Postprint: Inverse Model for Convective Heat Transfer at Indoor Building Boundaries

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Date: 2018-07-09T00:00:00+00:00

Abstract

To address the current limiting gap wherein inverse calculation models are yet unable to perform inverse calculations of boundary convective heat transfer in building interiors, the temperature contribution rate method is employed to represent the causal relationship between boundary convective heat transfer and indoor measurement point temperatures as a temperature contribution factor matrix. Based on computational fluid dynamics (CFD), and by combining least squares with Tikhonov regularization methods, an inverse problem mathematical model is established for solving boundary convective heat transfer based on discrete temperatures from several indoor measurement points. Experimental validation was conducted using both a three-dimensional ventilated cavity and an office room within a building. The root mean square error (RMSE) between model-calculated values and measured values was less than 80% in all cases, and the results demonstrate that the inverse calculation model can accurately solve for indoor boundary convective heat transfer.

Full Text

Inverse Method to Determine Wall Boundary Convective Heat Fluxes in Indoor Environments

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Abstract: Thus far, no research has inversely solved for boundary convective heat fluxes in indoor environments. This study proposed an inverse method based on Tikhonov regularisation and least-squares optimisation using computational fluid dynamics (CFD) to determine the wall boundary convective heat fluxes in indoor environments. The inverse model applied the CRI to establish a matrix to describe the cause-effect relation between the cabin wall boundary heat flux and the exhibited discrete temperature at certain points. To test and evaluate the proposed inverse model, the surface heat fluxes on walls in a three dimensional ventilation cavity and an office in a building were inversely solved. The RMS errors of the model were less than 80%. This study finds that the developed inverse method can accurately and efficiently determine the wall convective heat fluxes in indoor environments.

Key words: inverse modeling; thermal boundary condition; Tikhonov regularization; least square; CFD

When designing indoor environments, thermal boundary conditions, air supply locations, air supply velocities, and air supply temperatures must be considered as key environmental parameters. To achieve a comfortable thermal environment, proper configuration of thermal boundary conditions is a critical factor. Conventional methods for determining thermal boundary conditions require a time-consuming iterative guess-and-correction process. If the temperature distribution within a specified environment is known, then the corresponding thermal boundary conditions must be determined promptly, which would greatly accelerate the thermal environment design process. This represents a computational problem of determining thermal boundary conditions based on a target temperature distribution—an inverse problem that works "from effect to cause" by exploring unknown boundary conditions based on limited measured data.

Many researchers have conducted inverse modeling studies in indoor environments, but these have primarily focused on pollution source identification [1-2]. Reference [1] employed positive definite matrix factorization to inversely determine the release strength of indoor particulate pollution sources. Reference [2] used probability density distributions to inversely identify pollution source locations in ventilated environments. Reference [3] investigated the influence of indoor pollution source release locations on pollutant concentration distributions under different air supply methods. Building upon different air supply methods, Reference [4] added varying air supply velocities as a condition to study the combined effects of air supply methods and velocities on indoor formaldehyde concentration distributions. Reference [5] applied Fluent numerical simulation software to examine ventilation effectiveness between natural and mechanical ventilation in severe cold regions, and studied the effects of different air supply velocities and temperatures on formaldehyde concentrations at various indoor heights.

Research on inverse heat transfer problems has made progress in modern engineering contexts. Reference [6] constructed an inverse method to estimate boundary temperature curves during oven heating processes. Reference [7] proposed an inverse technique based on Green's functions and dynamic observations to predict tool interface temperatures and heat fluxes during drilling operations.

Additionally, due to its high computational accuracy [8], the conjugate gradient method is a commonly used approach for solving inverse heat transfer problems. References [9, 10] employed the conjugate gradient method to inversely determine indoor boundary temperature variation characteristics under mixed ventilation conditions. Reference [11] studied solid heat conduction inverse problems, applying the conjugate gradient method to inversely solve solid boundary conductive heat fluxes. Reference [12] used the conjugate gradient method to invert heat transfer characteristics during blast furnace lining erosion. Beyond the widely used conjugate gradient method, Reference [13] utilized genetic algorithms to inversely determine the effective thermal conductivity of alumina fiber thermal protection insulation based on temperature measurements.

These inverse problem models can all be solved using computational fluid dynamics (CFD) and can meet computational accuracy requirements. However, these methods are typically costly, require appropriate initial values for iterative loops, and often result in large computational loads and low efficiency. Moreover, the aforementioned inverse models are based on solving for single thermal boundary conditions and do not consider how multiple simultaneous thermal boundary conditions influence temperature distributions. To date, no research has conducted inverse modeling specifically for thermal boundary conditions in indoor environments, nor has there been a convenient and efficient method to simultaneously solve for multiple boundary heat fluxes. Since boundary convective heat fluxes significantly impact indoor thermal and humidity environments, research on determining these fluxes is important for analyzing heating, ventilation, and air conditioning (HVAC) systems and studying boundary heat transfer characteristics in air-conditioned spaces.

1. Inverse Calculation Model for Boundary Convective Heat Flux

1.1 Temperature Field Linear System

In indoor building environments, thermal boundary conditions appear as source terms in the energy equation. The steady-state energy equation derived from the first law of thermodynamics is as follows [15]:

$$\rho C_p \mathbf{u} \cdot \nabla T = \nabla \cdot (\Gamma \nabla T) + Q$$

where: T is air temperature (°C), $\mathbf{u}$ is air velocity (m/s), Γ is the diffusion coefficient (m^2/s), Q is the heat source (W), C_p is specific heat at constant pressure (J/kg·°C), ρ is air density (kg/m^3), div denotes the divergence operator, and grad denotes the gradient operator.

If only one heat source exists indoors, the steady-state energy equation formed by this heat source can be expressed as:

$$\rho C_p \mathbf{u} \cdot \nabla T_i = \nabla \cdot (\Gamma \nabla T_i) + Q_i$$

If the indoor air supply velocity remains constant and the supply air temperature remains constant or varies within a small range, the indoor airflow pattern can be considered a steady-state flow field. In this case, solving only the energy equation yields the temperature field. Under mechanical ventilation conditions, forced convection dominates, and if thermal buoyancy forces caused by changes in thermal boundary conditions cause minimal disturbance to the flow field, the temperature field can be assumed to be a linear system [16]. The temperature field under the combined action of n (n>1) heat sources equals the linear superposition of sub-temperature fields formed by individual heat sources, expressed as:

$$T = \sum_{i=1}^{n} T_i$$

where T is the total temperature field under n heat sources, and T_i is the sub-temperature field formed when only the i-th heat source Q_i exists.

Due to indoor turbulence characteristics, the temperature field is not strictly linear in the rigorous sense, but the nonlinearity of the time-averaged velocity field is minor compared to convection under the mean flow field. Therefore, assuming linearity of the indoor temperature field is reasonable. With the aid of the energy equation (1), equation (3) can be transformed into the following form:

$$\rho C_p \mathbf{u} \cdot \nabla T = \nabla \cdot (\Gamma \nabla T) + \sum_{i=1}^{n} Q_i$$

1.2 Objective Function Establishment

Changes in indoor thermal boundary conditions affect the temperature field. Since only radiative heat transfer occurs between solid walls and air (a transparent medium that does not absorb radiation), convective heat transfer between air and solid walls is the primary factor influencing indoor air temperature changes. Based on the linearity of the temperature field under steady-state flow conditions, the relationship between temperature changes at measurement points and changes in boundary convective heat flux can be expressed as:

$$\Delta T = A \Delta Q$$

where $\Delta T = [\Delta T_1, \Delta T_2, \cdots, \Delta T_n]^T$ represents temperature changes at measurement points, and $\Delta Q = [\Delta Q_1, \Delta Q_2, \cdots, \Delta Q_n]^T$ represents changes in boundary convective heat flux. Since the indoor flow field is assumed fixed and the relationship between temperature changes and boundary convective heat flux changes is linear, A is a linear matrix describing the causal relationship between boundary convective heat flux and measurement point temperatures.

The primary purpose of the inverse model is to determine thermal boundary conditions based on indoor measurement point temperatures. Since matrix A is ill-conditioned, solving equation (5) inversely represents an ill-posed problem lacking numerical stability. Therefore, direct inverse solution of equation (5) is not feasible. To obtain results that better reflect actual conditions, special treatment of equation (5) is required. This paper employs Tikhonov regularization to enhance solution stability by adding a regularization term during the solution process. Through Tikhonov regularization, equation (5) is transformed into a minimization objective function problem, expressed as follows [17]:

$$Z = \|A\Delta Q - \Delta T\|^2 + \lambda\|L\Delta Q\|^2$$

where $\|\cdot\|$ denotes the matrix 2-norm, λ is the regularization parameter, and L is the regularization matrix. The most common expression is:

$$L = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{(n-1)\times n}$$

According to equation (8), if the measurement point temperature T, linear matrix A, regularization matrix L, and regularization parameter λ are known, the boundary convective heat flux Q can be obtained.

Selection of an appropriate regularization parameter λ is key to solving the Tikhonov regularization method. The Fixed-point method (FP) is a simple algorithm that can obtain parameter λ at low computational cost [22-24]. The FP algorithm expression is:

$$\Psi_\mu(\lambda) = x(\lambda) - \mu y(\lambda)$$

where $x(\lambda)$ is a monotonically increasing function of λ, and $y(\lambda)$ is a monotonically decreasing function of λ. Based on this property, the following function expression for regularization parameter λ can be obtained:

$$\Phi_\eta(\lambda) = \frac{x(\lambda)}{\lambda^\eta} + \mu\lambda^\eta y(\lambda)$$

where μ is a parameter. Due to the monotonicity of functions $x(\lambda)$ and $y(\lambda)$, as λ iterates continuously, $\Psi_\mu(\lambda)$ has an extremum point, and the λ that achieves this extremum is the preferred local regularization parameter. Taking the first derivative of $\Psi_\mu(\lambda)$ and setting it equal to zero yields the iterative formula for regularization parameter λ:

$$\frac{d\Psi_\mu(\lambda)}{d\lambda} = 0$$

For variable μ, extensive numerical verification has shown that when $\mu = 1$, a reasonable regularization parameter λ can often be obtained. $\phi_1(\lambda)$ is a monotonically increasing function of λ that possesses the geometric characteristic of having one convex point and one concave point in the iteration interval of λ. The λ corresponding to the convex point is the locally optimal regularization parameter.

1.3 Construction of Temperature Contribution Factor Matrix A

The CRI method is employed to obtain the linear matrix A. The definition of CRI for thermal boundary i at measurement location X is as follows [13]:

$$CRI_i(X) = \frac{\Delta T_i(X)}{\Delta T_{P,i}} = \frac{\Delta T_i(X)}{\Delta T_i(X)} \cdot \frac{\rho C_p V}{Q_i}$$

where $\Delta T_i(X)$ is the temperature change (°C) at measurement point X caused by the change in convective heat flux ΔQ_i of boundary i, $\Delta T_{P,i}$ is the temperature change (°C) caused by the i-th thermal boundary Q_i when all thermal boundaries exist simultaneously, ρ is air density (kg/m^3), C_p is specific heat at constant pressure (J/kg · °C), and V is air flux (m^3/s).

Let the temperature change when a single thermal boundary exists independently be denoted as matrix X, and the temperature contribution rate when all thermal boundaries exist simultaneously be denoted as matrix C. According to the linear superposition property of the temperature field, the relationship between boundary convective heat flux changes and measurement point temperature changes is:

$$\Delta T = CX^{-1}\Delta Q$$

Substituting expression (15) into equation (16) yields the expression for temperature contribution factor matrix A:

$$A = CX^{-1}$$

According to the regularized problem (6), let:

$$\Delta Q = (A^T A + \lambda L^T L)^{-1} A^T \Delta T$$

The calculation results of the inverse model after Tikhonov regularization treatment are as follows:

$$\Delta Q = \sum_{k=1}^{K} \frac{\sigma_k^2}{\sigma_k^2 + \lambda} \frac{\mathbf{u}_k^T \Delta T}{\sigma_k} \mathbf{v}_k$$

where σ_k are the singular values of matrix A, and $\mathbf{u}_k$, $\mathbf{v}_k$ are the corresponding left and right singular vectors.

2. Experimental Validation Using 3D Ventilation Cavity

2.1 Experimental Setup Description

To evaluate the effectiveness of the inverse model, a three-dimensional ventilation cavity experimental platform was constructed with geometric dimensions of 1.04 m × 1.04 m × 0.7 m (X × Y × Z), as shown in Figure 1 [Figure 1: see original paper]. Air is supplied through a strip inlet 0.018 m wide at the upper left side and exhausted through an outlet 0.024 m wide at the lower right side of the cavity. Based on temperatures at four measurement points P1-P4 shown in Figure 1(b), the inverse model is applied to solve for the convective heat fluxes on the ceiling, floor, left wall, and right wall shown in Figure 1(a).

The prerequisites for applying the inverse model calculation are steady-state flow field, measurement point temperatures, boundary measurement values, thermophysical properties, and spatial geometric dimensions. CFD is a powerful tool for simulating indoor velocity distributions, temperature distributions, and pollutant diffusion. The Fluent (6.3.26) software was used to simulate the velocity and temperature fields in the three-dimensional ventilation cavity. Since the indoor airflow pattern is turbulent, the RNG k- turbulence model was selected. The momentum, turbulence, and energy equations were discretized using a second-order upwind scheme, and the Boussinesq model was adopted to approximate thermal buoyancy. Since radiative heat transfer does not directly affect air temperature, the radiation heat transfer model was not considered in the simulation. Convergence criteria were set at 1.0×10^{-3} for the continuity and momentum equations, and 1.0×10^{-2} for the energy equation. In the CRI method calculation, four thermal boundaries exist (ceiling, floor, left wall, right wall). After obtaining the indoor steady-state flow field, the CRI distributions for the four solid walls of the ventilation cavity can be acquired. Based on the CRI distributions of the four walls, the temperature contribution factor matrix A is constructed through equation (17). The matrix constructed by the CRI method describes the causal relationship between wall convective heat flux and measurement point temperature. Since this matrix is a function of the flow field and boundary type, it is not affected by the steady-state flow field and boundary heat fluxes.

The four measurement point temperatures shown in Figure 1(b) at positions P1-P4 on the center line of the z = 0.35 m cross-section are known information used to inversely solve for the convective heat fluxes of the four solid walls. A self-compiled MATLAB program was used to operate the inverse model. T is an R4$×$1 dimensional vector, the temperature contribution factor matrix $A \in \mathbb{R}^{4\times4}$, and L employs a second-order regularization matrix. As previously mentioned, the key to solving the inverse model is finding a suitable regularization parameter λ. Once T, L, and λ are determined, the boundary convective heat flux Q can be solved according to equation (8).

The purpose of the Tikhonov regularization method is to minimize the modified objective function. The FP method was applied to obtain $\lambda = 1807.17$ as the optimal regularization parameter, as shown in Figure 2 [Figure 2: see original paper]. The FP method is computationally efficient and robust. Substituting λ into equation (8) yields the convective heat fluxes of the four walls. Table 1 lists the actual convective heat fluxes of the four solid walls, the inverse model calculation results, and the errors between them. The results show that the actual values are close to the inverse model calculations, with a root mean square difference of 0.6°C. This demonstrates that the inverse model can determine boundary convective heat fluxes and has good practical value.

2.2 Experimental Validation Results

The inverse model enables rapid determination of convective heat fluxes for each wall surface based on measurement point temperatures.

3. Experimental Validation in an Office Building

3.1 Office Environment Description

To verify the feasibility of the inverse calculation model in practical applications, an office room in a building was selected for experimental testing. The three-dimensional view of the office is shown in Figure 2 Figure 2: see original paper, with geometric dimensions of 4.92 m × 4.32 m × 2.42 m (X × Y × Z). A file cabinet is placed in one corner of the office, two desks and four computers are placed in the other two corners, four people sit in front of the desks, six fluorescent lamps are installed on the ceiling, four diffusers measuring 0.5 m × 0.5 m are installed on the floor, and one exhaust outlet measuring 0.3 m × 0.3 m is installed on the ceiling. Based on six measurement point temperatures shown in Figure 2(b), the inverse model is applied to solve for the convective heat fluxes on the ceiling, floor, east wall, west wall, south wall, and north wall shown in Figure 2(a).

Using the same CFD simulation method as the previous three-dimensional ventilation cavity test, the vector T composed of six measurement points is an R6$×$1 dimensional vector, the temperature contribution factor matrix $A \in \mathbb{R}^{6\times6}$, and L employs a second-order regularization matrix. The FP method was applied to obtain $\lambda = 26.33$ as the optimal regularization parameter, as shown in Figure 4 [Figure 4: see original paper]. Through λ, the convective heat fluxes of six wall surfaces can be obtained. Table 2 lists the actual convective heat fluxes of the six wall surfaces, the inverse model calculation results, and the errors between them. The root mean square difference between actual values and inverse model calculations is 0.79°C, demonstrating the feasibility of the inverse model in practical applications.

3.2 Experimental Validation Results

The inverse model demonstrates practical applicability in real-world scenarios.

4. Conclusion

This paper proposes an inverse calculation model for determining boundary convective heat fluxes based on temperature measurements at several indoor points. The model combines Tikhonov regularization with least-squares optimization for inverse modeling. Selection of the regularization parameter is crucial for solving the inverse model. The FP method constructs the optimal regularization parameter based on the local convexity properties of the iterative function $\Phi_\eta(\lambda)$. The FP method obtains reliable results at low computational cost, which is a characteristic of the FP algorithm that makes implementation simpler and more efficient. The inverse model was validated using both a three-dimensional ventilation cavity experimental platform and an office room in a building. The root mean square differences between measured values and model calculations were less than 80%, demonstrating that the inverse model can accurately and effectively determine boundary convective heat fluxes.

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Note: Figure translations are in progress. See original paper for figures.