use pixel intensity values as features for recognition, represented by single-valued variables. However, such single-valued variables may fail to capture feature variations across images of the same subject and result in high-dimensional feature data. To develop more efficient systems with fewer features, researchers have adopted feature learning methods. Feature learning plays an important role in improving classification performance by learning discriminative representations from training image data. Consequently, many active learning methods have been proposed that combine representativeness and informativeness of training samples to achieve better classification accuracy. Among them, a supervised feature learning model was proposed for classifying low- and high-dimensional data, using discriminative regression to estimate similarity between test and training data. Nevertheless, classification becomes challenging when low- or high-dimensional image data suffer from occlusion, geometric variations, and illumination changes.

Research has shown that symbolic objects provide an effective method for representing variable information about facial images. This paper proposes a spectral domain feature extraction algorithm based on a novel appearance-based recognition method that effectively utilizes local spatial variations in facial images, such as illumination and expression changes. For feature extraction, the proposed method employs techniques related to Fourier analysis, where the frequency set equals the formulation proposed by Averbuch et al. when viewed in polar coordinates. Furthermore, FFT features generated in a polar grid are evaluated as one-dimensional equidistant polar FFT values and converted to 1D P-FFT (polar-FFT). The maximum magnitude from the 1D P-FFT is used as the feature value representing the facial image. Additionally, this work utilizes symbolic modeling methods to better represent extracted features, introducing a new symbolic modeling technique that relies solely on the appearance of raw data to reduce the dimensionality of 2D-DFT spectral features. Finally, classification is implemented using a fast and efficient linear regression classification algorithm. The accuracy of the proposed method is demonstrated through various experiments on the AR and GT databases.

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1 Spectral Analysis and Polar FFT Computation

For many image processing applications, fast and accurate estimation of the Fourier transform in polar coordinates has been a major challenge. Over the past decades, several techniques have been proposed to address these issues. The literature presents two approaches for computing polar or log-polar Fourier transforms, differing in whether interpolation is performed directly on the original 2D Cartesian image or on processed images. Due to interpolation errors, image registration techniques based on FFT cannot accurately recover transformed data, even failing to recover large-scale factors. During phase correlation, FFT methods often suffer from false peaks. Pseudo-polar FFT was therefore proposed and tested in image registration. Under certain conditions, these methods can significantly reduce interpolation errors while computing polar Fourier transforms with the same complexity as 2D FFT.

The pseudo-polar FFT algorithm proposed in the literature can restore scale up to a factor of 4 through an iterative process. Polar FFT can also be used to improve or simplify digital processing for computing frequency values in a polar grid. In the implementation of polar FFT, pseudo-polar grid points are first defined in the frequency domain and divided into vertical subsets V_S and horizontal subsets H_S. In the pseudo-polar grid, V_S and H_S points are centered on concentric circles and distributed non-equidistantly on rays. These square blocks have size N. Rays have uniform slopes, and H_S rays are similar but rotated 90° clockwise. Generally, V_S and H_S grid points can be expressed as:

$$V_S : \xi_k = \frac{k\pi}{N}, \quad \xi_l = \frac{l\pi}{N}, \quad \text{for } -\frac{N}{2} \le k \le \frac{N}{2}, \text{for } -N < l \le N$$
$$H_S : \xi_k = \frac{k\pi}{N}, \quad \xi_l = \frac{l\pi}{N}, \quad \text{for } -\frac{N}{2} \le k \le \frac{N}{2}, \text{for } -N < l \le N$$

In the fundamental region, sampling frequencies can be represented as:

$$V_S : \xi_k = \frac{k\pi}{N}, \quad \xi_l = \frac{l\pi}{N}, \quad \text{for } -\frac{N}{2} \le k \le \frac{N}{2}, \text{for } -N < l \le N$$
$$H_S : \xi_k = \frac{k\pi}{N}, \quad \xi_l = \frac{l\pi}{N}, \quad \text{for } -\frac{N}{2} \le k \le \frac{N}{2}, \text{for } -N < l \le N$$

To obtain geometrically straight lines, the pseudo-polar grid points ξ_k and ξ_l must be oversampled in both axial and radial directions, requiring uniformly distributed point sets in the pseudo-polar grid. This conversion involves two operations: obtaining uniformly sampled rays through rotating rays, and obtaining concentric circles by transforming circles into squares (as required in polar coordinates). To transform circles into squares, a constant is divided along each ray according to its angle ξ_k and ξ_l. Therefore, new grid points ξ_k^{new} and ξ_l^{new} are formed to replace ξ_k and ξ_l in the pseudo-polar grid, which can be calculated as:

$$\xi_k^{\text{new}} = \frac{k\pi}{N}, \quad \text{for } -\frac{N}{2} \le k \le \frac{N}{2}$$
$$\xi_l^{\text{new}} = \tan\left(\frac{l\pi}{2N}\right), \quad \text{for } -N < l \le N$$

The new pseudo-polar grid points ξ_k^{new} and ξ_l^{new} lie on lines and are distributed at different spacings. Using this formulation, new polar grid points for frequencies can be defined as:

$$\text{new } V_S : \xi_k^{\text{new}} = \frac{k\pi}{N}, \quad \xi_l^{\text{new}} = \tan\left(\frac{l\pi}{2N}\right), \quad \text{for } -\frac{N}{2} \le k \le \frac{N}{2}, \text{for } -N < l \le N$$
$$\text{new } H_S : \xi_k^{\text{new}} = \frac{k\pi}{N}, \quad \xi_l^{\text{new}} = \tan\left(\frac{l\pi}{2N}\right), \quad \text{for } -\frac{N}{2} \le k \le \frac{N}{2}, \text{for } -N < l \le N$$

Since the pseudo-polar grid is closer to the polar grid, converting pseudo-polar FFT to polar FFT involves 1D equidistant FFT with only a single 1D interpolation. For a given 2D facial image of size $N \times N$, the polar FFT algorithm produces a polar FFT with complexity $O(N^2 \log N)$. Therefore, implementing pseudo-polar FFT in the proposed work can compute facial features with higher accuracy and faster speed.

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2 Symbolic Data Modeling Method for Face Recognition

The recognition accuracy of any biometric system depends on discriminative features extracted from the training biometric set. In this work, frequency domain transforms are used to extract local variations in the spatial domain. Frequency domain features obtained using FFT represent different illumination, expression changes, and occlusion conditions in the frequency domain. Furthermore, low-frequency components are preserved to extract discriminative features required for classification. In the polar grid, generated FFT features are evaluated as one-dimensional equidistant polar FFT values and converted to 1D P-FFT. The maximum magnitude value in the 1D P-FFT is considered the feature descriptor for the facial image and is used to represent symbolic objects.

The proposed work uses a different feature representation compared to other appearance-based face recognition systems, employing a single feature value to represent the entire facial image. Due to spectral symmetry, quadrant positions can be diagonally swapped. By considering this property, low-frequency positions are diagonally exchanged so that they appear in the center of the image—i.e., even and odd quadrants are swapped to effectively compute low-frequency components. A fundamental property of the DFT of real images is its periodicity and complex conjugate symmetry. The spectrum repeats endlessly in both directions; therefore, only non-redundant coefficients of the DFT spectrum are considered for feature extraction.

In this work, 2D-FFT analysis of digital facial image data is performed using the periodicity and complex conjugate symmetry of FFT. The Fourier transform converts image information into the spatial frequency domain. The 2D Discrete Fourier Transform (2D-DFT) of a facial image $\tau(m,n)$ can be expressed as:

$$\Gamma(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \tau(m,n) e^{-j2\pi\left(\frac{mk}{M} + \frac{nl}{N}\right)}$$

where coordinates (k,l) represent the positions of 2D-DFT coefficients in the original spectrum, while coordinates (m,n) represent the grayscale values at corresponding positions in image τ. The efficient computational form of DFT is the Fast Fourier Transform (FFT), which converts image information to the frequency domain. The FFT algorithm is central to signal and image processing with broad applications in telecommunications, cryptography, medical imaging, and spectral analysis. An advantage of working in the frequency domain is that a few DFT coefficients with higher magnitude are sufficient to represent an image or part of an image.

In this work, the facial database is constructed by cropping face regions from original images of standard databases using the Viola-Jones algorithm and converting them to grayscale images of size 120×120 pixels, as shown in [Figure 2: see original paper]. FFT is then applied to processed images to generate 2D-DFT coefficients. In the generated coefficients, low-frequency components are distributed at the spectrum corners, representing edges/lines present in the image. Low-frequency components contain most of the information required to represent specific facial images and possess high discriminative power. High-frequency components, which help obtain finer details, are less important for recognition and do not contribute effectively to the recognition process. Therefore, this work utilizes spectral symmetry to efficiently preserve low-frequency components.

The covariance matrix obtained is approximately one-quarter the size of that obtained using grayscale values of facial images, potentially enabling more accurate estimation for feature extraction. Zero-padding is performed by adding a string of zeros at the end of the new synthetic covariance matrix to match the original size of facial images, i.e., $N \times N$.

By referencing the formulation, 2D-DFT can be defined for facial images $\tau(m, n)$ of size $N \times N$ (where N is even) to define a new spectrum. In this formulation, coordinates (k, l) represent the positions of 2D-DFT coefficients in the new spectrum, while coordinates (m, n) represent grayscale values at corresponding positions in image τ. Calculating new 2D-DFT values on the non-redundant coefficient spectrum is similar to calculating 2D-DFT on the original spectrum. The radius and phase angle of the non-redundant DFT spectrum (NRDFT) can be calculated using this formulation. Furthermore, the non-redundant Fourier spectrum can be computed through convolution of DFT spectrum coefficients with DC component moved to center and NRDFT radius. Non-redundant coefficients of the phase spectrum are used for symbolic representation of facial images. The new representation of the phase spectrum reduces the spectrum size to one-quarter compared to the original size obtained from raw grayscale facial images.

This chapter introduces feature extraction through polar Fourier transform (PFT) analysis of NRDFT. PFT generates rotation-invariant data particularly suitable for extracting orientation features of facial images with subtle variations.

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3 Spectral Domain Feature Extraction and Linear Regression Classification

3.1 Symbolic Representation of Facial Features

A person's facial image may vary in expression, illumination, and occlusion. Traditional systems using fewer feature values struggle to achieve accurate recognition under such conditions. Symbolic objects provide an effective method for representing this type of information. Symbolic objects are extensions of classical data types and can be categorized as assertion objects, hoard objects, and synthetic objects. An assertion object consists of a set of events belonging to a given object, where an event is a combination of logically linked feature variables and feature value/attribute pairs. A combination of one or more assertion objects is called a hoard object, and a combination of one or more hoard objects is called a synthetic object.

In this work, initial facial images are cropped from original images and used to extract polar FFT features (represented as assertion objects). Symbolic facial objects are described as hoard objects constructed from different assertion-type objects using max-min rules. Assuming the facial image dataset contains C image classes, each class consists of S image samples that vary in expression, occlusion, and illumination. Let $X_{i,j}$ denote the set of S sample images of a face class as a first-level object. Generally, all face image classes can be represented as $\Omega = \{X_{i,j} \mid i = 1 \text{ to } C, j = 1 \text{ to } S\}$.

Furthermore, using the maximum and minimum values of $X_{i,j}$ as symbolic interval data representation for class i can be calculated as:

$$FD_i^{MM} = \{\max(X_{i,j}), \min(X_{i,j}) \mid j = 1 \text{ to } S\}, \quad \forall i = 1 \text{ to } C$$

The symbolic knowledge base for class i can be represented as $SKB_i = \{FD_i^{MM}\}$, and the symbolic knowledge base matrix SKB representing all C classes can be expressed as:

$$SKB = \begin{bmatrix} FD_1^{MM} \\ FD_2^{MM} \\ \vdots \\ FD_C^{MM} \end{bmatrix}$$

where $SKB \in \mathbb{R}^{C\times 2}$, with 2 representing the maximum and minimum values of F1D-MM. In summary, matrix SKB contains the maximum and minimum values of S samples for each class i, with each row representing a gallery face image and each pair of values representing the S samples of class i. The proposed symbolic data modeling method uses only a few feature values to represent different variations of images.

3.2 Linear Regression Classification

Face recognition using linear regression methods directly employs downsampled facial images for linear regression analysis, followed by a nearest neighbor classifier for classification. The basic process is described as follows:

Input: Class models and test image vectors. Let C be the number of face classes, S_i be the number of training images for class i, and q be the length of the feature vector for each image.

Original training images can be represented as $X_i \in \mathbb{R}^{a\times b}$, where $a \times b$ is the original face image size. Images are downsampled to $c \times d$ ($c \times d < a \times b$) and concatenated column-wise into a 1D vector $x_i \in \mathbb{R}^q$. Thus, the class model can be represented as $X_i = [x_{i,1}, x_{i,2}, \dots, x_{i,S_i}] \in \mathbb{R}^{q\times S_i}$.

Output: The class label of test image $y \in \mathbb{R}^q$.

Computation Process: a) For each class model X_i, compute the parameter vector $\hat{\beta}_i = (X_i^T X_i)^{-1} X_i^T y$; b) Compute the prediction vector $\hat{y}_i = X_i \hat{\beta}_i$; c) Calculate the distance between the original vector and prediction vector: $d_i = \|y - \hat{y}_i\|_2$; d) The class of test image y is determined by the nearest neighbor criterion: if $d_j = \min\{d_i \mid i = 1, 2, \dots, C\}$, then y is classified into class j.

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4 Experimental Results

To evaluate the proposed method's performance, extensive experiments were conducted on public face databases including AR and Georgia Tech (GT). The implementation platform was a computer equipped with a 2.5 GHz Intel® B960 processor and 2 GB DDR3 RAM, using MATLAB version 2012a.

4.1 Performance Comparison Using AR Database

The AR database consists of face data from 120 individuals, with 13 images per session (see [Figure 3: see original paper]). All 26 facial image samples per person were captured across two sessions, exhibiting different illumination, expression, and occlusion conditions.

[Figure 3: see original paper] A typical sample from the AR database

This section analyzes the accuracy of the proposed symbolic modeling and recognition technique, comparing performance with state-of-the-art methods from recent literature. The method in [19] proposed a face recognition approach based on Local Approximation Gradient with LDA (LAG-LDA) technique and conducted various experiments on the AR database (exp1 through exp8). For experiments 1-4, they used images numbered 1-4 from session 1 for training, and for experiments 5-8, they used images numbered 8-13 from session 1 for training, testing different combinations across all AR database images.

Performance comparison between the proposed algorithm and LAG-LDA on AR database

The results in demonstrate the effectiveness of the proposed method, which achieves better performance than LAG-LDA under illumination variation (subset 1), expression variation (subset 2), and combined occlusion and illumination effects (subsets 4 and 5).

Another comparable method, the Extended Collaborative Neighbor Representation (ECNR) technique proposed by Waqas Jadoon et al. [19], achieved best recognition rates of 90.45%, 87.94%, 89.32%, and 83.42% for face images under illumination variation, expression variation, occlusion, and combined occlusion with illumination changes, respectively. lists the experimental results obtained by the proposed method and ECNR on the AR database.

Performance comparison between the proposed algorithm and ECNR on AR database

The results in show that the proposed method's best recognition rates are significantly superior to ECNR under expression variation (subset 2) and illumination with occlusion (subsets 4 and 5). Under illumination variation (subset 1), the proposed method achieves comparable recognition rates to ECNR. However, compared with ECNR, the proposed technique's recognition rate is slightly lower under occlusion-only conditions (subset 3).

The proposed method's accuracy is also comparable to some supervised learning methods such as the minimax framework, which handles distortion problems in high-dimensional AR image datasets. For experimental purposes, a similar setup was adopted as defined in [18], uniformly selecting facial images of 25 males and 25 females, downsampling all images to 32$×$32 pixels, and using 10-fold cross-validation across the entire dataset. lists the experimental results of the proposed method and other techniques mentioned in [18] (results taken from [18]).

Performance comparison between the proposed algorithm and minimax framework on AR database

shows that the proposed technology achieves the best recognition accuracy on the AR database compared with other existing classifiers: GMD, SRC, RF, GMD(R), SVM(P), SVM(R), KNN(E), KNN(C), KLDA(P), and KLDA(R) (except for the GMD(R) classifier).

From the results in and , the proposed symbolic similarity method for face recognition demonstrates higher recognition accuracy than ECNR, LAG-LDA, and minimax framework methods, exhibiting certain robustness to facial images under illumination variation, expression changes, and occlusion.

4.2 Performance Comparison Using GT Database

The performance of the proposed symbolic modeling and similarity analysis method was also compared with similar methods defined in [20] using similar experimental settings. The GT database includes 50 subjects with 15 images each, featuring variations in pose, expression, cluttered background, and illumination (see [Figure 4: see original paper]). Compared with AR database images, GT database images exhibit greater differences in expression, pose, occlusion, age, illumination, resolution, and background.

[Figure 4: see original paper] A typical subject sample from the GT database

In the proposed algorithm, all database images were initially cropped to 120$\times$120 pixel size using the Viola-Jones algorithm and converted to grayscale. For experiments, six images per person were randomly selected for training, with the remaining images used for testing. lists the experimental results of the proposed method and methods mentioned in [20] for GT database images.

Performance comparison between the proposed algorithm and methods from [20] on GT database

The experimental results in demonstrate that the proposed method outperforms Fisherfaces, Eigenfaces, Symbolic PCA, Symbolic ICA, Symbolic KPCA, and Symbolic LDA methods.

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5 Conclusion

This paper proposes a spectral domain feature extraction algorithm for face recognition that effectively utilizes local spatial variations in facial images. Initially, the Viola-Jones algorithm crops faces from images and converts them to grayscale images of size 120$\times$120 pixels. The 2D-DFT is performed on preprocessed images, and the main magnitude of 2D-DFT coefficients is calculated using polar Fourier transform techniques and represented as 1D P-FFT. Feature values are the maximum values in the 1D P-FFT amplitude, which are used to construct symbolic objects representing facial images. Finally, classification is implemented using a fast and efficient linear regression classification algorithm. Due to the use of polar FFT, the proposed method is not only effective but also more accurate than other methods based on Cartesian Fourier transforms.

The proposed symbolic modeling method provides a new approach for representing facial features and improves recognition accuracy. Experimental results obtained on AR and GT databases are analyzed in detail and compared with other recently reported methods, demonstrating satisfactory performance of the proposed method.

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Note: Figure translations are in progress. See original paper for figures.