

## Design Method for Optimal Diameter-to-Distance Ratio of Loosely Coupled Coils in ICPT Systems (Postprint)

**Authors:** Li Zhengtai, Wang Chunfang, Wei Zhihao, Li Dan

**Date:** 2018-06-15T00:00:00+00:00

### Abstract

In conventional Inductive Coupling Power Transfer (ICPT) systems, the optimal proportionality coefficient between the radius of the receiving coil Rx of a loosely coupled transformer and the distance h from the transmitting coil Tx to the receiving coil Rx—i.e., the radial-distance ratio—is typically obtained through system modeling to derive the mutual inductance between Tx and Rx, supplemented by extensive experimental validation. To address this lack of theoretical basis and the associated waste of manpower and material resources, this paper proposes a design methodology for determining the optimal proportionality coefficient between the radius of Rx and h by observing the variation pattern of current density in Rx via simulation. Firstly, a decoupled equivalent circuit model for a single-tube ICPT system with primary-side parallel, secondary-side parallel (PP) configuration is derived, establishing the relationship between the current density on Rx, mutual inductance M, and system transmission power. Through analytical formulation, the values of the ratio  $\alpha$  between the inductance L1 of Rx and the inductance L2 of Tx, as well as the system coupling coefficient  $k$ , are determined to ascertain L1 and L2 values at different frequencies, thereby constructing a simulation model. Finite element simulation software is then employed to investigate the optimal value of  $\alpha$ . Compared with coil optimization methods based on mutual inductance values, the current density in this work can be directly observed through software, offering visual and intuitive results while saving time and cost, thus effectively improving design efficiency. The comprehensive simulation results determine the optimal radial-distance ratio  $\alpha$  for loosely coupled coils, which aligns with empirical values summarized by enterprises through production practice.

## Full Text

### Preamble

DOI: 10.11985/2018.01.002

### A Design Method for Optimal Radius-Center Distance Ratio of Loosely Coupled Transformer Coils in ICPT Systems

Li Zhengtai<sup>1</sup>, Wang Chunfang<sup>1</sup>, Wei Zhihao<sup>1</sup>, Li Dan<sup>2</sup>

<sup>1</sup> School of Automation and Electrical Engineering, Qingdao University, Qingdao 266071, China

<sup>2</sup> Qingdao Lu Yu Energy Technology Co., Ltd, Qingdao 266071, China

---

### Abstract

In traditional Inductively Coupled Power Transfer (ICPT) systems, the optimal ratio coefficient between the radius of the receiving coil (Rx) of a loosely coupled transformer and the distance (h) between the transmitting coil (Tx) and receiving coil (Rx)—known as the radius-center distance ratio—is typically obtained through system modeling to derive the mutual inductance value between Tx and Rx, supplemented by extensive experiments. This approach lacks theoretical foundation and wastes significant manpower and material resources. To address this issue, this paper proposes a design method that determines the optimal proportion between Rx radius and h by simulating and observing the variation patterns of current density in Rx.

First, we derive the decoupled equivalent circuit model for a single-switch ICPT system with primary-parallel and secondary-parallel (PP) compensation structure. Based on this model, we establish the relationship among current density on Rx, mutual inductance M, and system transmission power. Through formula analysis, we determine the ratio a of Rx inductance L to Tx inductance L and the system's coupling coefficient, which allows us to determine L and L values at different frequencies. We then build a simulation model to investigate the optimal value of using finite element simulation software.

Compared with coil optimization methods based on mutual inductance values, the current density in our approach can be directly observed through software, providing a vivid and intuitive visualization that saves time and cost while effectively improving design efficiency. The comprehensive simulation results determine the optimal radius-center distance ratio for loosely coupled coils, which aligns with empirical values summarized from industrial production practice.

**Keywords:** Single-switch inverter, Inductively Coupled Power Transfer, radius-center distance ratio, current density, finite element simulation

## 1 Introduction

In recent years, Inductively Coupled Power Transfer (ICPT) technology has become a research hotspot among scientists and engineers [1-2], with successful applications in wireless power supply for smart appliances, electric vehicle wireless charging, and in-vivo charging for biological systems [3-5]. As a key component in ICPT systems, the structure and parameters of loosely coupled transformers directly affect system volume and transmission power [6-9]. Currently, research on the radius-center distance ratio of loosely coupled transformers in inductive coupling systems is scarce domestically and internationally, with parameters obtained primarily through empirical values from production practice, lacking theoretical analysis and validation.

To address this gap, this paper builds upon the single-switch inverter ICPT system topology studied by our research group [10-12]. We first derive the correspondence between current density on Rx and mutual inductance values, and analyze the optimal radius-center distance ratio of loosely coupled coils from the perspective of the relationship between mutual inductance and system voltage gain and transmission power.

---

## 2 Single-Switch ICPT System Structure

The topology of the single-switch inverter ICPT system is shown in [Figure 1: see original paper]. In the diagram,  $C_p$  and  $C_{p1}$  are primary-side compensation capacitors, while  $C_s$  and  $C_{s1}$  are secondary-side compensation capacitors. When  $C_p$  is connected, the primary side employs parallel compensation; when  $C_{p1}$  is connected, the primary side uses series compensation. Similarly, when  $C_s$  is connected, the secondary side employs parallel compensation; when  $C_{s1}$  is connected, the secondary side uses series compensation. This configuration allows four compensation topologies: primary-parallel secondary-parallel (PP), primary-parallel secondary-series (PS), primary-series secondary-parallel (SP), and primary-series secondary-series (SS).

This paper first investigates the minimum value of  $\eta$  using the PP compensation configuration shown in [Figure 1: see original paper], then extends the methodology to the other three compensation configurations and to half-bridge and full-bridge circuits. [Figure 2: see original paper] illustrates the equivalent circuit model for PP compensation, where [Figure 2a: see original paper] shows the primary-side equivalent model and [Figure 2b: see original paper] shows the secondary-side equivalent model. The AC220V/50Hz mains voltage, after full-bridge rectification and filtering by  $L_1$  and  $C_1$ , can be considered a constant voltage source  $U_{cp}$ .  $Z_f$  represents the impedance reflected from the secondary side to the primary side. [Figure 2b: see original paper] shows the secondary-side equivalent model, where  $U_{oc}$  is the induced electromotive force on the secondary side. For other topologies such as full-bridge, half-bridge, and

push-pull circuits [13], the equivalent circuit during switching processes remains the same as shown in [Figure 2: see original paper].

By obtaining the steady-state equivalent model and applying decoupling equivalence principles, the primary-side impedance is:

$$Z_1 = R_P + j\omega L_P$$

The impedance reflected from secondary to primary side is:

$$\frac{\omega^2 M^2}{Z_2}$$

where  $Z$  is the secondary-side impedance:

$$Z_2 = R_S + j\omega L_S + \frac{R_{eq}}{1 + \omega^2 C_S^2 R_{eq}^2} + j \frac{\omega L_S - \omega C_S R_{eq}^2}{1 + \omega^2 C_S^2 R_{eq}^2}$$

Since the system's resonant angular frequency is large, the internal resistance  $R_s$  of the secondary coil can be neglected.

The current density on Rx is:

$$\rho = \frac{I_{Ls}}{A} = \frac{U_{Rl}}{A \cdot R_{eq}} = \frac{U_{Ls}}{A \cdot |Z_2|} = \frac{U_{oc}}{A \cdot |Z_2|} \cdot \frac{\omega M}{|Z_2|} = \frac{U_{oc} \cdot \omega M}{A \cdot Z_2^2}$$

where  $U_{Rl}$  is the RMS output voltage,  $U_{Ls}$  is the RMS AC voltage after the resonant network,  $I_{Ls}$  is the RMS input current to the rectifier network from Rx, and  $A$  is the cross-sectional area of one turn of Rx. This equation shows that  $\rho$  is directly proportional to  $\frac{1}{Z_2^2}$ , meaning  $\rho$  is proportional to  $M$ .

Since  $\rho$  is positively correlated with  $M$ , and  $M$  directly affects system transmission power and efficiency, we can simulate the current density of Rx coils with different specifications to investigate the minimum  $\rho$  value for magnetically coupled coils.

---

### 3 Minimum Value of

#### 3.1 System Parameter Selection

The primary-side resonant angular frequency is:

$$\omega_1 = \frac{1}{\sqrt{L_P C_P}}$$

The secondary-side resonant angular frequency is:

$$\omega_2 = \frac{1}{\sqrt{L_S C_S}} = 2\pi f$$

From these equations, we derive the voltage gain:

$$M_v = \frac{U_R}{U_{cp}} = \frac{j\omega M}{(Z_2 - j\omega L_S)} \cdot \frac{Z_{eq}}{Z_1 + Z_f}$$

For analysis convenience, the load output power is approximated as equal to the system transmission power:

$$P_{PP} = \frac{R_{eq} M^2 \omega^2 U_{cp}^2}{(\omega L_P \omega L_S - \omega^2 M^2)^2 + (R_{eq} \omega M^2)^2}$$

System efficiency is:

$$\eta = \frac{Re(Z_2) I_P^2}{(Re(Z_2) + R_P) I_P^2} = \frac{1}{1 + R_P / Re(Z_2)} = \frac{1}{1 + Re(Z_f) R_P / (\omega^2 M^2)}$$

where Q is the system quality factor. Based on our system characteristics, Q = 0.9. Let a be the ratio of Tx inductance Lp to Rx inductance Ls (a = Lp/Ls). Substituting a into the equations yields the variation curves of voltage gain Mv and power P with respect to a, shown in [Figure 3: see original paper].

Since this system has high output power and requires high voltage gain, a is selected as 4.

To determine the optimal value, the transmission distance h should be optimized. According to ICPT system characteristics, the coupling coefficient generally ranges from 0 to 0.5, but in practical applications, is typically 0.2 to 0.3. Therefore, this paper calculates system parameters at frequencies of 200 kHz, 500 kHz, 800 kHz, 1000 kHz, and 1500 kHz for = 0.2 and = 0.25 to investigate the optimal value, as shown in .

\*\* Various parameters of the system at different frequencies\*\*

f/kHz	Lp/ H	Ls/ H	Cp/nF	Cs/nF ( =0.25)	Cs/nF ( =0.2)
200					
500					
800					
1000					
1500					

### 3.2 Relationship Between $\gamma$ and Mutual Inductance M

The radius-center distance ratio of loosely coupled coils in the system is:

$$\gamma = \frac{D}{h}$$

where D is the radius of Rx. Applying Neumann's formula:

$$M = \frac{\mu_0 N_1 N_2}{2} \sqrt{D_1 D_2} \left[ \left( \frac{2}{b} - b \right) K(b) - \frac{2}{b} E(b) \right]$$

where  $D_1$  is the Tx radius,  $D_2$  is the Rx radius, and b, K(b), and E(b) are:

$$b = \frac{4D_1 D_2}{(D_1 + D_2)^2 + h^2}$$

$$K(b) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - b^2 \sin^2 \theta}}$$

$$E(b) = \int_0^{\pi/2} \sqrt{1 - b^2 \sin^2 \theta} d\theta$$

Substituting these into the mutual inductance equation yields the variation curve of M with respect to  $\gamma$ , shown in [Figure 4: see original paper]. The results indicate that to achieve maximum mutual inductance and system power, the  $\gamma$  value corresponding to maximum mutual inductance approaches 1.3 to 1.5 as switching frequency increases. This suggests the optimal radius-center distance ratio  $\gamma$  should be in the range of 1.3 to 1.5.

---

## 4 Simulation Results

### 4.1 Relationship Between $\gamma$ and M

To analyze and verify the correspondence between current density on Rx and mutual inductance M, we use the mutual inductance calculation tool in finite element simulation software to obtain inter-coil mutual inductance values. The simulation is shown in [Figure 5: see original paper]. Figures 5a and 5b show  $\gamma$  and M values on an Rx coil with outer diameter 9 cm and inner diameter 7 cm as Tx dimensions vary. In Figure 5b, redder colors indicate larger mutual inductance values, and the Tx dimensions corresponding to the red region represent the optimal Tx size matching this Rx.

To further verify the relationship between coil current density and mutual inductance, an additional comparison was made for an Rx coil with outer diameter 9

cm and inner diameter 5 cm, shown in Figures 5c and 5d. The variation trends remain consistent. By comparing Rx coil current density and magnetic coupling coil mutual inductance values as Tx coil inner and outer radii change, we find that the Tx dimensions corresponding to both the minimum and maximum values of current density and mutual inductance are identical. That is, as Tx inner and outer diameters change, the current density on Rx and the mutual inductance of the coupled coils follow the same trend. The simulation verifies the positive proportional relationship between  $\rho$  and M on Rx.

**[Figure 5: see original paper] Current density and mutual inductance of Rx coil changing with Tx coil inner and outer radii**

#### 4.2 Relationship Between $\rho$ and M

For the  $\rho$  value, with a fixed coupling coefficient, when  $\rho$  is constant, the transmission distance h varies as coil radius D changes, but the ratio remains constant at  $\frac{h}{D}$ . To determine the minimum  $\rho$  value, we identify the coil radius D that yields the farthest transmission distance while maintaining normal system operation. This distance corresponds to the minimum  $\rho$  value.

Using finite element simulation software with parameters from [1], where the Rx coil has outer diameter 11 cm, inner diameter 9 cm, and average radius 10 cm, we employ the software's mutual inductance calculation function to determine the farthest transmission distance at which the inter-coil mutual inductance meets requirements for normal system operation, and then calculate  $\rho$ .

As shown in [1], for the same coupling coefficient, transmission distance h increases and  $\rho$  decreases as frequency increases. At the same frequency, smaller coupling coefficients result in longer transmission distances. The optimal  $\rho$  value is approximately 1.33, which basically matches the values obtained from [Figure 5: see original paper].

\*\* Farthest h and minimum  $\rho$  at different frequencies during normal system operation\*\*

f/kHz	$\rho = 0.25$	$\rho = 0.2$
	h (cm)	
200	6.1-6.2	1.61-1.64
500	6.1-6.3	1.58-1.61
800	6.2-6.3	1.58-1.61
1000	6.1-6.2	1.61-1.64

## 5 Conclusion

This paper proposes a design method to obtain the minimum  $\rho$  value by simulating and observing current density variation patterns in Rx. The conclusions

are:

1. This method is applicable to various primary-secondary compensation structures in full-bridge, half-bridge, push-pull, and single-switch inverter ICPT systems.
2. For ICPT systems, under the condition of Rx radius and transmission distance, the farthest transmission distance between the two coils is 1.33 times the Rx radius.
3. The optimized parameters obtained by the proposed design method basically match the results summarized from enterprise experiments and have been successfully applied in a Qingdao-based wireless charging product manufacturer, demonstrating promising prospects for broader application.

---

## References

- [1] Wang Zhenya, Wang Xuemei, Zhang Bo, et al. Advances of wireless charging technology in electric vehicle[J]. Journal of Power Supply, 2014, 12(3): 27-32.
- [2] Liao Chenglin, Li Junfeng, Tao Chengxuan, et al. A review on control methods for wireless power transfer system[J]. Journal of Electrical Engineering, 2015, 10(6): 1-6.
- [3] Gao Jianxin, Wu Xusheng, Gao Wei, et al. Review on inductive contactless power transfer technology[J]. Journal of Power Supply, 2017, 15(2): 166-178.
- [4] Cao Lingling, Chen Qianhong, Ren Xiaoyong, et al. Review of the efficient wireless power transmission technique for electric vehicles[J]. Transactions of China Electrotechnical Society, 2012, 27(8): 1-13.
- [5] Aditya K, Williamson S S. Design considerations for loosely coupled inductive power transfer(IPT)system for electric vehicle battery charging-A comprehensive review[C]. Transportation Electrification Conference and Expo, 2014: 1-6.
- [6] Jean-Romain Sibué, Kwimang G, Jean-Paul Ferrieux, et al. A global study of a contactless energy transfer system: analytical design, virtual prototyping, and experimental validation[J]. IEEE Transactions on Power Electronics, 2013, 28(10): 4690-4699.
- [7] Xia Chenyang, Xie Guangqing, Lin Kezhang, et al. Study of dual resonance point characteristics and maximum output power of ICPT based on double LCL compensation[J]. Proceedings of the CSEE, 2016, 36(19): 5200-5208, 5401.
- [8] Huang Xiaosheng, Chen Wei. A novel compensation network for ICPT systems[J]. Proceedings of the CSEE, 2014, 34(18): 3020-3026.
- [9] Yang Minsheng, Wang Yaonan. Transferred power regulating method with a dynamically detuning inductor for ICPT pickups[J]. Electric Machines and Control, 2012, 16(1): 72-78.

- [10] Xia Chenyang. Analysis and optimization research on energy efficiency characteristics of inductively coupled power transfer systems[D]. Chongqing University, 2010.
- [11] Wang Xuemei, Zhang Bo. Study of bifurcation and chaos in single-phase SPWM inverter[J]. Transactions of China Electrotechnical Society, 2009, 24(1): 101-107.
- [12] Wang Chunfang, Qi Fei. Optimal design on main circuit parameters of WPT system with single tube[J]. Advanced Technology of Electrical Engineering and Energy, 2015, 34(5): 59-62.
- [13] Hatanaka K, Sato F, Matsuki H, et al. Power transmission of a desk with a cord-free power supply[J]. IEEE Transactions on Magnetics, 2002, 38(5): 3329-3331.
- [14] Wang Chunfang, Chen Jiemin, Li Dan, et al. A zero-voltage turn-on and turn-off single-switch IPT power supply[J]. Transactions of China Electrotechnical Society, 2015, 30(4): 203-208.
- [15] Liu Xiuquan, Zeng Zhaorui, Huang Ping, et al. Numerical and experimental analysis on performances of coreless coil inductance[J]. Chinese Journal of Engineering Design, 2008, 15(2): 149-153.

---

*Note: Figure translations are in progress. See original paper for figures.*

*Source: ChinaXiv –Machine translation. Verify with original.*