

Spectrum Allocation in Cognitive Radio Networks Based on Binary Fireworks Optimization Algorithm: Postprint

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Abstract

To address the problem of scarcity and low utilization of wireless spectrum resources, a spectrum allocation method based on the binary fireworks optimization algorithm is proposed. Each firework individual performs distributed explosion search, and the explosion radius of the optimal firework is dynamically updated using an improved formula; in the mutation phase, to overcome the deficiency of insufficient information exchange among particles, crossover and mutation operators from genetic algorithms are introduced to further enhance population diversity; for the selected optimal individual, simulated annealing perturbation is performed using the Metropolis criterion to avoid falling into local optima. Simulation results demonstrate that the binary fireworks optimization algorithm exhibits high optimization accuracy and fast convergence speed in spectrum allocation for cognitive radio networks, effectively achieving the maximization of network utility and user proportional fairness.

Full Text

Preamble

Spectrum Allocation of Cognitive Radio Network Based on Binary Fireworks Optimization Algorithm

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Abstract: To address the scarcity and low utilization of wireless spectrum resources, this paper proposes a spectrum allocation method based on the binary fireworks optimization algorithm. Each firework individual performs a

distributed explosion search, and an improved formula is employed to dynamically update the explosion radius of the optimal firework. In the mutation phase, to overcome insufficient information exchange among particles, crossover and mutation operators from genetic algorithms are introduced to further enhance population diversity. The Metropolis criterion of simulated annealing is applied to perturb the selected optimal individual, preventing premature convergence into local optima. Simulation experiments demonstrate that the binary fireworks optimization algorithm exhibits high optimization accuracy and fast convergence speed in cognitive radio network spectrum allocation, effectively maximizing network utility and user proportional fairness.

Keywords: cognitive radio network; spectrum allocation; binary fireworks optimization algorithm; Metropolis criterion

0 Introduction

With the rapid development of wireless communication technology, the scarcity of wireless spectrum resources has become increasingly prominent, while spectrum utilization remains low, leaving numerous spectrum holes unused. Cognitive radio networks (CRN) are intelligent networks capable of identifying current network conditions and performing planning, decision-making, and response based on these states. Dynamic spectrum allocation methods proposed for CRN can effectively solve these problems. The core idea is that secondary users (cognitive users) can opportunistically use the spectrum of primary users (licensed users) through real-time sensing technology without affecting normal primary user communications, thereby efficiently allocating available spectrum resources to optimize overall network performance.

Extensive research on cognitive radio network spectrum allocation has been conducted by scholars worldwide. Existing spectrum allocation methods primarily include game theory, auction theory, and graph coloring theory. Considering the interference relationships among CRN nodes, graph coloring theory can visually represent user geographic locations, making it a convenient and effective approach for spectrum resource planning and management. Since cognitive radio network spectrum allocation must consider user fairness constraints, it constitutes a typical NP-hard problem, making swarm intelligence algorithms suitable for solution. In recent years, scholars have combined graph coloring theory with intelligent optimization algorithms for spectrum allocation, achieving significant results. References [6,7] proposed spectrum allocation solutions based on genetic algorithm (GA) and quantum genetic algorithm, but these suffer from premature convergence that affects final optimization results. References [8,9] introduced the immune clone selection algorithm (ICS) for cognitive radio network spectrum allocation, which effectively addresses the lack of diversity in later evolutionary stages but suffers from slow convergence. Reference [10] employed particle swarm optimization (PSO) for spectrum allocation, which converges quickly but easily falls into local optima. Additionally, many other classical intelligent algorithms have been widely applied to CRN spectrum allocation.

tion, such as ant colony algorithm, bee colony algorithm, and cuckoo algorithm. However, these methods struggle to balance convergence speed and optimization capability, ultimately affecting spectrum allocation results and network performance.

This paper builds upon the graph coloring spectrum allocation model, optimizing for network utility and proportional fairness among users. We improve the explosion radius and mutation operators of the traditional binary fireworks algorithm and incorporate the Metropolis criterion for elite selection, proposing a spectrum allocation solution based on the binary fireworks optimization algorithm (BFOA). Experimental simulations and comparative analysis demonstrate the effectiveness and superiority of BFOA in solving this problem.

1.1 Spectrum Allocation Graph Theory Model

As shown in [Figure 1: see original paper], the graph coloring model interprets cognitive radio network topology as an undirected graph, illustrating geographic relationships and mutual interference among users. In the figure, primary users have authorized spectrum in parentheses, while cognitive users have sensed available spectrum in parentheses. r_i^m and r_n^m represent respective signal coverage radii.

Assume a region of area $X \times Y$ contains M primary users and N cognitive users randomly distributed geographically, with K available spectrum bands. Primary users operate on fixed authorized spectrum. Cognitive users sense available primary user spectrum through CRN spectrum sensing technology and opportunistically access it. Under spectrum allocation rules, cognitive users may simultaneously use multiple spectrum bands. When primary user PU_m is assigned authorized spectrum B_m , its coverage on spectrum B_m is a circular area centered at (x_m, y_m) with radius r_m . If cognitive user CU_n opportunistically accesses spectrum B_m , its coverage is a circular area centered at (x_n, y_n) with radius r_{nm} . Assume interference between users depends solely on mutual geographic distance. If coverage areas of primary user PU_m and cognitive user CU_n , or cognitive users CU_n and CU_k on the same spectrum B_m , overlap, interference will occur.

1.2 Mathematical Modeling

Since the spectrum allocation period is extremely short relative to network environment changes, the network topology is assumed static during one allocation cycle. Cognitive radio network spectrum allocation can be expressed as a mathematical model using the following matrices:

- a) **Available Spectrum Matrix** $L = \{l_{nm}\}_{N \times M}$, where $l_{nm} \in \{0, 1\}$ indicates whether cognitive user CU_n can use spectrum B_m . $l_{nm} = 1$ means it can; otherwise $l_{nm} = 0$.
- b) **Benefit Matrix** $B = \{b_{nm}\}_{N \times M}$, where b_{nm} represents the benefit when

cognitive user CU_n uses spectrum B_m , with positive values. Network bandwidth is typically used as the benefit metric.

- c) **Interference Matrix** $C = \{c_{nkm}\}_{N \times N \times M}$, where $c_{nkm} \in \{0, 1\}$ indicates whether different cognitive users CU_n and CU_k interfere when using the same spectrum B_m . $c_{nkm} = 0$ means no interference; otherwise $c_{nkm} = 1$.
- d) **Interference-Free Allocation Matrix** $A = \{a_{nm}\}_{N \times M}$, where $a_{nm} \in \{0, 1\}$ indicates whether spectrum B_m can be allocated to cognitive user CU_n under constraints of matrices L and C . $a_{nm} = 1$ means it can; otherwise $a_{nm} = 0$. The interference-free allocation matrix must satisfy:

$$\begin{cases} a_{nm} \leq l_{nm} \\ a_{nm} + a_{km} \leq 1, & \text{if } c_{nkm} = 1 \end{cases}$$

From the final interference-free allocation matrix and benefit matrix, each cognitive user's benefit after spectrum allocation can be determined. The total network benefit is the sum of all cognitive users' benefits:

$$F_{\text{sum}} = \sum_{n=1}^N \sum_{m=1}^M a_{nm} \cdot b_{nm}$$

The average benefit is:

$$F_{\text{mean}} = \frac{1}{N} \sum_{n=1}^N \sum_{m=1}^M a_{nm} \cdot b_{nm}$$

To measure allocation fairness, i.e., whether benefits among cognitive users are balanced, the network's proportional fairness function is used:

$$F_{\text{fairness}} = \left(\prod_{n=1}^N \left(1 + \frac{R_n}{4} \right) \right)^{1/4}$$

where $R_n = \sum_{m=1}^M a_{nm} \cdot b_{nm}$.

These metrics can directly serve as objective functions during algorithm execution to evaluate cognitive radio network spectrum allocation results.

2 BFOA Algorithm Design

The traditional fireworks algorithm consists of explosion operators, mutation operators, and selection strategies, commonly applied to continuous variable problems. To adapt to discrete spectrum allocation, we propose a novel binary fireworks optimization algorithm. The mutation operator incorporates classic

genetic algorithm crossover and mutation operations to address insufficient particle information exchange, enhancing population diversity. Simulated annealing with the Metropolis criterion is applied to perturb selected optimal solutions, preventing local optima and improving convergence speed. Additionally, the fixed explosion radius of the optimal firework per iteration is changed to dynamic decrement, facilitating adaptive global optimum search.

2.1 Explosion Operator

Each individual in the population represents a firework that produces numerous sparks upon explosion. In spectrum allocation, high-quality fireworks (with larger fitness values) have smaller explosion ranges but produce more sparks, while low-quality fireworks have larger ranges with fewer sparks. The former exhibits strong local “exploitation capability,” while the latter demonstrates powerful global “exploration capability.” This balance enables the population to achieve equilibrium between global and local search capabilities.

Each firework’s explosion radius and spark count are determined by its own fitness value and those of other fireworks. For firework x_i , the explosion radius A_i and spark number S_i are:

$$A_i = \hat{A} \cdot \frac{f(x_i) - f_{\min} + \epsilon}{\sum_{j=1}^Z (f(x_j) - f_{\min}) + \epsilon}$$

$$S_i = \hat{S} \cdot \frac{f_{\max} - f(x_i) + \epsilon}{\sum_{j=1}^Z (f_{\max} - f(x_j)) + \epsilon}$$

where \hat{A} and \hat{S} are constants controlling total sparks, Z is population size, f_{\max} and f_{\min} are maximum and minimum fitness values, and ϵ is machine precision (10^{-6}).

When firework x_i explodes, the j -th spark is generated as:

$$x_j = x_i \oplus P(r_{ij})$$

where \oplus is a binary conversion operator that randomly selects r_{ij} elements from index set $\{1, 2, \dots, \hat{A}\}$ and flips the corresponding bits in the binary string x_i . The explosion step size r_{ij} is:

$$r_{ij} = \text{ceil}(A_i \cdot \text{rand}(\cdot))$$

where $0 < r_{ij} \leq A_i$ and $\text{rand}(\cdot)$ is a uniform random number in $[0, 1]$.

2.2 Improved Explosion Radius

The explosion radius of each firework is closely related to its quality. The optimal firework has the smallest radius, and its surrounding particles are closest to the global optimum, requiring enhanced deep “exploitation” to improve optimization precision. However, using the standard formula often results in an excessively small radius that prevents generating the most sparks.

To address this, we improve the optimal firework’s radius based on the Logistic function, a common S-curve reflecting bounded growth. Through integral simplification:

$$y = \frac{1}{1 + e^{-x}}$$

we derive the optimal firework radius formula:

$$A_g^o = r_{\min} + (r_{\max} - r_{\min}) \cdot \frac{1}{1 + e^{-\alpha(2g/G-1)}}$$

where A_g^o is the optimal firework radius at generation g , r_{\max} and r_{\min} are initial and final radius values, G is total generations, and α is a control parameter ($0 < \alpha < 1$). As generations increase, the optimal firework radius decreases nonlinearly—rapidly in early stages for broad search, then slowly approaching the minimum for precise local search near the global optimum.

2.3 Improved Mutation Operator

Traditional fireworks algorithms use Gaussian mutation, which poorly escapes local optima and suits real-number encoding. Moreover, fireworks algorithms perform parallel distributed explosion search with insufficient particle information exchange, limiting diversity. We introduce binary crossover and mutation operators from genetic algorithms to enhance information transfer and resource interaction, improving population diversity.

After explosion, the search space contains both fireworks and explosion sparks. The process is:

- a) Evaluate fitness of all fireworks and explosion sparks, then rank them.
- b) Select appropriate individuals for crossover and mutation with probabilities p_c and p_m :

$$x_{i,d}^c = \begin{cases} x_{i,d}, & \text{if } d \in \text{randperm}(\hat{A}, \text{dim}) \\ x_{i,d}, & \text{otherwise} \end{cases}$$

$$x_{i,d}^m = \begin{cases} 1 - x_{i,d}, & \text{if } \text{rand} \leq p_m \\ x_{i,d}, & \text{otherwise} \end{cases}$$

where $x_{i,d}^c$ and $x_{i,d}^m$ are post-crossover/mutation values, $\text{randperm}(\hat{A}, \text{dim})$ randomly selects \hat{A} dimensions, and p_c, p_m are crossover and mutation probabilities. Elements for crossover/mutation consist of 50% top-ranked individuals and 50% randomly selected from the remaining population. This produces high-quality individuals while preserving excellent genes from less fit ones, enhancing information sharing.

2.4 Selection Strategy

After the above processes, candidates must be selected to pass excellent information to the next generation. The candidate set includes fireworks, explosion sparks, and mutation sparks. First, elitism deterministically preserves the best firework/spark. Remaining individuals are selected via roulette wheel from the candidate set, where individual x_i is selected with probability:

$$p(x_i) = \frac{D(x_i)}{\sum_{j=1}^K D(x_j)}$$

where $D(x_i) = \sum_{j=1}^K d(x_i, x_j)$ is the sum of Hamming distances between x_i and other candidates, and K is the candidate pool size.

2.5 Metropolis Acceptance Criterion

To avoid local optima, we introduce simulated annealing's Metropolis criterion after explosion and mutation to further optimize elite individuals. Simulated annealing solves combinatorial optimization by simulating thermodynamic cooling, randomly perturbing the current state to generate a new state, and accepting it via the Metropolis criterion. Since spectrum allocation is a maximization problem, the criterion is:

$$p = \begin{cases} 1, & \text{if } f(x_j) \leq f(x_i) \\ \exp\left(-\frac{f(x_j)-f(x_i)}{T}\right), & \text{if } f(x_j) > f(x_i) \end{cases}$$

where x_i is the current solution, x_j is the new solution, $f(\cdot)$ is the objective function, and T is current temperature. The Metropolis criterion accepts both improving solutions and, with some probability, worsening solutions. At high temperatures, worsening solutions are more likely accepted; as temperature decreases, this probability diminishes to zero, helping escape local optima. We use the cooling function:

$$T(t) = \frac{T_0}{1 + \sqrt{t}}$$

where β is the cooling coefficient and t is iteration count. This achieves rapid early cooling with slower later descent. A “best so far” memory unit prevents losing the best solution found due to probabilistic acceptance of worsening solutions.

3 Spectrum Allocation Scheme Based on BFOA

In our proposed BFOA-based spectrum allocation method, each firework and generated spark (explosion or mutation spark) represents a potential spectrum allocation strategy. After interference constraint processing, objective functions evaluate them to select the optimal allocation.

3.1 Population Encoding

Cognitive radio network spectrum allocation uses binary encoding. When available spectrum matrix element $l_{nm} = 1$, the corresponding interference-free allocation matrix element a_{nm} may be 0 or 1 after constraint processing. However, when $l_{nm} = 0$, a_{nm} must be 0, indicating initially unavailable spectrum. Thus, we only encode elements where $L = 1$, improving computational speed.

Each firework/spark individual is a mapping from spectrum allocation matrix to binary string, representing a possible allocation scheme. String length is $\sum_{n=1}^N \sum_{m=1}^M l_{nm}$, following encoding order of incrementing n first, then m .

3.2 Interference Constraint Handling

Not all allocation schemes are feasible due to potential interference conflicts when multiple cognitive users access the same spectrum. When cognitive users CU_n and CU_k interfere on spectrum B_m (i.e., $c_{nkm} = 1$), only one user may access B_m . We check allocation matrix A 's column m rows n and k : if both elements are 1, we compare row sums $\sum_{m=1}^M a_{nm}$ and $\sum_{m=1}^M a_{km}$. The larger sum retains its value; the other is set to 0. If equal, one is randomly set to 0. The processed matrix A becomes a valid solution satisfying constraints L and C .

3.3 Algorithm Implementation Flow

The BFOA-based spectrum allocation procedure is:

- a) **Generate matrices:** Derive available spectrum matrix L , benefit matrix B , and interference matrix C from the randomly generated CRN topology.
- b) **Initialize parameters:** Input population size Z , maximum iterations G , crossover/mutation probabilities p_c and p_m , initial temperature T_0 , etc. Randomly generate initial firework population P_0 and compute fitness.
- c) **Termination check:** If maximum iterations G is reached, map the current best firework to allocation matrix and output the optimal scheme;

otherwise continue to step d.

- d) **Explosion operation:** Compute explosion radius A_i and spark count S_i for each firework to generate explosion sparks x_j .
- e) **Mutation operation:** Select the top 50% individuals from fireworks and explosion sparks, plus 50% randomly chosen from the remainder. Perform crossover and mutation with probabilities p_c and p_m to generate mutation sparks.
- f) **Constraint handling and selection:** Process interference constraints for all fireworks and sparks, compute fitness, and select the best individual.
- g) **Global search:** Perturb the best individual using Metropolis criterion, process constraints, and search for the global optimum firework for the next round.
- h) **Selection update:** Replace the best individual from step f) with the perturbed optimum from g), combine with remaining individuals, and use roulette wheel to select Z individuals for the next generation. Return to step c).

4 Simulation and Results Analysis

MATLAB is used to establish the simulation environment. On identical network topologies, we compare BFOA with traditional GA, PSO, and ICS algorithms, analyzing performance through network utility and user proportional fairness, as well as algorithm behavior under varying network parameters. This validates BFOA's feasibility and superiority for cognitive radio network spectrum allocation.

In a given rectangular network area, M primary users, N cognitive users, and K available spectrum bands are randomly distributed. Cognitive users dynamically share spectrum with primary users. Extensive experiments yield these parameters: total generations $G = 200$, population size $Z = 10$, total fireworks = 100, $r_{\max} = 0.25$, $r_{\min} = 0.01$, $\alpha = 0.05$, crossover probability $p_c = 0.6$, mutation probability $p_m = 0.1$, cooling coefficient $\beta = 0.98$.

4.1 Algorithm Performance Comparison

This experiment compares optimization capability and convergence. [Figure 2: see original paper] shows network total benefit optimization results across 40 different user location distributions (primary and cognitive users), with $M = N = K = 15$. compares sum and mean values of the 40 runs. Results show BFOA achieves maximum network total benefit with significant improvement over other algorithms.

[Figure 3: see original paper] and [Figure 4: see original paper] illustrate optimization processes for network total benefit and proportional fairness during

one allocation, with $M = N = K = 10$. BFOA consistently outperforms others. In optimization accuracy, BFOA finds the best values, followed by PSO, ICS, and GA. BFOA's final network total benefit and proportional fairness exceed GA's by 8.3% and 8.9%, respectively. Due to its distributed explosion search characteristic, BFOA starts with higher initial values. Convergence occurs around generation 80 for benefit and generation 60 for fairness—the fastest among all algorithms—effectively escaping local optima through simulated annealing perturbation of the best firework each iteration.

4.2 Impact of Available Spectrum Quantity

Algorithm performance varies with available spectrum quantity. [Figure 5: see original paper] and [Figure 6: see original paper] show changes in network average benefit and proportional fairness with different spectrum counts (K from 5 to 30), with $N = 15$ and $M = K$. Both metrics improve as spectrum quantity increases because more spectrum reduces conflicts and interference. Initially, with few spectrum bands, performance gaps among algorithms are small. As spectrum increases, BFOA and PSO achieve significantly higher average benefit and fairness than ICS and GA, with BFOA outperforming PSO, proving its superiority.

4.3 Impact of Cognitive User Quantity

To further compare algorithms, we examine performance under varying cognitive user quantities. [Figure 7: see original paper] and [Figure 8: see original paper] show curves of network average benefit and proportional fairness versus cognitive user count (N from 5 to 30), with $M = K = 15$. As cognitive users increase, network load and interference grow, causing both metrics to decline. However, BFOA maintains the highest benefit and fairness values, achieving optimal spectrum allocation better than PSO, ICS, and GA, with more pronounced advantages at higher user counts.

5 Conclusion

To address spectrum resource scarcity and low utilization in wireless communications, this paper optimizes the binary fireworks algorithm and applies it to cognitive radio network spectrum allocation, achieving excellent results. The improved BFOA resolves slow convergence and local optima issues in traditional fireworks algorithms. Simulation results demonstrate that BFOA obtains better network utility and user proportional fairness than GA, PSO, and ICS, enhancing overall network performance.

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