# Terminal Sliding Mode Synchronization of Uncertain Fractional-Order Sprott-C Systems with Unknown Parameters (Postprint)

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**Date:** 2018-08-13T00:00:00+00:00

## Abstract

Based on the excellent performance of sliding mode control, this paper investigates the finite-time synchronization problem for fractional-order chaotic systems using terminal sliding mode control, and presents synchronization results for achieving synchronization of fractional-order Sprott-C drive-response systems (with order $0<\alpha<1$) with unknown parameters and disturbances. By constructing appropriate sliding mode surfaces, suitable fractional-order controllers and parameter adaptation laws are designed for two cases: when the upper bounds of the system' s unknown parameters are known and when they are unknown. Combining relevant theories of fractional-order differential equations and finite-time stability theorems, the conclusion that synchronization control for this system can be achieved is proven, and accurate estimates of the upper bounds of unknown parameters and disturbances are obtained. Finally, by selecting appropriate parameters and through numerical simulations, the validity and feasibility of the presented conclusions are verified.

## Full Text

### Terminal Sliding-Mode Synchronization of Uncertain Fractional-Order Sprott-C System with Unknown Parameters

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**Abstract:** Based on the excellent performance of sliding-mode control, this paper investigates the finite-time synchronization of fractional-order chaotic systems using Terminal sliding-mode control. Synchronization results are presented for fractional-order Sprott-C drive-response systems (with order $0<\alpha<1$) containing unknown parameters and disturbances. By constructing appropriate sliding-mode surfaces, suitable fractional-order controllers and parameter adaptive laws are designed for two scenarios: when the upper bounds of unknown parameters are known and when they are unknown. Combining fractional-order differential equation theory with finite-time stability theorems, the synchronization control of the system is proven, and accurate estimates of the unknown parameters and disturbance bounds are obtained. Finally, numerical simulations with appropriate parameters verify the effectiveness and feasibility of the proposed conclusions.

**Keywords:** fractional-order chaotic system; Terminal sliding-mode control; synchronization; parameter estimation

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## 0 Introduction

Chaotic systems are deterministic nonlinear dynamical systems that exhibit disordered behavior yet contain internal rules, widely existing in natural science and social fields. Over two decades ago, Ott, Grebogi, and Yorke successfully stabilized chaotic systems onto unstable periodic orbits. Subsequently, Ditto and Roy completed experimental verification of chaos control, opening avenues for research on chaotic circuit control, secure communication, and fluid turbulence analysis. Currently, main approaches for chaos control include adaptive control, fuzzy control, optimal control, neural network control, and sliding-mode control. Among these, sliding-mode control has attracted significant attention from scholars in recent years.

In conventional sliding-mode control, linear sliding surfaces are typically selected. To achieve better performance, Xu et al. proposed the Terminal sliding-mode control strategy, which enables systems to achieve complete tracking of desired states within finite time. Literature investigated the design of nonsingular Terminal sliding-mode controllers, achieving asymptotic synchronization of chaotic systems with uncertainties. Tao et al. employed fuzzy rules to design the switching term of Terminal sliding-mode controllers and adaptively adjusted it through adaptive algorithms, realizing Terminal sliding-mode control for mismatched uncertain time-varying systems. Zhuang et al. addressed cases where the upper bounds of uncertainties such as parameter perturbations and external disturbances are unknown, achieving adaptive Terminal controller design for MIMO systems. Yang et al. proposed a nonsingular Terminal sliding-mode control method, improving the convergence speed when the system reaches and maintains the sliding surface.

In recent years, fractional-order calculus has developed rapidly and been applied to fields such as anomalous diffusion, signal processing, quantum mechanics, and system control. Compared with traditional integer-order theory, fractional-order nonlinear systems exhibit richer response characteristics and robustness. Sliding-mode control also offers advantages including fast response, strong robustness, and simple physical implementation. Therefore, applying sliding-mode control to synchronize fractional-order chaotic systems can greatly improve control efficiency. Based on this, literature studied sliding-mode control problems for integer-order chaotic systems with modeling uncertainties and external disturbances. Deng et al. achieved synchronization of fractional-order hyperchaotic systems using output feedback sliding-mode control. Literature analyzed adaptive sliding-mode control for fractional-order (hyper)chaotic systems. Li et al. designed a sliding-mode controller using fractional-order calculus theory to achieve synchronization control of two uncertain fractional-order chaotic systems while identifying bounds of uncertainties and external disturbances. Literature investigated sliding-mode chaotic synchronization of fractional-order Genesio-Tesi systems based on Lyapunov stability theory and Laplace transforms of fractional-order calculus. Gao et al. combined adaptive control theory with sliding-mode control theory, proposing a Terminal sliding-mode surface with faster convergence than traditional ones, where the designed adaptive sliding-mode control law ensured system convergence to the sliding surface within finite time. Thus, using sliding-mode control to synchronize fractional-order chaotic systems holds important research value.

For practical systems, random and external disturbances objectively exist due to modeling errors and operating environments. Considering these factors, this paper proposes a new fractional-order sliding surface and designs a Terminal sliding-mode synchronization algorithm. It proves that the error dynamical system can converge to equilibrium within finite time, thereby achieving synchronization of fractional-order Sprott-C drive-response chaotic systems with unknown parameters and disturbances, while simultaneously estimating the unknown parameters and disturbance bounds online. Finally, MATLAB numerical simulations verify the correctness and effectiveness of the proposed synchronization scheme.

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## 1 Model Establishment and Preliminaries

This paper studies a generalized fractional-order Sprott-C chaotic drive system with unknown parameters:

$$\begin{cases} D^\alpha x_1 = -ax_2 \\ D^\alpha x_2 = x_1 - x_3 \\ D^\alpha x_3 = ax_1 + x_2^2 - x_3 \end{cases}$$

where $x_1, x_2, x_3$ are system state variables, and the fractional order $\alpha$ satisfies $0 < \alpha < 1$.

The corresponding fractional-order response system with unknown bounded random disturbances and external perturbations is selected as:

$$\begin{cases} D^\alpha y_1 = -\hat{a} y_2 + r_1(t) + u_1(t) + \omega_1(t) \\ D^\alpha y_2 = y_1 - y_3 + r_2(t) + u_2(t) + \omega_2(t) \\ D^\alpha y_3 = \hat{a} y_1 + y_2^2 - y_3 + r_3(t) + u_3(t) + \omega_3(t) \end{cases}$$

where $y = (y_1, y_2, y_3)$, $r_i(t)$ and $\omega_i(t)$ (for $i = 1, 2, 3$) represent unknown bounded random disturbances and external perturbation terms with unknown upper bounds, and $u_i(t)$ are sliding-mode synchronization controllers.

For the uncertain terms and external disturbances, we make the following assumption:

**Assumption 1.** There exist unknown bounded positive constants $r_i$ and $\omega_i$ such that the random disturbances and external perturbation terms satisfy:

$$|r_i(t)| \le r_i, \quad |\omega_i(t)| \le \omega_i, \quad i = 1, 2, 3$$

where $\|\cdot\|$ denotes the Euclidean norm. This assumption is reasonable in practical applications since all real signals are bounded.

Let $\hat{r}_i$ and $\hat{\omega}_i$ be the estimated values of the unknown upper bounds of random and external disturbances, satisfying the following adaptive laws:

$$\begin{cases} \dot{\hat{r}}_i = k_{1i} s_i, & \hat{r}_i(0) = \hat{r}_{i0} \\ \dot{\hat{\omega}}_i = k_{2i} s_i, & \hat{\omega}_i(0) = \hat{\omega}_{i0} \end{cases}, \quad i = 1, 2, 3$$

where $k_{1i}$ and $k_{2i}$ are adaptive gain coefficients.

The estimation errors are defined as:

$$\tilde{r}_i = r_i - \hat{r}_i, \quad \tilde{\omega}_i = \omega_i - \hat{\omega}_i, \quad i = 1, 2, 3$$

The error between drive-response systems (1) and (2) is:

$$e(t) = y(t) - x(t)$$

Thus, the error dynamical system is:

$$\begin{cases} D^\alpha e_1 = -\hat{a} e_2 + (\hat{a} - a) x_2 + r_1(t) + u_1(t) + \omega_1(t) \\ D^\alpha e_2 = e_1 - e_3 + r_2(t) + u_2(t) + \omega_2(t) \\ D^\alpha e_3 = \hat{a} e_1 + (\hat{a} - a) x_1 + y_2^2 - x_2^2 - e_3 + r_3(t) + u_3(t) + \omega_3(t) \end{cases}$$

For convenience, we introduce the following assumptions and lemmas:

**Assumption 2.** The unknown parameters $a_i$ satisfy the constraint condition:

$$|a_i| \le \bar{a}_i, \quad i = 1, 2, 3$$

where $\bar{a}_i$ are positive constants.

**Lemma 1 (Finite-time Stability Theorem).** Assume there exists a continuous positive definite function $V(t)$ satisfying the differential inequality:

$$\dot{V}(t) \le -\varepsilon V^{\alpha}(t), \quad \forall t \ge t_0, \quad V(t_0) \ge 0$$

where $\varepsilon > 0$ and $0 < \alpha < 1$. Then for any given $t_0$, $V(t)$ satisfies:

$$V^{1-\alpha}(t) \le V^{1-\alpha}(t_0) - \varepsilon(1-\alpha)(t - t_0), \quad t_0 \le t \le t_{\text{fin}}$$

and

$$V(t) \equiv 0, \quad \forall t \ge t_{\text{fin}}$$

where the finite time $t_{\text{fin}}$ is given by:

$$t_{\text{fin}} = t_0 + \frac{V^{1-\alpha}(t_0)}{\varepsilon(1-\alpha)}$$

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## 2.1 Finite-Time Terminal Sliding-Mode Control

Referring to the design of traditional integer-order sliding surfaces in literature, this paper improves and extends it to fractional-order cases by designing the following Terminal sliding surface:

$$s_i(t) = D^{\alpha-1} e_i(t) + \theta_i \int_0^t \text{sgn}(e_i(\tau)) |e_i(\tau)|^{\eta_i} d\tau$$

where $\theta_i > 0$, $0 < \eta_i < 1$ are gain coefficients. Differentiating this expression yields:

$$\dot{s}_i(t) = D^{\alpha} e_i(t) + \theta_i \text{sgn}(e_i(t)) |e_i(t)|^{\eta_i}$$

The unknown parameter upper bounds $\bar{a}_i$ are divided into two cases: known and unknown. For different scenarios, appropriate controllers will be designed respectively to achieve sliding-mode synchronization control of the fractional-order drive-response Sprott-C system.

**Case 1: Known Upper Bounds of Drive System Parameters**

The adaptive update law for unknown parameters is designed as:

$$\dot{\hat{a}}_i = -c_i s_i, \quad \hat{a}_i(0) = \hat{a}_{i0}, \quad i = 1, 2, 3$$

where $c_i$ are positive constants.

**Theorem 1.** On the Terminal sliding surface (6), the trajectory of error system (5) reaches the equilibrium point in finite time:

$$t_{\text{fin}} \leq \frac{\theta_{\min}}{\theta_{\min} - \eta_{\max}} \left( \sum_{i=1}^{3} |e_i(0)|^{\theta_{\min}} \right)^{\frac{\theta_{\min} - \eta_{\max}}{\theta_{\min}}}$$

where $\theta_{\min} = \min\{\theta_1, \theta_2, \theta_3\}$ and $\eta_{\max} = \max\{\eta_1, \eta_2, \eta_3\}$.

**Proof.** When the error system moves on the sliding surface, the sliding surface and its derivative must satisfy $s_i(t) = 0$ and $\dot{s}_i(t) = 0$. From equation (8), we obtain:

$$D^\alpha e_i(t) = -\theta_i \text{sgn}(e_i(t)) |e_i(t)|^{\eta_i}$$

Choose the Lyapunov function:

$$V_1(t) = \frac{1}{2} \sum_{i=1}^{3} e_i^2(t)$$

Taking its time derivative yields:

$$\dot{V}_1(t) = \sum_{i=1}^{3} e_i(t) D^\alpha e_i(t) = -\sum_{i=1}^{3} \theta_i |e_i(t)|^{\eta_i + 1} \leq -\theta_{\min} \sum_{i=1}^{3} |e_i(t)|^{\eta_{\max} + 1}$$

Using the inequality $(\sum_{i=1}^n |x_i|^p)^{1/p} \leq \sum_{i=1}^n |x_i|$ for $0 < p < 1$, we have:

$$\dot{V}_1(t) \leq -\theta_{\min} \left( \sum_{i=1}^{3} e_i^2(t) \right)^{\frac{\eta_{\max}+1}{2}} = -\theta_{\min} (2V_1(t))^{\frac{\eta_{\max}+1}{2}}$$

According to Lemma 1, the error system reaches the equilibrium point in finite time $t_{\text{fin}}$.

**Theorem 2.** Assume the upper bounds $\bar{a}_i$ of unknown parameters in the drive system are known. Under the adaptive laws (4) and (13) for unknown parameters and disturbances, and the control law (14), the drive-response systems (1) and (2) achieve sliding-mode synchronization control. The control law is:

$$\begin{cases} u_1(t) = -\hat{a}e_2 + \hat{a}x_2 - \mathrm{sgn}(s_1)\left(\hat{r}_1 + \hat{\omega}_1 + \eta_1|e_1|^{\theta_1}\right) \\ u_2(t) = -e_1 + e_3 - \mathrm{sgn}(s_2)\left(\hat{r}_2 + \hat{\omega}_2 + \eta_2|e_2|^{\theta_2}\right) \\ u_3(t) = -\hat{a}e_1 - \hat{a}x_1 - y_2^2 + x_2^2 + e_3 - \mathrm{sgn}(s_3)\left(\hat{r}_3 + \hat{\omega}_3 + \eta_3|e_3|^{\theta_3}\right) \end{cases}$$

where the gain coefficients $\eta_i > 0$ can adjust the synchronization convergence speed. The estimation errors are denoted as $\tilde{a}_i = a_i - \hat{a}_i$.

**Proof.** Construct the Lyapunov energy function:

$$V_2(t) = \frac{1}{2}\sum_{i=1}^{3} s_i^2 + \frac{1}{2k_{1i}}\tilde{r}_i^2 + \frac{1}{2k_{2i}}\tilde{\omega}_i^2 + \frac{1}{2c_i}\tilde{a}_i^2$$

Taking its time derivative and substituting the parameter update law (13) and disturbance adaptive law (4) yields:

$$\dot{V}_2(t) \leq -\sum_{i=1}^{3} \eta_i|s_i| \leq -\eta_{\min}\sum_{i=1}^{3}|s_i|$$

where $\eta_{\min} = \min\{\eta_1, \eta_2, \eta_3\}$. Using the inequality relation, we obtain:

$$\dot{V}_2(t) \leq -\eta_{\min}\sqrt{2}V_2^{1/2}(t)$$

According to the finite-time stability theorem, the error system reaches the equilibrium point in finite time $t_{\text{fin}}$, meaning the error system (7) reaches the sliding surface in finite time.

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## 2.2 Terminal Sliding-Mode Control

The above theorems guarantee that the error system can reach the equilibrium point in finite time. This section discusses the stability of the error system on the sliding surface.

### Case 2: Unknown Upper Bounds of Drive System Parameters

When the upper bounds of unknown parameters are unknown, the adaptive update law for unknown parameters can be designed as:

$$\dot{\hat{a}}_i = -c_i s_i - l_i \mathrm{sgn}(s_i), \quad \hat{a}_i(0) = \hat{a}_{i0}, \quad i = 1, 2, 3$$

where $l_i > 0$ are feedback parameters, and the meanings of other parameters are consistent with those in adaptive law (13).

**Theorem 3.** Under the adaptive laws (4) and (15) for unknown parameters and disturbances, and the control law (16), the drive-response systems (1) and (2) achieve sliding-mode synchronization control. The control law is:

$$\begin{cases} u_1(t) = -\hat{a}e_2 + \hat{a}x_2 - \operatorname{sgn}(s_1)\left(\hat{r}_1 + \hat{\omega}_1 + \eta_1|e_1|^{\theta_1} + l_1\right) \\ u_2(t) = -e_1 + e_3 - \operatorname{sgn}(s_2)\left(\hat{r}_2 + \hat{\omega}_2 + \eta_2|e_2|^{\theta_2} + l_2\right) \\ u_3(t) = -\hat{a}e_1 - \hat{a}x_1 - y_2^2 + x_2^2 + e_3 - \operatorname{sgn}(s_3)\left(\hat{r}_3 + \hat{\omega}_3 + \eta_3|e_3|^{\theta_3} + l_3\right) \end{cases}$$

**Proof.** Construct the same Lyapunov energy function as in Theorem 2:

$$V_3(t) = \frac{1}{2}\sum_{i=1}^{3} s_i^2 + \frac{1}{2k_{1i}}\tilde{r}_i^2 + \frac{1}{2k_{2i}}\tilde{\omega}_i^2 + \frac{1}{2c_i}\tilde{a}_i^2$$

Taking its time derivative and substituting the disturbance adaptive law (4), parameter update law (15), sliding surface derivative (9), and controller (16) yields:

$$\dot{V}_3(t) \le -\sum_{i=1}^{3}\left(\eta_i|s_i| + l_i|s_i|\right) \le -l_{\min}\sum_{i=1}^{3}|s_i|$$

where $l_{\min} = \min\{l_1, l_2, l_3\}$. Therefore:

$$\dot{V}_3(t) \le -l_{\min}\sqrt{2}V_3^{1/2}(t)$$

Integrating both sides with respect to time and applying the comparison principle gives:

$$V_3(t) \le \left(\sqrt{V_3(0)} - \frac{l_{\min}}{\sqrt{2}}t\right)^2$$

Thus, when $t \ge \frac{\sqrt{2V_3(0)}}{l_{\min}}$, $V_3(t)$ decreases monotonically to zero. According to LaSalle' s invariance principle, when $t \to +\infty$, $s_i(t) \to 0$. Combining Barbalat' s lemma, we know $e_i(t) \to 0$ for $i = 1, 2, 3$. This means sliding-mode synchronization of systems (1) and (2) is achieved even when the upper bounds of unknown parameters in the drive system are unknown.

Furthermore, from fractional-order calculus theory, $D^\alpha e_i(t) \to 0$, which implies that the drive-response systems (1) and (2) achieve Terminal sliding-mode synchronization.

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## 3 Numerical Simulation

This chapter presents numerical simulations to verify the above results. To avoid chattering phenomena in sliding-mode control, two methods are commonly used in practical engineering applications: (1) dividing the controller into continuous and switching components to eliminate switching amplitude; (2) using high-gain methods to replace the $\mathrm{sgn}(\cdot)$ term with saturation functions or similar functions. In the simulation experiments, this paper adopts the second method, using $\frac{s_i}{|s_i|+\varepsilon}$ (where $\varepsilon = 0.001$) to replace the $\mathrm{sgn}(s_i)$ term, achieving good control effects.

When the fractional order $\alpha = 0.97$, the Sprott-C system exhibits chaotic behavior with two equilibrium points $E_1 = (10, 10, 1.7)$ and $E_2 = (-10, -10, 1.7)$. The chaotic attractor is shown in [Figure 1: see original paper]. For simulation convenience, the uncertain terms in the system are taken as:

$$r_1(t) = 0.2\sin(3t), \quad r_2(y) = 0.15\sin(2y), \quad r_3(t) = 0.3\cos(t)$$

External disturbances are:

$$\omega_1(t) = 0.2\sin(t), \quad \omega_2(t) = 0.3\cos(t), \quad \omega_3(t) = 0.4\sin(t)$$

In the simulation, disturbance initial values are $\hat{r}(0) = (0.1, 0.2, 0.3)$ and $\hat{\omega}(0) = (0.7, 0.5, 0.3)$. Without loss of generality, the disturbance bound tuning parameters are $k_{11} = k_{12} = k_{13} = 1.1$ and $k_{21} = k_{22} = k_{23} = 0.9$. In sliding surface (8), parameters are $(\eta_1, \eta_2, \eta_3) = (0.5, 0.5, 0.4)$ and $\theta = 0.2$.

**Case 1: Known Upper Bounds of Drive System Parameters**

The upper bounds are taken as $\bar{a}_1 = 10$, $\bar{a}_2 = 100$, $\bar{a}_3 = 1.7$. System initial values are $x(0) = \{0.8, 0.2, -1.9\}$ and $y(0) = (1.6, 1, 4)$. Unknown parameter initial values are $\hat{a}_1(0) = 7.41$, $\hat{a}_2(0) = 97.9$, $\hat{a}_3(0) = -1.92$. Parameter adaptive law coefficients are $c_i = 1$ for $i = 1, 2, 3$.

Using the predictor-corrector numerical algorithm, the synchronization simulation results are shown in [Figure 2: see original paper] and [Figure 3: see original paper]. [Figure 1: see original paper] shows the chaotic attractor of the fractional-order Sprott-C system at $\alpha = 0.97$. When controller (14) is applied to the drive-response systems (1) and (2), the system errors are shown in [Figure 2: see original paper]. The errors converge to zero within finite time, indicating that the drive-response network (1) and (2) achieves Terminal sliding-mode synchronization. Meanwhile, the unknown parameters of the drive system are accurately estimated, as shown in [Figure 3: see original paper], where all unknown parameters converge to their true values.

In fact, when the fractional order $\alpha = 0.98$, the Sprott-C system remains chaotic, coexisting with two unstable equilibrium points $E_1$ and $E_2$. Selecting initial values $x(0) = (0.5, 0.3, 1.2)$ and $y(0) = (3, 1.5, 5)$, with unknown parameter initial values $(\hat{a}_{10}, \hat{a}_{20}, \hat{a}_{30}) = (10.57, 99.7, 0.53)$ and other parameters unchanged, the simulation results are shown in [Figure 4: see original paper] and [Figure 5: see original paper].

**Case 2: Unknown Upper Bounds of Drive System Parameters**

When the upper bounds of unknown parameters are unknown, synchronization of systems (1) and (2) can be achieved using controller (16). Fixing the same parameters as in Case 1, the feedback parameters are $l_1 = 0.2$, $l_2 = 0.15$, $l_3 = 0.2$, with initial values $x(0) = (2.6, 1.2, 3)$ and $y(0) = (1, 0.2, 0.2)$. Unknown parameter initial values are $(\hat{a}_{10}, \hat{a}_{20}, \hat{a}_{30}) = (12.3, 102.8, 3.16)$, and $c_i = 0.1$ for $i = 1, 2, 3$. With system order $\alpha = 0.97$, the error evolution curves and unknown parameter estimates are shown in [Figure 6: see original paper] and [Figure 7: see original paper].

[Figure 8: see original paper] shows the evolution curve of the system gradually approaching equilibrium point $E_1$. With fixed feedback parameters and system initial values, unknown parameter initial values are selected as $(\hat{a}_{10}, \hat{a}_{20}, \hat{a}_{30}) = (4.56, 95.6, 2.15)$ with $c_i = 2$ for $i = 1, 2, 3$. The system can stabilize to equilibrium point $E_2$, with simulation results shown in [Figure 9: see original paper]. The unknown parameter estimation curves and error evolution curves are shown in [Figure 10: see original paper] and [Figure 11: see original paper].

Through the above numerical simulations, we observe that the estimated values of unknown parameters and random (external) disturbances all approach their true values, and the drive system (1) and response system (2) achieve finite-time synchronization, fully demonstrating the effectiveness of the designed synchronization controller and unknown parameter identification rules. Moreover, the synchronization time in simulations matches the theoretical calculation from Theorem 1.

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## 4 Conclusion

The purpose of studying chaos control is to better manipulate the dynamics of nonlinear systems within a larger scope for human benefit. This paper employs fractional-order controllers to synchronize fractional-order controlled objects, investigates the Terminal sliding-mode control problem for uncertain fractional-order Sprott-C systems with unknown parameters and bounded disturbances, designs sliding-mode dynamic surface functions, achieves sliding-mode synchronization control of the constructed drive-response system, and estimates unknown parameters. Numerical simulations finally verify the validity of the conclusions. The methods and conclusions in this paper have certain universality and can provide references for synchronization control research of similar fractional-order chaotic systems.

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## References

[1] Ahamad H, Mojtaba H, Duitru B. On the adaptive sliding mode controller for a hyperchaotic fractional-order financial system [J]. Physica A, 2018, 497(1): 139-153.

[2] Muthukumar P, Balasubramamiam P, Rzthnavelu K. Sliding mode control design for synchronization of fractional order chaotic systems and its application to a new cryptosystem [J]. International Journal of Dynamics and Control, 2017, 5(1): 115-123.

[3] Fu Hongrui, Shi Hongtao, Zhang Jiangang. Chaos synchronization of complex networks based on the novel four-wing chaotic system and its application in secure communication [J]. Journal of Sichuan University: Natural Science Edition, 2017, 54(5): 965-970.

[4] Xu Ruiping, Gao Mingmei. Synchronization of chaotic systems with uncertainty using an adaptive terminal sliding mode controller [J]. Control Engineering of China, 2016, 23(5): 715-719.

[5] Ayub K, Arti T. Fractional order disturbance observer based adaptive sliding mode synchronization of commensurate fractional order Genesio-Tesi system [J]. International Journal of Electronic Communication (AEÜ), 2017(82): 346.

[6] Liu Heng, Li Shenggang, Sun Yeguo, et al. Adaptive fuzzy synchronization for uncertain fractional order chaotic systems with unknown non-symmetrical control gain [J]. Acta Physica Sinica, 2015, 64(7): 070503.

[7] Tao Siyan, Lin Da, Zeng Xiaohui. Robust optimal control of unified chaotic system with uncertain parameters [J]. Journal of Measurement and Control Technology, 2016, 35(2): 96-98.

[8] Lin Feifei, Zeng Zhezhao. Synchronization of uncertain fractional order chaotic systems with time delay based on adaptive neural network control [J]. Acta Physica Sinica, 2017, 66(9): 33-42.

[9] Sun Ning, Zhang Huaguang, Wang Zhiliang. Projective synchronization of uncertain fractional order chaotic system using sliding mode controller [J]. Journal of Zhejiang University: Engineering Science, 2010, 44(7): 1288-1291.

[10] Ni Junkang, Liu Chongxin, Liu Kai, et al. Finite-time sliding mode synchronization of chaotic systems [J]. Chinese Physics B, 2014, 23(10): 100503.

[11] Yu Xinghuo, Man Zhihong. Model reference adaptive control systems with terminal sliding modes [J]. International Journal of Control, 2012, 64(6): 1165-1176.

[12] Deng Wei, Fang Jie, Wu Zhengjun, et al. Adaptive modified function projective synchronization of a class of chaotic systems with uncertainties [J]. Acta Physica Sinica, 2012, 61(14): 191-199.

[13] Tao Chinwang, Taur Jinshiuh, Chan Meilang. Adaptive fuzzy terminal sliding mode controller for linear systems with mismatched time-varying uncertainties [J]. IEEE Trans on Systems, Man and Cybernetics, Part B, 2004, 34(1): 1-6.

[14] Zhuang Kaiyu, Su Hongye, Zhang Keqing, et al. Adaptive terminal sliding mode control for high order nonlinear dynamic systems [J]. Zhejiang University Science, 2003, 4(1): 58-63.

[15] Yang Liang, Yang Jianying. Nonsingular fast terminal sliding mode control for nonlinear dynamical systems [J]. International Journal of Robust and Nonlinear Control, 2011, 21(16): 1865-1879.

[16] Aghababa M P, Feizi H. Design of a sliding mode controller for synchronizing chaotic systems with parameter and model uncertainties and external disturbances [J]. Transactions of the Institute of Measurement and Control, 2012, 34(8): 990-997.

[17] Deng Liwei, Song Shenmin. Synchronization of fractional order hyperchaotic systems based on output feedback sliding mode control [J]. Acta Automatica Sinica, 2014, 40(11): 2420-2427.

[18] Pan Guang, Wei Jing. Design of an adaptive sliding mode controller for synchronization of fractional-order chaotic systems [J]. Acta Automatica Sinica, 2015, 64(4): 41-47.

[19] Mohadeszadeh M, Delavari H. Synchronization of fractional order hyperchaotic systems based on a new adaptive sliding mode control [J]. International Journal Dynamical Control, 2015, 5(1): 1-11.

[20] Li Te, Yuan Jianbao, Wu Ying. A method of adaptive sliding mode control for synchronization of one class of uncertain fractional-order chaotic systems [J]. Journal of Dynamics and Control, 2017, 15(2): 110-118.

[21] Mao Beinxing, Li Qiaoli. Sliding mode chaos synchronization of a class of fractional-Genesio-Tesi systems [J]. Acta Scientiarum Naturalium Universitatis Sunyatseni, 2017, 56(2): 76-79.

[22] Gao Junshan, Song Ge, Deng Liwei. Finite-time sliding mode synchronization control of chaotic systems with uncertain parameters [J]. Control and Decision, 2017, 32(1): 149-156.

[23] Zhao Yuhan, Wu Ranchao. Chaos and synchronization of a new fractional order system with only two stable equilibria [J]. International Journal of Dynamical Systems and Differential Equations, 2016, 6(3): 187-194.

[24] Podlubny. Fractional differential equation [M]. New York: Academic Press, 1999.

*Note: Figure translations are in progress. See original paper for figures.*