Accuracy Analysis of Integrated CAPS, BDS, and GPS Satellite Positioning Postprint

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Abstract

With the construction and improvement of global satellite positioning systems such as GPS, GLONASS, Galileo, and BDS, the simultaneous utilization of signals from two or more positioning systems for multi-system integrated positioning has emerged as a research hotspot in the navigation field. Compared with traditional single-system positioning, multi-system integration provides superior positioning, velocity measurement, and timing performance, along with enhanced environmental adaptability. Through static single-point joint positioning solution of three systems—CAPS, BDS, and GPS—integrated positioning of transponder-based and direct broadcast positioning systems was achieved, thereby verifying the feasibility of the China Area Positioning System participating in multi-system GNSS positioning and analyzing the performance and positioning accuracy of the integrated solution. Analysis demonstrates that the participation of the China Area Positioning System in multi-system joint positioning solution is entirely feasible, with single-point positioning accuracy better than 3m and potential for further improvement. The incorporation of the China Area Positioning System enriches the selection of GNSS systems and holds research significance and application value for enhancing the diversity of GNSS integrated positioning and improving integrated positioning performance.

Full Text

Analysis of Combined Satellite Positioning Accuracy Based on CAPS, BDS, and GPS

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Abstract

With the construction and completion of global satellite positioning systems such as GPS, GLONASS, Galileo, and BDS, using signals from two or more positioning systems for multi-system combined positioning has become a research hotspot in the navigation field. Compared with traditional single-system positioning, multi-system combination offers better positioning, velocity measurement, and timing performance, along with stronger environmental adaptability. Through combined static single-point positioning solution using CAPS, BDS, and GPS, we achieved joint positioning between a transponding positioning system and direct-sequence positioning systems, verified the feasibility of the Chinese Area Positioning System participating in multi-system GNSS positioning, and analyzed the performance and accuracy of combined positioning. The analysis demonstrates that Chinese Area Positioning System participation in multi-system joint positioning solution is entirely feasible, with single-point positioning accuracy better than 3 meters and room for further improvement. The addition of CAPS enriches the selection of GNSS systems and holds research significance and application value for improving the diversity of GNSS combined positioning and enhancing positioning performance.

Keywords: Global Navigation Satellite System; Chinese Area Positioning System; Combined positioning

1. Combined Positioning Method

The schematic diagram of combined positioning using BDS, GPS, and CAPS is shown in [Figure 1: see original paper]. The navigation satellites from the three systems collectively form the space GNSS constellation. A dedicated GNSS receiver platform receives navigation messages from all visible satellites across the three systems, measures code pseudoranges, and stores the raw positioning data to a computer. Finally, offline data processing is performed on the computer using the least squares method for position solution, yielding combined positioning results and enabling analysis of positioning accuracy.

During combined positioning solution, the CAPS system serves as a supplement and enhancement to traditional satellite positioning systems GPS and BDS. Due to its transponding positioning characteristics that utilize communication satellites for positioning, CAPS can provide advantages in satellite quantity, spatial layout, flexibility, and anti-jamming capabilities for multi-system combined positioning. CAPS inclusion increases the effective number of satellites in the constellation, optimizes the spatial layout, and reduces Position Dilution of Precision (PDOP) values. Furthermore, CAPS navigation signals employ ranging codes with a chip rate of 10.23 Mcps, which yields more precise pseudorange measurements compared to GPS C/A code at 1.023 Mcps and BDS at 2.046 Mcps. Moreover, CAPS system time is uniformly provided by high-precision atomic clocks at ground stations, eliminating inter-satellite clock errors and thereby improving positioning accuracy.

1.1 Observation Equations and Linearization Let ρ_i^k denote the geometric distance between satellite k and receiver i, c the speed of light, dt^k the clock bias of satellite k relative to its system time, dt^i the receiver clock bias, e_i^k the pseudorange measurement error including ionospheric delay, tropospheric delay, measurement noise, etc., and dt^S the inter-system clock bias, including dt^B or dt^C, which represent the time differences of system times BDT and CAPST relative to GPST. The basic pseudorange observation equation L_i^k is:

$$L_i^k = \rho_i^k + c(dt^i - dt^S - dt^k) + e_i^k, \tag{1}$$

where the geometric distance between satellite and receiver ρ_i^k can be calculated by:

$$\rho_i^k = \sqrt{(X^k - X_i)^2 + (Y^k - Y_i)^2 + (Z^k - Z_i)^2}, \tag{2}$$

Substituting into the basic observation equation (1) yields:

$$L_i^k = \sqrt{(X^k - X_i)^2 + (Y^k - Y_i)^2 + (Z^k - Z_i)^2} + c(dt^i - dt^S - dt^k) + e_i^k, \tag{3}$$

where satellite position (X^k, Y^k, Z^k) and satellite clock bias dt^k can be obtained from satellite ephemeris [3-5].

During solution, for BDS and CAPS systems, the receiver clock bias and system clock bias are solved jointly, resulting in six unknowns: $X_i, Y_i, Z_i, dt^i, (dt^i - dt^B)$, and $(dt^i - dt^C)$. When the number of satellites receiving signals in the combined system is not less than the number of systems plus three (i.e., at least 4 satellites for a single system, 5 for dual systems, and 6 for triple systems), combined positioning can be achieved [6].

Equation (3) is nonlinear with respect to receiver position (X_i, Y_i, Z_i), so it must be linearized before using the least squares method. The nonlinear portion is:

$$f(X_i, Y_i, Z_i) = \sqrt{(X^k - X_i)^2 + (Y^k - Y_i)^2 + (Z^k - Z_i)^2}, \tag{4}$$

An initial receiver position $(X_{i,0}, Y_{i,0}, Z_{i,0})$ must be selected for linearization; here the Earth's center $(0, 0, 0)$ is chosen as the initial position. Define increments $\Delta X, \Delta Y, \Delta Z$ to update the approximate receiver coordinates, where p is the iteration count, and each iteration uses the updated position from the previous calculation as its initial position:

$$\begin{aligned} X_{i,p+1} &= X_{i,p} + \Delta X_i \\ Y_{i,p+1} &= Y_{i,p} + \Delta Y_i \qquad p = 0, 1, 2 \cdots, \\ Z_{i,p+1} &= Z_{i,p} + \Delta Z_i \end{aligned} \tag{5}$$

Expanding equation (4) using Taylor series at each iteration's initial position $(X_{i,p}, Y_{i,p}, Z_{i,p})$ yields:

$$f(X_{i,p+1}, Y_{i,p+1}, Z_{i,p+1}) = f(X_{i,p}, Y_{i,p}, Z_{i,p}) + \frac{\partial f(X_{i,p}, Y_{i,p}, Z_{i,p})}{\partial X_{i,p}} \Delta X_i + \frac{\partial f(X_{i,p}, Y_{i,p}, Z_{i,p})}{\partial Y_{i,p}} \Delta Y_i + \frac{\partial f(X_{i,p}, Y_{i,p}, Z_i}{\partial Z_{i,p}} \tag{6}$$

where the partial derivatives are:

$$\frac{\partial f(X_{i,p}, Y_{i,p}, Z_{i,p})}{\partial X_{i,p}} = -\frac{X^k - X_{i,p}}{\rho^k_{i,p}}$$
$$\frac{\partial f(X_{i,p}, Y_{i,p}, Z_{i,p})}{\partial Y_{i,p}} = -\frac{Y^k - Y_{i,p}}{\rho^k_{i,p}}$$
$$\frac{\partial f(X_{i,p}, Y_{i,p}, Z_{i,p})}{\partial Z_{i,p}} = -\frac{Z^k - Z_{i,p}}{\rho^k_{i,p}}$$

Thus, equation (3) becomes a first-order linear observation equation:

$$L^k_i = \rho^k_{i,p} - \frac{X^k - X_{i,p}}{\rho^k_{i,p}} \Delta X_i - \frac{Y^k - Y_{i,p}}{\rho^k_{i,p}} \Delta Y_i - \frac{Z^k - Z_{i,p}}{\rho^k_{i,p}} \Delta Z_i + c(dt^i - dt^S - dt^k) + e^k_i, \tag{8}$$

where $\rho^k_{i,p}$ is the distance calculated from the approximate receiver position:

$$\rho^k_{i,p} = \sqrt{(X^k - X_{i,p})^2 + (Y^k - Y_{i,p})^2 + (Z^k - Z_{i,p})^2}. \tag{9}$$

1.2 Least Squares Method The least squares method is typically used when a system of linear equations has more equations than unknowns and thus no exact solution [3]. For an unsolvable equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where $\mathbf{A}$ has m rows and n columns with $m > n$, the number of observations $b_1, \cdots, b_m$ exceeds the number of parameters $x_1, \cdots, x_n$. The error vector is $\hat{\mathbf{e}} = \mathbf{b} - \mathbf{A}\hat{\mathbf{x}}$, and its squared value $\|\mathbf{e}\|^2 = (\mathbf{b} - \mathbf{A}\mathbf{x})^T(\mathbf{b} - \mathbf{A}\mathbf{x})$ represents the sum of squares of m individual errors. To obtain the optimal solution $\hat{\mathbf{x}}$ that minimizes the error vector length, we minimize this quadratic equation to get:

$$\mathbf{A}^T\mathbf{A}\hat{\mathbf{x}} = \mathbf{A}^T\mathbf{b} \quad \text{or} \quad \hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}. \tag{10}$$

Thus, equation (8) can be rewritten in vector form as:

$$L_i^k = \rho_{i,p}^k + \left[-\frac{X^k - X_{i,p}}{\rho_{i,p}^k}, -\frac{Y^k - Y_{i,p}}{\rho_{i,p}^k}, -\frac{Z^k - Z_{i,p}}{\rho_{i,p}^k}, c(dt^i - dt^s) \right] - cdt^k + e_i^k. \tag{11}$$

Rearranging this into the standard form of a least squares problem $\mathbf{Ax} = \mathbf{b}$ yields:

$$\frac{X^k - X_{i,p}}{\rho_{i,p}^k}\Delta X_i + \frac{Y^k - Y_{i,p}}{\rho_{i,p}^k}\Delta Y_i + \frac{Z^k - Z_{i,p}}{\rho_{i,p}^k}\Delta Z_i + c(dt^i - dt^S) = L_i^k - \rho_{i,p}^k + cdt^k - e_i^k. \tag{12}$$

A unique solution cannot be obtained from a single equation, so we combine observation equations from all visible navigation system satellites to form a system of equations. Let $b_i^k = L_i^k - \rho_{i,p}^k + cdt^k - e_i^k$. The final solution can be obtained from equation (13), where the first m rows of coefficient matrix $\mathbf{A}$ correspond to GPS satellites with coordinates (X_G, Y_G, Z_G); the middle n rows correspond to BDS satellites with coordinates (X_B, Y_B, Z_B); and the last l rows correspond to CAPS satellites with coordinates (X_C, Y_C, Z_C).

$$\begin{bmatrix} -\frac{X_{G1}-X_{i,p}}{\rho_{i,p}^{G1}} & -\frac{Y_{G1}-Y_{i,p}}{\rho_{i,p}^{G1}} & -\frac{Z_{G1}-Z_{i,p}}{\rho_{i,p}^{G1}} & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{X_{Gm}-X_{i,p}}{\rho_{i,p}^{Gm}} & -\frac{Y_{Gm}-Y_{i,p}}{\rho_{i,p}^{Gm}} & -\frac{Z_{Gm}-Z_{i,p}}{\rho_{i,p}^{Gm}} & 1 & 0 & 0 \\ -\frac{X_{B1}-X_{i,p}}{\rho_{i,p}^{B1}} & -\frac{Y_{B1}-Y_{i,p}}{\rho_{i,p}^{B1}} & -\frac{Z_{B1}-Z_{i,p}}{\rho_{i,p}^{B1}} & 1 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{X_{Bn}-X_{i,p}}{\rho_{i,p}^{Bn}} & -\frac{Y_{Bn}-Y_{i,p}}{\rho_{i,p}^{Bn}} & -\frac{Z_{Bn}-Z_{i,p}}{\rho_{i,p}^{Bn}} & 1 & 1 & 0 \\ -\frac{X_{C1}-X_{i,p}}{\rho_{i,p}^{C1}} & -\frac{Y_{C1}-Y_{i,p}}{\rho_{i,p}^{C1}} & -\frac{Z_{C1}-Z_{i,p}}{\rho_{i,p}^{C1}} & 1 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{X_{Cl}-X_{i,p}}{\rho_{i,p}^{Cl}} & -\frac{Y_{Cl}-Y_{i,p}}{\rho_{i,p}^{Cl}} & -\frac{Z_{Cl}-Z_{i,p}}{\rho_{i,p}^{Cl}} & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta X_{i,1} \\ \Delta Y_{i,1} \\ \Delta Z_{i,1} \\ cdt_{i,1} \\ c(dt^i - dt^B) \\ c(dt^i - dt^C) \end{bmatrix} = \mathbf{b} \tag{13}$$

When $m+n+l \geq 6$, there is a unique solution: $\Delta X_{i,p+1}, \Delta Y_{i,p+1}, \Delta Z_{i,p+1}$. This solution can be added to the receiver's initial approximate position to obtain a more accurate approximate position for the next step:

$$\begin{aligned} X_{i,p+1} &= X_{i,p} + \Delta X_{i,p+1} \\ Y_{i,p+1} &= Y_{i,p} + \Delta Y_{i,p+1} \\ Z_{i,p+1} &= Z_{i,p} + \Delta Z_{i,p+1} \end{aligned} \tag{14}$$

The process can continue by substituting $X_{i,p+1}, Y_{i,p+1}, Z_{i,p+1}$ for $X_{i,p}, Y_{i,p}, Z_{i,p}$ into equation (13) for iterative calculation until the solution $\Delta X_{i,p+q}, \Delta Y_{i,p+q}, \Delta Z_{i,p+q}$

reaches the desired error magnitude after q iterations, where q is the number of least squares iterations. Typically, four to five iterations are sufficient to meet the target.

2. Experiment and Data Processing

2.1 Test Scheme The combined satellite positioning experiment based on BDS, GPS, and CAPS employed static single-point positioning. The test receiver was installed on the rooftop of the National Astronomical Observatories, Chinese Academy of Sciences. Since the CAPS system operates in the C-band, different from the L-band used by the other two systems, separate receiving antennas were placed close together to improve signal reception performance. After navigation RF signals were converted to intermediate frequency, they were captured, tracked, and demodulated by the CAPS receiver and BDS/GPS dual-system single-frequency receiver baseband respectively, with raw positioning data from the three systems recorded synchronously in real-time and stored to a computer. Finally, MATLAB software was used for data processing and combined positioning solution, and the test results were analyzed to statistically evaluate positioning accuracy. The block diagram of the positioning test system is shown in [Figure 2: see original paper].

2.2 Test Procedure

1. A BDS/GPS dual-system positioning receiver was placed on the observatory rooftop for 48 hours of continuous measurement. The average of its positioning solutions was taken to obtain the static reference point coordinates (x_0, y_0, z_0) as (-2173764.5307, 4383137.0366, 4078283.8735).
2. At the same location, raw positioning data were collected over a long period using both BDS/GPS single-frequency receiver and CAPS receiver, including satellite ephemeris, satellite clock bias, message correction information, code pseudorange measurements, etc., for all visible GPS, BDS, and CAPS satellites.
3. Satellite positioning information from the same time period for each system was selected. Satellite position coordinates were calculated for each epoch from satellite ephemeris, and positioning solutions were performed using all observable satellites from the three systems with the least squares method.
4. The positioning result data were processed to analyze and statistically evaluate the accuracy and precision of combined positioning results, which were then compared with BDS/GPS dual-system positioning results to draw experimental conclusions.

2.3 Data Statistical Method When the equipment is in normal operation and all three systems have successfully acquired and tracked signals, n ($n > 1000$) epochs of positioning pseudorange measurement data and navigation messages during this period are recorded continuously. Positioning solutions are performed for these n epochs to obtain positioning results for each epoch. The calculation steps are as follows:

1. For each epoch, calculate the positioning deviation in each direction and the three-dimensional total error:

$$\begin{aligned}
\Delta x_i &= x_i - x_0 \\
\Delta y_i &= y_i - y_0 \\
\Delta z_i &= z_i - z_0 \\
\Delta p_i &= \sqrt{\Delta x_i^2 + \Delta y_i^2 + \Delta z_i^2} \quad i = 1, 2, \cdots, n
\end{aligned} \tag{15}$$

where x_i, y_i, z_i are the three-dimensional coordinates of the i-th epoch positioning result in meters, and x_0, y_0, z_0 are the static reference point coordinates in meters.

2. Calculate the mean error and standard deviation error for each direction and the three-dimensional total error:

$$\overline{\Delta p} = \frac{\sum \Delta p_i}{n}, \tag{16}$$

$$S_{\Delta p} = \sqrt{\frac{1}{n-1} \sum (\Delta p_i - \overline{\Delta p})^2}, \tag{17}$$

where $\overline{\Delta p}$ is the mean error of three-dimensional total error for n epochs in meters, and $S_{\Delta p}$ is the standard deviation error of three-dimensional total error for n epochs in meters.

3. Test Results

The test selected measurement data from 18:19 on September 11, 2017 to 10:34 on September 12, 2017, using all visible GPS, BDS, and CAPS satellite data. At each moment, at least 16 satellites could be received simultaneously. Details of CAPS system satellites used in this test are shown in .

A MATLAB program for CAPS/BDS/GPS multi-system solution was developed for position calculation. The solution results are plotted as a horizontal scatter diagram in [Figure 3: see original paper].

The PDOP (Position Dilution of Precision) values for dual-system and triple-system combinations are shown in [Figure 4: see original paper].

Statistical results of positioning accuracy are presented in .

4. Conclusions

The above results show that in static single-point positioning scenarios, the inclusion of CAPS provides richer satellite selection, improves the spatial satellite constellation configuration, and reduces PDOP values, thereby enhancing positioning accuracy—particularly with noticeable improvement in horizontal positioning accuracy. In future research, further refinement of error elimination or incorporation of carrier phase measurements in the solution could yield substantial improvements in positioning accuracy.

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Note: Figure translations are in progress. See original paper for figures.