Multi-level Threshold Image Segmentation Algorithm Based on Particle Swarm Optimization and Fuzzy Entropy (Postprint)

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Abstract

To address the issue of substantial computational cost arising from exhaustive search for optimal thresholds in existing thresholding segmentation algorithms, this paper proposes a multi-level thresholding image segmentation algorithm based on Particle Swarm Optimization (PSO) and fuzzy entropy. Image segmentation constitutes a critical preprocessing step in image analysis. In the proposed methodology, Shannon entropy and fuzzy entropy are first selected as objective functions for the optimization technique. Subsequently, a multi-level image thresholding framework based on the PSO algorithm is established to perform image segmentation by maximizing Shannon entropy or fuzzy entropy. Finally, "Lena", "Baboon", and "Airplane" are selected from an image segmentation database as test images for performance analysis (including robustness, efficiency, and convergence), and compared with several existing thresholding segmentation algorithms. Experimental results demonstrate that the proposed algorithm achieves higher PSNR values and fewer classification errors, thereby validating its efficacy as an efficient multi-level thresholding image segmentation algorithm.

Full Text

Multi-level Threshold Image Segmentation Algorithm Based on Particle Swarm Optimization and Fuzzy Entropy

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Abstract: To address the high computational cost caused by existing threshold segmentation algorithms that use exhaustive search to find optimal thresholds, this paper proposes a multi-level threshold image segmentation algorithm based on particle swarm optimization and fuzzy entropy. Image segmentation is a crucial preprocessing step in image analysis. In the proposed method, Shannon entropy and fuzzy entropy are first selected as the objective functions for the optimization technique. Then, a multi-level image threshold segmentation framework based on particle swarm optimization is established, which performs image segmentation by maximizing Shannon entropy or fuzzy entropy. Finally, "Lena," "Baboon," and "Airplane" are selected from an image segmentation database as test images for performance analysis (including robustness, efficiency, and convergence) and compared with several existing threshold segmentation algorithms. The results demonstrate that the proposed algorithm achieves higher PSNR values and lower classification errors, proving it to be an efficient multi-level threshold image segmentation algorithm.

Key words: image segmentation; particle swarm optimization; fuzzy entropy; Shannon entropy; robustness; objective function

0 Introduction

Image segmentation is a frontier research area in computer vision and a very important preprocessing step in image analysis [1~3]. The goal of segmentation is to simplify or change the representation of an image into something more meaningful and easier to analyze. Over the years, this field has achieved numerous research results. Generally, current image segmentation methods can be summarized into four categories: region-based methods, boundary-based methods, clustering-based methods [4,5], and threshold-based methods [6,7]. Threshold technique is one of the most popular methods for segmenting various types of images. Image thresholding extracts objects in a scene from the background, which helps analyze and interpret images. Traditional multi-level thresholding methods are effective for bi-level thresholding because of their simplicity and robustness. However, due to the use of exhaustive search to find optimal thresholds, they incur substantial computational cost. In recent years, researchers have proposed many excellent threshold segmentation techniques [8], attempting to apply evolutionary algorithms and intelligent optimization methods to obtain optimal thresholds.

In the literature, various approaches have been explored. Reference [9] classifies images by calculating thresholds based on the class variance of pixel intensity [10]. This method belongs to bi-level threshold classification [11] and is effective for two thresholds, but its computational complexity becomes very high for multi-level thresholding. Reference [12] proposes a multi-level image threshold segmentation approach using tabu search algorithm to optimize fuzzy entropy. Reference [13] applies the Bacteria Foraging Algorithm (BFA) to optimize the objective function for efficient image segmentation, which further improves convergence speed and global search capability. This method also makes minor modifications to BFA by adaptively changing the step size of bacteria instead of using a fixed step size, and then performs standard bacteria foraging to segment brain magnetic resonance images [14]. Reference [15] combines fuzzy entropy with the Bat Algorithm (BA) and compares results with Artificial Bee Colony (ABC), Particle Swarm Optimization (PSO), and Genetic Algorithm (GA), demonstrating the effectiveness of the proposed approach. Reference [16] applies the Firefly Algorithm (FA) to multi-level image thresholding, which is simple and effective. However, when extended to multi-level thresholds, the computational load is significant because they perform exhaustive search for optimal thresholds.

Based on this, this paper establishes a multi-level image threshold segmentation based on particle swarm optimization algorithm, which performs image segmentation by maximizing Shannon entropy or fuzzy entropy, and compares the results with several existing threshold segmentation methods. For performance analysis of image thresholding based on particle swarm optimization, metrics including objective function, standard deviation, structural similarity index, peak signal-to-noise ratio, misclassification error, and computational complexity are considered. Overall, compared with several existing methods, the proposed algorithm achieves higher PSNR values and lower classification errors, demonstrating the effectiveness of the proposed approach.

1 Shannon Entropy and Fuzzy Entropy

Image thresholding is the process of converting a grayscale input image into a binary image using optimal thresholds. Thresholding may be local or global, but these methods are computationally intensive, so optimization techniques are needed to optimize the objective function, thereby reducing the computation time of local or global methods. Optimization techniques find optimal thresholds by maximizing the objective function, enabling the segmented image to clearly distinguish between background and foreground.

This method selects Shannon entropy and fuzzy entropy as the objective functions for the optimization technique. Assume an image containing L gray levels, with these gray levels ranging from 0 to $L-1$. Let N_i represent the number of pixels corresponding to gray level i; N represents the total number of pixels in the image equal to the sum of N_i.

1.1 Concept of Shannon Entropy

The concept of Shannon entropy states that the greater the uncertainty of a variable, the greater its entropy, and thus the greater the amount of information needed to understand the variable [17]. Let X be a discrete random variable with n elements $\{X_1, X_2, \dots, X_n\}$, then the probability mass function $P(X)$ is given. Let $I(X)$ represent the information content, which is also a random variable. Furthermore, Shannon entropy is rewritten as equation (2) and considered as the objective function to be optimized by the optimization technique.

$$H(X) = E[I(X)] = E[-\ln(P(X))]$$

$$H(X) = \sum_{i=1}^{n} P(x_i) I(x_i) = -\sum_{i=1}^{n} P(x_i) \log_b P(x_i)$$

where b in the algorithm is generally equal to 2. If $P(x_i) = 0$, then the multiplier is considered zero, which is consistent with the limit.

The equations are described for discrete values of X, and are adapted for continuous values of X by replacing summation with integration.

1.2 Concept of Fuzzy Entropy

Let D be the image domain defined as $D = \{(x, y) : x = 0, 1, 2, \dots, M-1; y = 0, 1, 2, \dots, N-1\}$, where M is the image width and N is the image height; L is the number of gray levels in the image.

This paper divides the domain of the original image into three regions (such as dark, gray, and bright), and assumes two thresholds T_1 and T_2. The dark region covers pixels with intensity values less than T_1, the gray region contains pixels with intensity between T_1 and T_2, and the bright region covers pixels with intensity greater than T_2.

Let p_k be the unknown probability partition of D, with its probability distribution given in reference [18]. μ_d, μ_m, and μ_b are the membership functions of dark, gray, and bright regions respectively, requiring six parameters: a_1, b_1, c_1, a_2, b_2, and c_2. Thresholds T_1 and T_2 are variable based on the membership functions. For each k, the membership functions are calculated as follows:

The conditional probabilities that a pixel with gray value k belongs to the dark class (E_d), gray class (E_m), and bright class (E_b) are $p_d(k), p_m(k)$, and $p_b(k)$ respectively, then the following equations hold:

$$p_d(k) = \frac{p_k}{D} \times p_d, \quad p_m(k) = \frac{p_k}{M} \times p_m, \quad p_b(k) = \frac{p_k}{B} \times p_b$$

The fuzzy membership functions $\mu_d(k), \mu_m(k)$, and $\mu_b(k)$ are calculated as:

$$\mu_d(k) = \begin{cases} \frac{k-a_1}{b_1-a_1} & \text{if } a_1 < k < b_1 \\ \frac{c_1-k}{c_1-b_1} & \text{if } b_1 < k < c_1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_m(k) = \begin{cases} \frac{k-a_1}{b_1-a_1} & \text{if } a_1 < k < b_1 \\ \frac{c_1-k}{c_1-b_1} & \text{if } b_1 < k < c_1 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_b(k) = \begin{cases} \frac{k-a_2}{b_2-a_2} & \text{if } a_2 < k < b_2 \\ \frac{c_2-k}{c_2-b_2} & \text{if } b_2 < k < c_2 \\ 0 & \text{otherwise} \end{cases}$$

where $0 \le a_1 < b_1 < c_1 \le 255$ and $0 \le a_2 < b_2 < c_2 \le 255$.

The fuzzy entropy functions for each class can be expressed as:

$$H_d = -\sum_{k=0}^{255} \frac{\mu_d(k)p_k}{p_d} \ln\left(\frac{\mu_d(k)p_k}{p_d}\right)$$

$$H_m = -\sum_{k=0}^{255} \frac{\mu_m(k)p_k}{p_m} \ln\left(\frac{\mu_m(k)p_k}{p_m}\right)$$

$$H_b = -\sum_{k=0}^{255} \frac{\mu_b(k)p_k}{p_b} \ln\left(\frac{\mu_b(k)p_k}{p_b}\right)$$

The total fuzzy entropy is calculated by summing the fuzzy entropy of each class:

$$H(a_1, b_1, c_1, a_2, b_2, c_2) = H_d + H_m + H_b$$

The above equation is an objective function to be optimized by the optimization technique. The optimization technique optimizes or maximizes the H function by changing the parameters. Once these values are optimized, the thresholds are then calculated using the following equations.

Research shows that two-level thresholding can be extended to three or more levels, or even limited to a single level. For two thresholds, the number of parameters to be optimized is six, and as the threshold level increases, the number of parameters to be optimized also increases, so fuzzy entropy requires considerable time to converge. Therefore, two-level image thresholding methods using Shannon entropy and fuzzy entropy have proven to be efficient and effective, but for multi-level thresholding, both entropy techniques require significant convergence time that grows exponentially with the threshold level. The drawback of Shannon entropy and fuzzy entropy is the convergence time. To further improve the performance of these methods and reduce convergence time, researchers have applied optimization techniques such as differential evolution, particle swarm algorithm, bat algorithm, and firefly algorithm to image thresholding and subsequent image segmentation.

2 Particle Swarm Optimization Algorithm

2.1 Overview of Particle Swarm Optimization Algorithm

Particle Swarm Optimization (PSO) is a commonly used evolutionary optimization technique that originated from studies on bird flock predation behavior. Its basic idea is to find the optimal solution through collaboration and information sharing among individuals in the population.

In PSO, the bird flock evolves into a swarm of particles without mass or volume, extending into N-dimensional space. In N-dimensional space, each particle's position is represented by vector $X_i = (x_1, x_2, \ldots, x_N)$, while its flight velocity is represented by vector $V_i = (v_1, v_2, \ldots, v_N)$. Each particle corresponds to a fitness value, which is set according to the objective function.

In this process, each particle knows the best position it has currently found ($pbest$) and its current position. Moreover, each particle also knows the best position found by all particles in the current swarm ($gbest$, which is the best value among all $pbest$). Each particle determines its movement direction based on its own position information combined with other particles' position information. PSO initialization creates a swarm of random particles, i.e., random solutions. During each iteration, particles continuously change their movement direction by analyzing the two extremes ($pbest$ and $gbest$).

Particles update their velocity v_i and position x_i in real-time using the following formulas to seek the optimal solution during iteration:

$$v_i = \omega \times v_i + c_1 \times rand() \times (pbest - x_i) + c_2 \times rand() \times (gbest - x_i)$$

$$x_i = x_i + v_i$$

where $rand()$ is a random number between 0 and 1; ω is the inertia factor; c_1 and c_2 are learning factors, typically set to $c_1 = c_2 = 2$.

2.2 Fuzzy Entropy Method Based on Particle Swarm Optimization Algorithm

This section demonstrates the process of performing image segmentation by maximizing Shannon entropy or fuzzy entropy through the particle swarm algorithm. The following illustrates the particle swarm optimization algorithm for image thresholding using Shannon entropy or fuzzy entropy.

Input: Initialize swarm size (N), maximum iterations (k), threshold level (Th). Initialize randomization parameters (c_1 and c_2), inertia factor (ω), and random numbers ($rand()$).

Output: Optimized thresholds and segmented image.

Procedure: 1. Initialize all required parameters and their corresponding dimensions and time $t = 0$. 2. Calculate the fitness value I_i for each solution X_i using the equation. 3. For each particle in the population: - Initialize particle velocity (i.e., swap sequence) - For each path in the population: - Calculate the distance of each path - Evaluate new solutions and update particle position and velocity 4. Compare the fitness values before and after updating, update the fitness value when they are equal, and return the best solution and corresponding segmented image with selected thresholds when maximum iterations are reached and the global final position satisfies the fuzzy entropy objective function.

3 Experiments and Results

3.1 Experimental Setup

Researchers selected Lena, Baboon, and Airplane from the image segmentation database [19,20] as test images for performance analysis (including robustness, efficiency, and convergence), as shown in [Figure 1: see original paper]. All images are in JPG format with a size of 225×225.

All experiments were conducted on an Intel Core i5 processor with 2 GB RAM using MATLAB 2014a. Generally, if an image's histogram has high, symmetric peaks that are well-separated, it is easy to segment. It is observed that the histogram peaks of Baboon and Airplane images are high and symmetric, while the histogram peak of the Lena image is not high, making it difficult to segment using ordinary methods. Therefore, this paper proposes an image thresholding method based on particle swarm optimization, which achieves efficient image segmentation of these key images by optimizing Shannon entropy and fuzzy entropy.

The maximum iteration count for the particle swarm optimization algorithm is 30. The total number of particles (solutions) is 10 times the number of thresholds (i.e., if $thresholds = 2$, then $total = 10 \times 2$). The performance of the PSO algorithm depends on two adjustment parameters: acceleration constants (c_1 and c_2) and inertia weight factor (ω). Typically, c_1 and c_2 are set to 2, with a fixed weight of 0.5.

3.2 Results Analysis

This section focuses on the visual clarity of segmented images with different threshold levels (e.g., $Th = 2, Th = 3, Th = 4$, and $Th = 5$) using Shannon entropy and fuzzy entropy, as well as algorithms from references [9], [12], [13], [15], and [16]. Segmented images with different threshold levels obtained by applying several different algorithms to three test images are shown in [Figure 2: see original paper].

From these figures, it can be seen that segmented images with a higher number of thresholds ($Th = 5$) have better visual quality compared to $Th = 4, Th = 3$, and

$Th = 2$. For the Lena image, when the number of thresholds is extended to 5, the background becomes distinguishable. Similar to Figure 2: see original paper where the Baboon image mixes with background objects, but as the number of thresholds increases to 5, the Baboon image becomes clearly distinguishable. Additionally, it can be observed that segmented images with different threshold levels obtained by the proposed method have superior visual quality compared to the contrast methods.

The proposed method selects Shannon entropy and fuzzy entropy as the objective functions for the optimization technique. Using fuzzy entropy as the objective function for optimization is a common approach in current research and performs better. All algorithms are optimized to maximize the objective function.

3.2.1 Stability Analysis The results of optimization techniques are essentially random because randomness is involved in the procedure itself, and the results are not unique for each run. Therefore, algorithm performance is verified through multiple runs using different initial values. If an algorithm's results are acceptable under the same conditions (i.e., no difference from one run to another), the algorithm is considered robust. Consequently, each algorithm was run 50 times, and the results were considered over 50 independent runs.

Algorithm stability is measured by mean and standard deviation. Generally, an optimization technique is considered better if it has a higher stability factor among all techniques, meaning the objective function value should be the same for each run. The mean and standard deviation [21] are calculated by equations (22) and (23):

$$\mu = \frac{1}{N}\sum_{j=1}^{N} \mu_j$$

$$\sigma = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(\mu_j - \mu)^2}$$

where μ_j is the objective function value/fitness value at the j-th run; N is the number of runs.

[Figure 3: see original paper] shows the comparison of standard deviations for several different segmentation algorithms. From the figure, it can be seen that all segmented images obtained by the proposed method have lower standard deviation values, indicating that the particle swarm optimization algorithm is stable and robust compared to other algorithms.

3.2.2 Computational Complexity Computational complexity is a measure of the convergence time of optimization techniques, which varies for different threshold numbers. The computational complexity of Shannon entropy and fuzzy entropy is $O(L^{Th})$, which grows exponentially with the number of thresholds (Th) and the number of gray levels (L). The convergence time of the proposed particle swarm optimization algorithm depends on image size and maximum iteration count.

[Figure 4: see original paper] lists the convergence time/computational complexity of the proposed method compared with other methods, as well as Shannon entropy and fuzzy entropy for different images with different threshold numbers. It is observed that the average convergence time of the proposed particle swarm optimization algorithm is less than that of other algorithms. Except for certain cases highlighted in [Figure 4: see original paper], the runtime for all images increases with the number of thresholds.

3.2.3 Peak Signal-to-Noise Ratio (PSNR) PSNR indicates the dissimilarity between the segmented image and the input image, serving as a measure of visual difference between two images, with units in decibels (dB). Higher PSNR values indicate better quality of segmented or reconstructed images. PSNR is given in equation (24):

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right)$$

where MSE is given by equation (25):

$$MSE = \frac{1}{M \times N}\sum_{i=1}^{M}\sum_{j=1}^{N}[f(i,j) - \hat{f}(i,j)]^2$$

where $M \times N$ is the image size; $f(i,j)$ and $\hat{f}(i,j)$ represent pixel values of the original and reconstructed images respectively. In this experiment, the researchers have obtained square images with $M = N = 225$.

[Figure 5: see original paper] shows the PSNR values obtained by different algorithms, where the proposed algorithm achieves higher PSNR values.

3.2.4 Misclassification Rate The misclassification rate is a measure of the uniformity of segmented images for comparing the performance of optimization techniques. The misclassification rate is given by equation (26):

$$\sigma = \frac{\sum_{j=1}^{Th}\sum_{i\in R_j}(I_i - \mu_j)^2}{(I_{\max} - I_{\min}) \times \sum_{j=1}^{Th} N_j}$$

where Th is the number of thresholds used for image segmentation; R_j is the j-th segmented region; I_i is the intensity level of pixels in that particular segmented region; μ_j is the mean of the j-th segmented region of the image; $I_{\max}$ and $I_{\min}$ are the maximum and minimum intensities of the image. Typically, the misclassification rate is between 0 and 1, with lower rates indicating better algorithm performance. Therefore, the uniformity measure is assessed as the difference between the maximum value 1 (better image quality) and the minimum value 0 (worse image quality).

[Figure 6: see original paper] shows the misclassification rates of the proposed method and other techniques, where the proposed method is proven to have smaller classification errors and achieves better visual quality.

4 Conclusion

Image segmentation is a very important preprocessing step in image analysis. Traditional multi-level thresholding methods are effective for bi-level thresholding due to their simplicity, robustness, and low convergence time and accuracy. However, because they use exhaustive search to find optimal thresholds, they require substantial computational cost, leading to more research applying evolutionary algorithms to obtain optimal thresholds. The main purpose of image segmentation is to separate the foreground from the background. This paper establishes a multi-level image threshold segmentation method based on particle swarm optimization.

Multi-level image threshold segmentation based on particle swarm optimization has effectively produced the desired output for image segmentation. The particle swarm optimization algorithm efficiently and effectively maximizes Shannon entropy and fuzzy entropy for image thresholding. The proposed algorithm was tested on natural images to demonstrate its advantages. The results of the proposed method were compared with other optimization techniques, using Shannon entropy and fuzzy entropy as objective functions.

It is observed that compared with contrast methods, the proposed algorithm has higher fitness values and higher PSNR values, thereby improving the quality of segmented images. It can be concluded that the proposed algorithm outperforms contrast methods across all performance measurement parameters.

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Note: Figure translations are in progress. See original paper for figures.