Postprint: Passivity-Based Control of Buck Converters via Optimal Damping Injection

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Abstract

To address the issue that the fixed damping injection DI in existing passivity-based controllers for Buck converters cannot achieve satisfactory control performance, this paper proposes a method for calculating and optimizing DI. First, the transfer function of the Buck converter system under passivity-based control is established. Under step input conditions, the system' s output current response and its overshoot are derived. By correcting the overshoot of the system current response, the value range of DI is ultimately determined. Within this determined range, the genetic algorithm is employed to optimize DI and obtain the optimal DI. The passivity-based controller based on this optimal DI enables the Buck converter to achieve excellent performance. Computer simulation results demonstrate the feasibility of the proposed passivity-based control strategy based on optimal DI.

Full Text

Preamble

Study of Passivity-Based Control of Buck Converter Based on Optimal Damping Injection

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Abstract

Existing passivity-based controllers for Buck converters suffer from the limitation that the determined damping injection (DI) cannot achieve optimal control performance. To address this problem, this paper proposes a method for calculating and optimizing DI. First, the transfer function of the Buck converter system under passivity-based control is established. Under step input conditions, the system' s output current response and its overshoot are derived. By correcting the overshoot of the system' s current response, the value range of DI is ultimately determined. Within this determined range, the DI is optimized using a genetic algorithm to obtain the optimal DI. The passivity-based controller based on optimal DI enables the Buck converter to achieve excellent performance. Computer simulation results demonstrate the feasibility of the proposed passivity-based control strategy based on optimal DI.

Keywords: Buck converter, passivity-based control, optimal damping injection, genetic algorithm

1 Introduction

With the development of new energy sources and distributed power generation, DC/DC converters are widely applied in various applications requiring voltage step-up or step-down. Due to their inherent nonlinear characteristics, traditional linear control methods based on small-signal processing exhibit significant limitations [1-2]. Consequently, numerous nonlinear control methods have been proposed, such as feedback linearization control, sliding mode variable structure control, active disturbance rejection control, and passivity-based control (PBC). Feedback linearization requires full-state measurement, exact dynamic cancellation, introduces controller singularities, exhibits strong parameter dependence, and involves complex control implementation. Sliding mode control suffers from chattering issues. Active disturbance rejection control requires a set of satisfactory nonlinear functions and corresponding parameters, involves heavy computational load, and results in long control periods with poor real-time performance [3]. None of these methods achieve essential nonlinear control of converters. In contrast, PBC theory approaches the problem from an energy perspective and achieves nonlinear control through damping injection, representing an essential nonlinear control method with advantages including simple control structure, fast response, and zero overshoot. Therefore, PBC theory has found increasingly widespread application in power converter control. For instance, references [4-5] designed passivity-based controllers for DC/DC converters including Buck converters, achieving stable operation through damping injection (DI) and obtaining good control performance.

However, there is currently no effective method for determining DI. Typically, trial-and-error methods are employed, but these are inefficient and cannot guarantee optimal DI. Reference [4] constrained the DI range using the Routh-Hurwitz stability criterion by solving inequalities. Although relatively simple, this method is not applicable for inherently stable converter systems such as Buck converters, as the Routh-Hurwitz criterion no longer provides effective constraints.

Based on these considerations, this paper takes the Buck converter as an example and proposes a method to constrain the DI range by correcting the system' s overshoot. The optimal DI value is then obtained through optimization using a genetic algorithm (GA). DI not only improves converter performance but also suppresses overshoot in the converter' s current response, thereby reducing the adverse effects of inrush current on the system. Computer simulation results verify the feasibility of the passivity-based control strategy based on optimal DI.

2.1 EL Model of the Buck Converter

The main circuit of the Buck converter is shown in [Figure 1: see original paper]. In the diagram, VT is the switching transistor, VD is the diode, i_L is the inductor current, u_C is the capacitor voltage, d is the duty cycle of the drive signal for switching transistor VT, R is the load resistance, L is the inductance, C is the capacitance, and U_S is the input source voltage.

To simplify model development, all circuit components are assumed to be ideal. Selecting the switching-cycle-averaged inductor current i_{Lav} and switching-cycle-averaged capacitor voltage u_{Cav} as state variables, the mathematical model of the Buck converter can be derived from Figure 1 as:

$$L\frac{di_{Lav}}{dt} = dU_S - u_{Cav} \tag{1}$$

$$C\frac{du_{Cav}}{dt} = i_{Lav} - \frac{u_{Cav}}{R} \tag{2}$$

Under continuous conduction mode (CCM), let $x_1 = i_{Lav}$ and $x_2 = u_{Cav}$. According to the Buck converter mathematical model, we have:

$$\dot{x}_1 = \frac{1}{L}(dx_2 - x_2) \tag{3}$$

$$\dot{x}_2 = \frac{1}{C}\left(x_1 - \frac{x_2}{R}\right) \tag{4}$$

Expressing equation (2) in matrix form yields the EL model of the Buck converter:

$$M_{Bu}\dot{x} + J_{Bu}x + R_{Bu}x = du$$

where $M_{Bu} = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}$, $J_{Bu} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (an antisymmetric matrix satisfying $J_{Bu} = -J_{Bu}^T$), $R_{Bu} = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}$, and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

First, we analyze the passivity of the Buck converter with resistive load. In Figure 1, assuming the source-side port voltage is u and current is i, we have $i = dx_1$. Multiplying the first equation in (1) by i_{Lav} and the second by u_{Cav} gives:

$$Li_{Lav}\frac{di_{Lav}}{dt} = di_{Lav}U_S - i_{Lav}u_{Cav} \tag{5}$$

$$Cu_{Cav}\frac{du_{Cav}}{dt} = i_{Lav}u_{Cav} - \frac{u_{Cav}^2}{R} \tag{6}$$

Combining and substituting $i = di_{Lav}$ yields:

$$Li_{Lav}\frac{di_{Lav}}{dt} + Cu_{Cav}\frac{du_{Cav}}{dt} = iU_S - \frac{u_{Cav}^2}{R}$$

Integrating both sides of equation (5) gives:

$$\int_0^t Li_{Lav}(\tau)\frac{di_{Lav}(\tau)}{d\tau}d\tau + \int_0^t Cu_{Cav}(\tau)\frac{du_{Cav}(\tau)}{d\tau}d\tau = \int_0^t i(\tau)U_S d\tau - \int_0^t \frac{u_{Cav}^2(\tau)}{R}d\tau$$

Rearranging equation (6) yields:

$$\frac{1}{2}Li_{Lav}^2(t) + \frac{1}{2}Cu_{Cav}^2(t) = \int_0^t i(\tau)U_S d\tau - \int_0^t \frac{u_{Cav}^2(\tau)}{R}d\tau + \frac{1}{2}Li_{Lav}^2(0) + \frac{1}{2}Cu_{Cav}^2(0)$$

Equation (7) can be rewritten as:

$$H(t) - H(0) = \int_0^t i(\tau)U_S d\tau - \int_0^t \frac{u_{Cav}^2(\tau)}{R}d\tau$$

where $H(t) = H_L(t) + H_C(t)$ represents the total energy stored in the circuit at time t, with $H_L(t) = \frac{1}{2}Li_{Lav}^2(t)$ and $H_C(t) = \frac{1}{2}Cu_{Cav}^2(t)$. According to the definition of passivity [6], if $R > 0$, the Buck converter with resistive load is passive and can operate stably.

2.2 Passivity-Based Controller Based on EL Model

Let the desired state vector be $x^* = \begin{bmatrix} x_1^* & x_2^* \end{bmatrix}^T$, and the state vector error be $e = x - x^*$. Here, x_2^* is determined by load requirements. When the Buck converter operates at equilibrium, $x_1^* = x_2^*/R$.

From equation (3), we obtain:

$$M_{Bu}\dot{e} + J_{Bu}e + R_{Bu}e = du - [M_{Bu}\dot{x}^* + J_{Bu}x^* + R_{Bu}x^*]$$

To make the error energy storage function $H_e(x) = \frac{1}{2}e^T M_{Bu}e$ converge rapidly to zero, a DI term is added. Adding the DI term $R_a e$ to both sides of equation (9), where $R_a = \begin{bmatrix} R_{a1} & 0 \\ 0 & 1/R_{a2} \end{bmatrix}$ with $R_{a1}, R_{a2} > 0$, yields:

$$M_{Bu}\dot{e} + J_{Bu}e + R_{Bu}e + R_a e = du - [M_{Bu}\dot{x}^* + J_{Bu}x^* + R_{Bu}x^*] + R_a e$$

The passivity-based controller is designed as:

$$du = [M_{Bu}\dot{x}^* + J_{Bu}x^* + R_{Bu}x^*] - R_a e$$

Since x^* is a constant reference, $\dot{x}^* = 0$. Expanding equation (12) yields the duty ratio for the passivity-based controller:

$$d = \frac{1}{U_S}\left[x_2^* + R_{a1}(x_1 - x_1^*)\right]$$

The system structure of the Buck converter under passivity-based control is shown in [Figure 2: see original paper]. Here, i_L^* is the inductor current reference and u_C^* is the capacitor voltage reference. The passivity-based controller takes i_L and u_C as input signals, performs simple algebraic operations to output the duty ratio d, which is then compared with a triangular wave to generate PWM signals that drive the switching transistor through a driver circuit to achieve the control objective. The passivity-based controller features simple implementation and enables overshoot-free output voltage control.

Since DI does not consume energy, equation (13) remains valid for the closed-loop control system with DI. Therefore, the Buck converter system with resistive load can operate stably under the designed passivity-based controller.

3.1 Determination of DI Range

The passivity-based controller controls the inductor current. When the inductor current reaches its reference value (i.e., the circuit operates at equilibrium), the capacitor voltage also reaches its reference value based on the relationship between capacitor voltage and inductor current at Buck converter equilibrium. Thus, the passivity-based controller achieves capacitor voltage control by regulating the inductor current. Under this controller, the converter output voltage exhibits no overshoot and fast response. However, inrush current occurs when the circuit starts, which adversely affects the circuit. This can be mitigated by using DI to limit the inductor current overshoot, thereby reducing the inrush current magnitude and its detrimental effects.

To establish a quantitative relationship between DI and the inductor current overshoot $\sigma\%$, the transfer function of the Buck converter system under PBC is developed. The control system block diagram is shown in [Figure 3: see original paper], where $G(s)$ is the transfer function from duty ratio $D(s)$ to inductor current $I_L(s)$, and $I_L^*(s)$ is the Laplace transform of the inductor current reference. From Figure 3, the closed-loop transfer function $\Phi(s)$ can be derived, enabling analysis of the system' s step response and overshoot. The DI range can then be determined by constraining the overshoot.

From equation (1), we obtain:

$$G(s) = \frac{I_L(s)}{D(s)} = \frac{(RCs+1)U_S}{RLCs^2 + Ls + R}$$

The closed-loop transfer function $\Phi(s)$ is:

$$\Phi(s) = \frac{I_L(s)}{I_L^*(s)} = \frac{(R_{a1}+R)G(s)}{U_S + R_{a1}G(s)}$$

Substituting equation (16) into (17) yields:

$$\Phi(s) = \frac{R+R_{a1}}{RCs+1} \cdot \frac{RCs+1}{\frac{L+RCR_{a1}}{R+R_{a1}}s+1} = \frac{R+R_{a1}}{s^2 + \frac{L+RCR_{a1}}{R+R_{a1}}s+1}$$

Let the system input be $i_L^*(t) = I_L^*$, so $I_L^*(s) = I_L^*/s$. The system output $I_L(s)$ is:

$$I_L(s) = I_L^*(s)\Phi(s) = I_L^* \frac{R+R_{a1}}{s\left(s^2 + \frac{L+RCR_{a1}}{R+R_{a1}}s+1\right)}$$

Taking the inverse Laplace transform of equation (19) gives:

$$i_L(t) = I_L^* - I_L^* e^{-\alpha t}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right)$$

where $\alpha = \frac{L+RCR_{a1}}{2(R+R_{a1})}$ and $\omega_d = \sqrt{1-\alpha^2}$.

Differentiating equation (20) and setting the derivative to zero yields the peak time t_p:

$$t_p = \frac{\pi}{\omega_d}$$

Since overshoot occurs at the peak time, substituting equation (22) into (20) gives the peak value:

$$i_L(t_p) = I_L^* - I_L^* e^{-\alpha t_p} \left(\cos\pi + \frac{\alpha}{\omega_d} \sin\pi \right) = I_L^* \left(1 + e^{-\alpha t_p}\right)$$

The overshoot is therefore:

$$\sigma\% = e^{-\alpha t_p} \times 100\%$$

For a second-order system under unit step input with damping ratio $\zeta = 0.4 \sim 0.8$, the overshoot $\sigma\%$ ranges between 1.5% and 25.4% [7]. Using this standard to correct the system:

$$1.5\% \leq e^{-\alpha t_p} \leq 25.4\%$$

The circuit parameters are selected as shown in . Substituting these parameters into equation (25) and solving yields:

$$0.766\,\Omega \leq R_{a1} \leq 48\,\Omega$$

The relationship curve between $\sigma\%$ and R_{a1} is shown in [Figure 4: see original paper]. As R_{a1} increases, the overshoot $\sigma\%$ decreases, and vice versa.

3.2 Determination of Optimal DI

After correcting the system using overshoot constraints, the allowable range for R_{a1} is obtained as shown in equation (26). The GA is then used to optimize and determine the final DI value. The GA program parameters are listed in , and the program flowchart is shown in [Figure 5: see original paper]. The objective function is selected as:

$$\text{Objval} = \sum_{gen=1}^{20} \sum_{i=1}^{10} \left(u_C^*(gen, i) - u_C(gen, i)\right)^2$$

Through GA optimization, the minimum value of the objective function is obtained, yielding the optimal DI value of $7.5137\,\Omega$.

4 System Analysis After Damping Injection

The equivalent circuit of the Buck converter after damping injection is shown in [Figure 6: see original paper]. Here, R_{a1} is equivalent to being in series with inductor L, and R_{a2} is equivalent to being in parallel with capacitor C [4].

The closed-loop transfer function can be rewritten as:

$$\Phi(s) = \frac{RCs+1}{\frac{L+RCR_{a1}}{R+R_{a1}}s^2 + s + \frac{R+R_{a1}}{L+RCR_{a1}}}$$

Substituting the circuit parameters into equation (28) and plotting the Bode diagram yields [Figure 7: see original paper]. In the low-frequency region, the system' s magnitude response is 0 dB, indicating that the system is inherently stable and can track the input. As frequency increases, overshoot phenomena occur, but these are suppressed as the DI value increases. [Figure 7: see original paper] also shows that as DI increases, the system bandwidth increases, enhancing the ability to track input signals but reducing the ability to suppress high-frequency interference at the input. Therefore, a compromise must be made in selecting the DI value.

[Figure 8: see original paper] shows the closed-loop pole distribution. As R_{a1} increases, the system poles move away from the imaginary axis, transitioning the system from underdamped to overdamped. However, the poles always remain in the left half-plane, confirming that the system remains stable at all times and validating the stability analysis presented in this paper.

5 Simulation Study

Using the circuit parameters from with a switching frequency of 10 kHz, simulations were performed using the optimal DI value of 7.5137 Ω. For comparison, simulations were also conducted with DI values of 0.766 Ω and 48 Ω. The results are shown in [Figure 9: see original paper], and system performance metrics are listed in .

[Figure 9a: see original paper] shows the voltage response waveforms. The results demonstrate that the optimal DI obtained through GA optimization enables the system output voltage to track the reference voltage excellently, with virtually no overshoot and fast response. The output voltage decreases as the DI value increases.

[Figure 9b: see original paper] shows the current response waveforms. The current response overshoot decreases as DI increases, consistent with the conclusion from [Figure 4: see original paper] that increasing DI reduces current overshoot and suppresses inrush current. Additionally, at the optimal DI, the current ripple is minimized and tracks the reference value. As DI increases, the rise time t_r decreases, indicating faster system response.

6 Conclusion

This paper employs passivity-based control for Buck converters. PBC offers simple implementation and enables fast, overshoot-free voltage response. The closed-loop transfer function of the system was established, revealing the relationship between R_{a1} and current overshoot. By correcting the current overshoot, the DI range was determined and the optimal DI was obtained through GA optimization. The optimal DI not only achieves overshoot-free, zero-steady-state-error output voltage for the Buck converter but also suppresses current response overshoot, thereby mitigating the adverse effects of inrush current. Simulation experiments verify the effectiveness of the proposed method for determining the optimal DI. Bode diagram analysis reveals the impact of DI on the passivity-based controlled Buck converter system. While adding the DI term enhances the closed-loop system' s ability to reproduce input signals, it simultaneously reduces the ability to suppress high-frequency interference at the input. Thus, increasing DI improves input tracking capability but is detrimental to high-frequency interference rejection, necessitating a compromise in DI selection.

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Note: Figure translations are in progress. See original paper for figures.