Application of Multi-Population Genetic Algorithm and Truncated Singular Value Decomposition Method in Switching Arc Inversion: Postprint

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Abstract

By utilizing the correspondence between current and spatial magnetic field, the arc is modeled as a collection of multiple current lines with variable arc diameters. Based on the Biot-Savart law, the magnetic field distribution in space generated by the combined arc current lines is obtained through forward analysis. To simplify the problem complexity, considering the stage prior to arc entry into the splitter plates, a multi-population genetic algorithm is employed to optimize the objective function and solve the nonlinear equation system of the electromagnetic inverse problem, thereby determining the spatial position of the arc. Building upon this foundation, to address the ill-posedness of the inverse problem, the truncated singular value decomposition method is utilized to invert the arc current distribution, with the regularization parameter obtained via the GCV criterion. The inversion results contribute to understanding the physical characteristics of switching arcs.

Full Text

Application of Multi-Population Genetic Algorithm and Truncated Singular Value Decomposition in Switching Arc Inversion

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Abstract: By exploiting the correspondence between current and spatial magnetic field, the arc is modeled as a collection of multiple current lines with variable arc diameters. Based on the Biot-Savart law, the magnetic field distribution generated by the combined arc current lines in space is obtained through forward analysis. To simplify problem complexity, this study focuses on the phase before the arc enters the splitter plates. The electromagnetic inverse problem' s nonlinear equations are solved using a multi-population genetic algorithm (MPGA) to optimize the objective function and determine the arc' s spatial position. Building upon this foundation, the truncated singular value decomposition (TSVD) method is employed to invert the arc current distribution, with regularization parameters obtained via the GCV criterion. The inversion results contribute to understanding the physical characteristics of switching arcs.

Keywords: Switching arc inverse problem, ill-posedness, MPGA, TSVD, GCV criterion

1 Introduction

Circuit breakers serve as control and protection devices in power systems, disconnecting lines during fault conditions to protect personnel and system safety. As contact switches, circuit breakers inevitably produce arcs when contacts separate during current interruption. Arc combustion is a complex nonlinear process under multi-physical field coupling, exhibiting randomness and chaotic characteristics. In commonly used low-voltage circuit breakers, where air serves as the arc-quenching and insulating medium, arcs provide conductive paths for system circuits while simultaneously eroding electrodes and structural components within the arc-quenching system, thereby affecting switch lifespan. Research on the spatiotemporal evolution of arc behavior significantly impacts the breaking capacity and service life of switching devices.

Current experimental research methods for switching arcs primarily include high-speed charge-coupled device (CCD) imaging, optical fiber arrays, spectral testing, and magnetic testing methods. Optical-based testing methods affect the integrity of experimental prototypes, consequently influencing the dynamic characteristics of arc plasma within the arc-quenching chamber. Magnetic testing technology employs non-contact measurement approaches, using the spatial and temporal distribution of magnetic fields generated by arc currents to invert the possible spatial positions and morphologies of the arc.

Previous studies have compared various optimization algorithms for arc reconstruction, with simulated annealing algorithms accurately obtaining arc morphology and validating feasibility through magnetic imaging measurement devices. Other research has discretized the arc occurrence region and linearized the electromagnetic forward operator, employing regularization methods to invert arc current. Comparative studies have evaluated the accuracy of various regularization parameter selection strategies for reconstructing low-voltage switching arc currents, analyzing the influence of sampling distance and sampling point quantity on inversion accuracy.

2.1 Model and Related Assumptions

The two-dimensional simplified model of a low-voltage circuit breaker arc-quenching system is shown in [Figure 1: see original paper]. When the moving and stationary contacts separate, an arc forms between the contacts. As the stroke increases, the arc elongates until complete opening when the moving contact reaches maximum separation. Under the action of arc-blowing gas flow, the arc moves to a certain position in the arc-quenching chamber. According to previous research, the arc motion trajectory is constrained to the xOz plane due to arc-quenching chamber wall limitations.

The arc model simplifies the arc as a collection of multiple current lines, with each current represented as a three-segment polyline whose nodes lie on equally spaced lines between electrodes. Since current density is high in near-electrode regions, creating strong constriction pressure on the arc column, the arc radius in near-electrode regions can be temporarily neglected. If the node coordinates are A(x , 0, z), B(x_b, 0, z_b), C(x_c, 0, z_c), and D(x_d, 0, z_d) as shown in [Figure 2: see original paper], the z-coordinates of each node are known when the electrode spacing is determined. According to literature, when the magnetic field sampling plane is more than 15 mm from the arc occurrence plane, the error in arc inversion accuracy caused by splitter plates and other magnetic materials is less than 5%. Therefore, a sampling distance of 15 mm is adopted. To verify the method' s feasibility, this study focuses only on the phase before the arc enters the splitter plates, i.e., region Ω enclosed by dashed lines in [Figure 1: see original paper], with an area of 50 mm × 30 mm. Magnetic sampling points are distributed in a 4×6 array as shown in [Figure 2: see original paper].

In summary, the low-voltage switching arc model assumes a variable-diameter multi-current line assembly. Based on the correspondence between arc current and magnetic field, solving the nonlinear equation system is transformed into an optimization problem, using MPGA to obtain arc position and morphology distribution for establishing the forward operator.

2.2 Magnetic Field of Current Elements

According to the Biot-Savart law, the magnetic flux density B generated by a spatial current element is given by:

$$B = \frac{\mu_0 I}{4\pi d_0}(\cos\theta_1 - \cos\theta_2)$$

where d , , and are as shown in [Figure 3: see original paper], and is the vacuum permeability.

In [Figure 3: see original paper], the spatial current line AB lies in the xOz plane, while point S is located in plane EFGH at distance d from AB. Angles and are the angles between endpoints A(x , 0, z) and B(x__b, 0, z__b) and the line to sampling point S(x__s, 0, z__s), respectively. The derived components B , B__y, and B__z at point S are:

$$B_x = \frac{\mu_0 I}{4\pi} \cdot \frac{b_1}{b}(\cos\theta_1 - \cos\theta_2)$$

$$B_y = \frac{\mu_0 I}{4\pi} \cdot \frac{b_2}{b}(\cos\theta_1 - \cos\theta_2)$$

$$B_z = \frac{\mu_0 I}{4\pi} \cdot \frac{b_3}{b}(\cos\theta_1 - \cos\theta_2)$$

In equations (2)-(4), with d fixed, b , b , b , b, cos , and cos are nonlinear expressions of endpoints A and B coordinates. Equations (2)-(4) can be expressed in matrix form as:

$$Ax = b$$

To solve for the current distribution x = [I , I , ···, I__i, ···, I__n], the coordinates of each current line node must first be determined. When the total arc current value I is known, each branch current can be initially assumed as I__i = I/n, where n is the number of current lines. Intelligent optimization methods are then used to search for arc positions (i.e., coordinates of each current line), thereby determining operator A. Subsequently, current distribution x is obtained by solving the linear equation system. Since splitter plates and other ferromagnetic materials cause minimal disturbance to B and B has relatively larger values compared to other magnetic flux density components, the B component distribution is adopted for electromagnetic forward analysis.

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3 Multi-Population Genetic Algorithm Optimization

To overcome the ill-posedness of inversion, the truncated singular value decomposition method is employed, utilizing the GCV criterion to obtain truncation parameters and capture current distribution through magnetic field distribution. The results confirm that this method can accurately reconstruct arc current and aid in designing high-performance, reliable switching devices.

The multi-population genetic algorithm (MPGA) improves upon the standard genetic algorithm (GA) in three key aspects:

1. **Co-evolution of multiple populations**: Each population uses different control parameters. In standard GA, crossover operators determine global search capability while mutation operators determine local search capability, with fixed crossover and mutation probabilities. In MPGA, different populations have randomly generated crossover probabilities P_c and mutation probabilities P_m:

$$P_c = 0.7 + (0.9 - 0.7) \times \mathrm{rand}(N, 1)$$

$$P_m = 0.001 + (0.05 - 0.001) \times \mathrm{rand}(N, 1)$$

2. **Information exchange through immigration operators**: During evolution, the optimal individual from the previous population replaces the worst individual in the subsequent population, enabling information exchange between populations.

3. **Elite population preservation**: In each generation, an artificial selection operator chooses the optimal individuals from each population to store in an elite population. The elite population undergoes no selection, crossover, or mutation operations, only updates, ensuring optimal individuals are preserved.

The MPGA flowchart is shown in [Figure 4: see original paper], where t represents the current generation. The algorithm initializes N = 10 populations, each with M = 40 individuals, and a maximum generation T = 800, using binary encoding.

Due to the ill-posed nature of arc magnetic field inversion, current line positions may cross. Therefore, constraints must be imposed on algorithm variables. Equation (5) can be transformed into the following optimization problem:

$$\min \|Ax - b\|$$

In equation (6), analyzing the description of min‖Ax - b‖, the optimization process for searching operator A in feasible domain F involves multiple parameters and complex operational expressions. Traditional nonlinear optimization methods rely heavily on extensive calculations for multivariable extremum points when solving such problems. Heuristic random search methods can overcome this limitation, among which genetic algorithms (GA) are random search methods inspired by biological evolution principles (survival of the fittest) and represent parallel global optimization approaches. However, standard GA suffers from premature convergence—when one individual' s fitness far exceeds others, the population becomes dominated by that individual, causing evolutionary stagnation.

To improve solution quality, MPGA is adopted. After 500 generations of optimization, the objective function evolution curves for MPGA and GA are shown in [Figure 5: see original paper]. The relative error curves for optimizing each current line node coordinate are shown in [Figure 6: see original paper]. MPGA achieves an average relative error of 2.44% for all variables, while GA' s maximum relative error reaches 3.92%. Moreover, MPGA converges to the optimal value of 0.0290 more rapidly, whereas GA converges to 0.0753, demonstrating that MPGA enhances algorithmic precision.

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4 Truncated Singular Value Decomposition for Arc Current Inversion

Through the intelligent optimization process of MPGA, operator A in equation (5) can be determined. Since the optimized operator deviates from the original operator and electromagnetic inverse problems are ill-posed, solving for current using traditional linear equation methods amplifies errors. To assess the problem's ill-posedness, the discrete Picard condition is applied. Performing singular value decomposition on operator A:

$$A = \sum_{i=1}^{n} u_i \sigma_i v_i^T$$

If the Fourier coefficients |u_i^T b| decay to zero faster than the singular values _i, the equation satisfies the discrete Picard condition, enabling optimal approximate solutions via regularization methods. The discrete Picard condition verification for equation Ax = b is shown in [Figure 7: see original paper]. The Fourier coefficients exhibit fluctuations in the latter portion, indicating that components corresponding to these singular values are significantly affected by perturbations and must be truncated via regularization methods to obtain optimal solutions. The truncation parameter, also called the regularization parameter, is determined through parameter selection strategies.

To transform the constrained problem into an unconstrained extremum problem, a penalty function reduces the fitness of infeasible solutions to decrease their selection probability. The general constrained optimization problem is:

$$\min f(x), \quad x \in \mathbb{R}^n$$

subject to:

$$h_i(x) = 0, \quad i \in E = \{1, \dots, l\}$$
$$g_i(x) \le 0, \quad i \in I = \{1, \dots, m\}$$

The feasible domain F is defined as:

$$F = \{x \in \mathbb{R}^n \mid h_i(x) = 0(i \in E), g_i(x) \leq 0(i \in I)\}$$

The penalty function is constructed as:

$$p(x, \sigma) = f(x) + \sigma \cdot p(x)$$

where f(x) is the original objective function and:

$$p(x) = \sum_{i \in E} h_i^2(x) + \sum_{i \in I} [\min\{0, g_i(x)\}]^2$$

with penalty factor = 1.

The truncation parameter k critically affects the final solution. When k is too small, denoising is excessive, yielding overly smooth solutions; when k is too large, the solution includes unstable noise-affected components. Common methods for selecting truncation parameters include the L-curve method and GCV criterion, particularly when noise levels are unknown. The L-curve method plots the logarithmic coordinates of the regularized solution' s residual norm versus solution norm:

$$\eta = \log \|A_k x - b\|, \quad \rho = \log \|x\|$$

The point of maximum curvature corresponds to the truncation parameter. However, literature notes that L-curves can sometimes be too smooth for accurate curvature identification.

The GCV criterion defines the following expression, where the k minimizing the value is selected as the truncation parameter:

$$\mathrm{GCV}(k) = \frac{\|A_k x - b\|^2}{[\mathrm{trace}(I - AA_k^+)]^2}$$

where A_k is the regularized operator and trace denotes the matrix trace. For TSVD, A_k replaces small singular values causing perturbations with zeros. According to A_k' s pseudoinverse A_k^+, the regularized solution is:

$$x_k = A_k^+ b$$

The GCV criterion selection of truncation parameter is shown in [Figure 8: see original paper], yielding k = 4. Substituting k into equation (14) produces the inverted current distribution shown in [Figure 9: see original paper].

To demonstrate algorithmic accuracy, the least squares method solution x_LS for Ax = b is compared. The least squares solution is:

$$x_{LS} = (A^T A)^{-1} A^T b$$

Equation (15) becomes the traditional least squares solution when k equals A' s rank. However, least squares methods are vulnerable to ill-posedness, degrading solution stability. As shown in [Figure 9: see original paper], the least squares solution x_LS deviates significantly from the preset current, whereas TSVD with GCV criterion-selected regularization parameters accurately reflects the preset current distribution trend.

The relative error of arc current inversion is presented in the table below, where relative error _r is defined as:

$$\delta_r = \frac{\|x_{rec} - x_{ref}\|}{\|x_{ref}\|} \times 100\%$$

with x_rec being the inverted current and x_ref the preset current. Compared to least squares, TSVD demonstrates superior noise resistance for electromagnetic inverse problems' ill-posedness, achieving higher current reconstruction accuracy and validating regularization methods for arc inversion problems.

The error in regularized current inversion primarily stems from intelligent optimization algorithm preprocessing. Since the operator is ill-posed, even slight deviations can cause solutions to diverge significantly from true values, preventing approximate solutions via regularization. Therefore, intelligent optimization algorithm results substantially impact the equation' s ill-posedness degree.

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5 Conclusion

Addressing the ill-posedness of low-voltage circuit breaker arc inversion, this study employs multiple arc current lines to represent variable-diameter arcs, using electromagnetic correspondence to invert arc characteristics and simplify computational complexity. Arc position and current distribution are inverted through the combination of MPGA and TSVD, with regularization parameters selected via the GCV criterion.

The results demonstrate that this method yields small errors for reconstructing arc currents, a class of multivariable ill-posed inverse problems. Regularization accuracy depends on the quality of nonlinear intelligent optimization results. Future work will focus on enhancing method feasibility by reducing the influence of nonlinear materials such as splitter plates on inversion results.

References

[1] Zhao Hongchen. Research on switching arc inversion based on electromagnetic characteristic analysis [D]. Shenyang: Shenyang University of Technology, 2015.

[2] Leng Xue, Liu Li, Liu Xiaoming, et al. Chaotic characteristics analysis of high voltage SF6 circuit breaker under different breaking conditions [J]. Journal of Electrical Engineering, 2015, 10(11): 27-32.

[3] Wang Weizong, Wu Yi, Rong Mingzhe, et al. Research on air switch arc simulation technology and its application [J]. Low Voltage Electrical Apparatus, 2010(5): 7-11.

[4] Yang Fei, Rong Mingzhe, Wu Yi, et al. Numerical analysis of the influence of splitter-plate erosion on an air arc in the quenching chamber of a low-voltage circuit breaker [J]. Journal of Physics D: Applied Physics, 2010, 43: 1-12.

[5] Cai Bin, Chen Degui, Wu Rui, et al. Online detecting and protection system for internal faults arc in switchgear [J]. Transactions of China Electrotechnical Society, 2005, 20(10): 83-87.

[6] Ding Xinzhi, Liu Zhukui, Yan Yue, et al. Analysis of characteristics and application for arc spectral components [J]. Journal of Electrical Engineering, 2015, 10(5): 75-81.

[7] Brdys C, Toumazet J P, Velleaud G, et al. Study of the low-voltage electric breaking arc restrike by means of an inverse method [J]. IEEE Transactions on Plasma Science, 1999, 27(2): 595-603.

[8] Debellut E, Gary F, Cajal D, et al. Study of re-strike phenomenon in a low-voltage breaking device by means of the magnetic camera [J]. Journal of Physics D: Applied Physics, 2001, 34: 1665-1674.

[9] Brdys C, Toumazet J P, Laurent A, et al. Optical and magnetic diagnostics of the electric arc dynamics in a low voltage circuit breaker [J]. Measurement Science and Technology, 2002, 13: 1146-1153.

[10] Toumazet J P, Brdys C, Laurent A, et al. Combined use of an inverse method and a voltage measurement: variations [J]. Measurement Science and Technology, 2005, 16: 1525-1533.

[11] Zhang Pengfei, Zhang Guogang, Yang Boyu, et al. Investigations on intelligent inversion methods of current density distribution of electrical arc in air [J]. Proceedings of the CSEE, 2013, 33(33): 160-168.

[12] Zhang Pengfei, Zhang Guogang, Dong Jinlong, et al. Non-intrusive magneto-optic detecting system for investigations of air switching arcs [J]. Plasma Science and Technology, 2014, 16(7): 661-668.

[13] Luca G, Daniele P, Luca D R. Current density reconstruction in vacuum arcs by inverting magnetic field data [J]. IEEE Transaction on Magnetics, 2012,

48(8): 2324-2333.

[14] Dong Jinlong, Zhang Guogang, Zhang Zhiqiang, et al. Inverse problem solution and regularization reconstruction in switching arcs by inverting magnetic fields [J]. Mathematical Problems in Engineering, 2018: 1-11.

[15] Zhao Hongchen, Liu Xiaoming, Li Longnv. Arc inversion with genetic algorithm for low voltage switch [J]. High Voltage Apparatus, 2017, 53(4): 25-30.

[16] Lin Yang, Zhao Huan, Ding Han. Solution of inverse kinematics for general robot manipulators based on multiple population genetic algorithm [J]. Journal of Mechanical Engineering, 2017, 53(3): 1-8.

[17] Ye Manyuan, Zhou Qiqi, Cai Hong, et al. Multiple population genetic algorithm based on multi-band SHEPWM control technology for multi-level inverter [J]. Transactions of China Electrotechnical Society, 2015, 30(16): 111-119.

[18] Hansen P C, O' Leary D P. The use of the L-curve in the regularization of discrete ill-posed problems [J]. SIAM Journal of Scientific Computing, 1993, 14(6): 1487-1503.

[19] Ma Chao, Hua Hongxing. Impact force identification based on improved regularization technique [J]. Journal of Vibration and Shock, 2015, 34(12): 164-170.

Note: Figure translations are in progress. See original paper for figures.