Enhancement of the Prediction Accuracy of Pole Coordinates with Empirical Mode Decomposition Postprint

Authors: Zhao Danning, Lei Yu, CaiHongbing

Date: 2018-05-28T00:00:00+00:00

Abstract

This paper is aimed at separation treatment of low- and high-frequency components in polar motion forecasting and then improving time-series predictions. For the purpose, the empirical mode decomposition (EMD) is employed as a filter to extract low- and high-frequency signals from original pole coordinate data. The decomposition of the pole motion observations between 1986 and 2015 from the International Earth Rotation and Reference Systems Service (IERS) C04 series illustrates that the low-frequency fluctuations including inter-decadal, inter-annual, Chandler and annual wobbles and shorter-period high-frequency oscillations can be separated from the observed time-series by the EMD. On the basis of separation, the least-squares (LS) extrapolation of models for annual and Chandler wobbles and for the linear trend is used for deterministic prediction of the low-frequency fluctuations, while the autoregressive (AR) technology is applied to forecasting the high-frequency oscillations plus LS fitting residuals. Pole coordinate forecasts are calculated as the sum of LS extrapolation and AR predictions (LS+AR). We have evaluated the accuracy of our long-term predictions (up to 1 year in the future) in comparison with the IERS official predictions in terms of year-by-year statistics of 5 years. It is shown that the accuracy of the LS+AR method can be significantly improved using a combination of the EMD and LS+AR (EMD+LS+AR). Also, the proposed prediction strategy overall outperforms the IERS solutions. In addition, the predictions are compared with those from the Earth Orientation Parameters Prediction Comparison Campaign (EOP PCC). The comparison demonstrates that the developed scheme is a very accurate approach to predict polar motion. According to this study, it is concluded that polar motion predictions may be enhanced through separation treatment of different time-scale fluctuations and thus such processing seems to be necessary in pole coordinate prediction.

Full Text

Enhancement of the Prediction Accuracy of Pole Coordinates with Empirical Mode Decomposition

Zhao Danning[1,2], Lei Yu[1,3], Cai Hongbing[1,3]

[1] National Time Service Center, Chinese Academy of Sciences, Xi' an 710600, China

[2] University of Chinese Academy of Sciences, Beijing 100049, China

[3] Key Laboratory of Time and Frequency Primary Standards, Chinese Academy of Sciences, Xi' an 710600, China

Abstract: This paper addresses the challenge of separating low- and high-frequency components in polar motion forecasting to improve time-series predictions. To this end, empirical mode decomposition (EMD) is employed as a filter to extract low- and high-frequency signals from original pole coordinate data. Decomposition of pole motion observations from the International Earth Rotation and Reference Systems Service (IERS) C04 series between 1986 and 2015 demonstrates that EMD can separate low-frequency fluctuations—including inter-decadal, inter-annual, Chandler and annual wobbles—from shorter-period high-frequency oscillations in the observed time-series. Based on this separation, least-squares (LS) extrapolation of models for annual and Chandler wobbles and linear trend is used for deterministic prediction of low-frequency fluctuations, while autoregressive (AR) technology is applied to forecast high-frequency oscillations plus LS fitting residuals. Pole coordinate forecasts are calculated as the sum of LS extrapolation and AR predictions (LS+AR). We evaluated the accuracy of our long-term predictions (up to one year ahead) against IERS official predictions through year-by-year statistics over five years. Results show that the accuracy of the LS+AR method can be significantly improved by combining it with EMD (EMD+LS+AR). The proposed prediction strategy overall outperforms IERS solutions. Additionally, predictions are compared with those from the Earth Orientation Parameters Prediction Comparison Campaign (EOP PCC), demonstrating that the developed scheme represents a highly accurate approach for polar motion prediction. This study concludes that polar motion predictions may be enhanced through separation of different time-scale fluctuations, suggesting such processing is necessary for pole coordinate prediction.

Keywords: Polar Motion; Prediction; Empirical Mode Decomposition; Least-squares (LS); Autoregressive (AR) Model

---

1. Introduction

The Earth orientation parameters (EOP)—comprising universal time, pole coordinates, and nutation-precession corrections—describe the irregularities of Earth' s rotation. Technically, these parameters provide the transformation between the International Terrestrial Reference System (ITRS) and International Celestial Reference System (ICRS) as a function of time. EOP derived from modern space geodetic technologies such as Very Long Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR), and Global Navigation Satellite System (GNSS) are not available for real-time applications due to complex data processing requirements. Since rapid and accurate EOP predictions are essential for various fields associated with reference frames—including interplanetary spacecraft tracking and navigation, positional astronomy, geodesy, and precise orbit determination of artificial Earth satellites—it is crucial to achieve high-accuracy EOP predictions at least several days into the future [?]. This paper focuses on predicting pole coordinates up to 365 days ahead.

Numerous approaches have been employed to improve polar motion prediction accuracy. Most methods utilize only information within the pole coordinate data itself, such as the combination of least-squares (LS) extrapolation and autoregressive (AR) model (LS+AR) [?], and the combination of LS extrapolation and neural network (LS+NN) [?]. The LS+AR combination has been reported as one of the most accurate techniques for pole coordinate forecasting [?].

A fundamental challenge in time-series forecasting is the necessity of separating high- and low-frequency variations [?]. This problem can be addressed by combining empirical mode decomposition (EMD) with the LS+AR model. Our work presents a hybrid EMD+LS+AR method for enhancing pole variation prediction. In this approach, low-frequency components—including long-term trend, Chandler and annual wobbles determined by EMD—are first fitted and predicted by LS extrapolation of linear polynomial and harmonic models. Subsequently, short-period and non-cyclic high-frequency components together with LS fitting residuals are modeled using AR techniques. The predicted high-frequency components and LS residuals are then added to the LS extrapolation to obtain the final predicted pole coordinate values. Comparison with existing prediction methods demonstrates that the EMD+LS+AR approach is a powerful tool for pole coordinate prediction.

2.1 Empirical Mode Decomposition

EMD decomposes a signal into a limited number of intrinsic mode functions (IMFs) whose instantaneous amplitude and frequency are physically meaningful. The method operates on the simple assumption that, at any given time, the data may contain many coexisting simple oscillatory modes of remarkably different frequencies, superimposed upon one another. Each component is defined as an IMF satisfying two conditions [?]: (1) Throughout the entire dataset, the number of zero crossings and extrema must either be equal or differ by at most one; (2) At any data point, the mean value of the envelope defined by local minima and the envelope defined by local maxima equals zero.

Based on this IMF definition, a signal can be decomposed into multiple IMFs through the following sifting process:

Step 1: Identify all local extrema in the time-series.

Step 2: Connect all local minima (maxima) with a cubic spline line to form the lower (upper) envelopes.

Step 3: Calculate the mean value of the lower and upper envelopes, $m(t)$, and compute the difference between the original data $x(t)$ and $m(t)$ as a proto-IMF, $h(t)$, i.e., $h(t) = x(t) - m(t)$.

Step 4: Examine whether $h(t)$ satisfies the IMF conditions. If it does, $h(t)$ can be regarded as an IMF component. If not, the sifting process is repeated, treating $h(t)$ as the data in the next iteration: $h_k(t) = h_{k-1}(t) - m_k(t)$. After k iterations, $h_k(t)$ becomes the first IMF, i.e., $c_1(t) = h_k(t)$.

Step 5: Compute the residual $r_1(t) = x(t) - c_1(t)$ and treat it as new data.

Step 6: Repeatedly apply the decomposition procedure to all subsequent residuals $r_j(t)$.

The procedure terminates when the residual becomes a monotonic function or a function with only one extremum from which no further IMF can be extracted. Through this process, the original data are decomposed into n IMFs and one residual $r_n(t)$, which can be either a constant or the adaptive trend, i.e., $x(t) = \sum_{i=1}^{n} c_i(t) + r_n(t)$.

2.2 Least-squares Extrapolation

Based on EMD-separated low-frequency components of pole variation data, an LS model may be fitted. Since the sampling interval of the International Earth Rotation and Reference Systems Service (IERS) EOP C04 series is one solar day, we employ the following polynomial-sinusoidal function for LS fitting and extrapolation:

$$x(t) = a + bt + \sum_{i=1}^{2} [A_i \cos(2\pi f_i t) + B_i \sin(2\pi f_i t)]$$

where the bias a and drift b of the linear trend, and amplitudes A_i, B_i and phases ϕ_i of the annual and Chandler wobbles must be estimated by the LS method using the separated low-frequency fluctuation data, with $f_1 = 1/365.25$ cycles/day and $f_2 = 1/433$ cycles/day.

The model is applied to derive LS fitting residuals $r_{LS}(t) = x_{obs}(t) - x_{LS}(t)$ for all data used to fit the LS model. In the prediction process, $x_{LS}(t)$ is extrapolated to obtain a deterministic prediction of the low-frequency fluctuations.

2.3 Autoregressive Model

Stationary time-series may be modeled by an AR model of order p [AR(p)]. This technique captures temporal dependence within stationary time-series. A stationary, zero-mean stochastic process x_t (in our case the residuals) is said to be AR(p) if it satisfies:

$$x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \varepsilon_t$$

where ε_t is white noise with zero mean and variance σ^2, and ϕ_i are the AR coefficients.

The order p can be selected using the Akaike Information Criterion (AIC) [**?**]. The AIC for an AR(p) is based on the statistic:

$$\text{AIC}(p) = -2 \ln L(\hat{\phi}_p, \hat{\sigma}_p^2) + 2(p+1)$$

where L is the likelihood function, $\hat{\phi}_p$ is the vector of AR coefficients for AR(p), and $\hat{\sigma}_p^2$ is the estimated variance. The order p that minimizes the AIC statistic is selected. Several approaches exist for estimating AR coefficients; we employ the Yule-Walker procedure.

The fitted AR model is applied to forecast LS fitting residuals plus high-frequency oscillations of polar motion using linear prediction operators. Forecasts for multiple steps ahead are computed recursively.

2.4 Prediction Strategy

For prediction of x_p and y_p pole coordinates, an integrated prediction strategy referred to as EMD+LS+AR is applied as described in the flowchart of Figure 1. The strategy comprises the following stages: In the first stage, low-frequency components–including annual wobble, Chandler wobble, and long-term trend –and high-frequency signals containing shorter-period and non-cyclic fluctuations are extracted from original data using the EMD algorithm as described in Section 2.2. In the second stage, the extracted low-frequency oscillations are fitted and extrapolated by the LS model. Next, the AR technique models the LS fitting residuals plus separated high-frequency components of pole variations, i.e., $r_{LS}(t) + x_{HF}(t)$. The subsequently forecasted values are then added to the LS extrapolation to obtain final pole coordinate predictions.

[Figure 1: see original paper]

3.1 Data Description

The IERS publishes daily EOP values with a latency of several days after processing observations from all advanced space geodetic techniques including GNSS, VLBI, and SLR gathered at permanent tracking stations worldwide. The EOP are estimated together with station coordinates and velocities from each technique. The subsequently estimated EOP from all techniques are then combined to derive the EOP C04 solutions [?]. These combined solutions are regularly published on the official IERS website. In this work, x_p and y_p pole coordinate data from the IERS EOP 08 C04 series serve as the basis for numerical experiments. The current accuracy of x_p and y_p pole coordinates is approximately 30 as for the 08 C04 solutions.

3.2 Separation of Low- and High-Frequency Oscillations in Pole Coordinates

Taking the x_p pole coordinate as an example, Figure 2 shows the decomposed results of 30-year pole variations from January 1, 1986 to December 31, 2015. The pole coordinate can be decomposed into a limited number of IMFs using the EMD algorithm, indicating that oscillations of different time-scales in polar motion are characterized by a small number of IMFs. As shown in Figure 2, two modes—c_9 and c_{10}—exhibit noticeable seasonal oscillations in the range of a few hundred milliarcseconds, representing the two dominant oscillations along the x_p and y_p directions: annual wobble and Chandler wobble. Additionally, IMFs c_6, c_7, c_8 and residuals r_{10} from EMD analysis can be identified as inter-annual and inter-decadal signals and the long-term "natural"trend in observations, while modes c_1, c_2, c_3, c_4, and c_5 with smaller amplitudes represent shorter-period oscillations and very high-frequency non-wobbling components. It is noteworthy that due to mode mixing in EMD, component separation may be difficult when signals contain adjacent periods, as occurs for Chandler and annual wobbles. However, since we focus only on separating low- and high-frequency oscillations rather than extracting a particular component, this mode mixing problem does not limit our analysis.

A spectral analysis of IMFs c_9 plus c_{10} based on fast Fourier transform (FFT) is exhibited in Figure 3. For comparison, a similar analysis of original data is also provided [Figure 3: see original paper]. Visually, c_9 plus c_{10} indeed contains the two dominant oscillating modes—Chandler and annual wobbles—with the same periods and amplitudes as the original observations. These findings confirm that oscillatory features hidden in polar motion can be visually characterized by specific IMFs with particular physical meaning. To extract high-frequency components of pole coordinates, IMFs c_1 through c_5 are reconstructed, while other modes are summed to obtain low-frequency oscillations including long-term trend, Chandler and annual wobbles. Figure 4 depicts the separated low-frequency time-series, high-frequency time-series of x_p and y_p pole coordinates, and the raw data. These plots suggest that low-frequency components can be described by a low-order polynomial and harmonic model, while the AR technique is suitable for modeling high-frequency terms.

[Figure 2: see original paper]
[Figure 3: see original paper]
[Figure 4: see original paper]

3.3 Prediction Results

The EOP 08 C04 series serves as the data basis with a base data length of 10 years. Predictions of x_p and y_p pole coordinates are generated using two schemes: LS+AR and EMD+LS+AR. All predictions have a length of 365 days. To compare with official predictions from IERS Bulletin A—which contains EOP predictions for one year ahead at daily intervals and is issued by the IERS Rapid Service/Prediction Centre at the U.S. Naval Observatory (USNO)—all 365-day predictions are calculated weekly at different starting epochs from January 7, 2011 to December 31, 2015. Comparison of polar motion predictions is carried out through: (1) absolute differences between EOP 08 C04 pole coordinate data and their forecasts, and (2) mean absolute error (MAE) defined by [**?**]:

$$\mathrm{MAE}(k) = \frac{1}{N}\sum_{i=1}^{N} |\hat{x}_i(k) - x_i(k)|$$

where t_0 is the starting prediction epoch, N is the number of k-step predictions, $\hat{x}_i(k)$ is the k-step prediction of polar motion, and $x_i(k)$ is the corresponding observation from the IERS EOP C04 series. In our analysis, $N = 52$.

The maximum absolute differences between pole coordinate data and LS+AR predictions calculated at different starting epochs exceed 98 mas, with larger differences occurring for forecasts with prediction lengths greater than 5 months (Figure 5). For IERS Bulletin A predictions, maximum differences do not exceed 91 mas, which are smaller than those from our LS+AR scheme (Figure 6). For the tested data span, the EMD+LS+AR combination produces polar motion predictions with maximum absolute differences less than 87 mas, with extreme perturbations significantly smaller than both LS+AR and IERS solutions (Figure 7).

Representative MAE statistics and corresponding error bars for our predictions compared with IERS official predictions are shown year-by-year in Figure 8. Results indicate that LS+AR accuracy can be remarkably improved by incorporating EMD, especially for long-term predictions. Compared with IERS predictions, our EMD+LS+AR results appear comparable for near-term prediction but noticeably outperform IERS solutions for long-term forecasts, except for individual prediction days, as clearly shown in Figure 8.

To further demonstrate the effectiveness of the developed EMD+LS+AR strategy, results are also compared with those from the EOP PCC (October 1, 2005 to February 28, 2008), where prediction period and validation scheme were clearly specified in advance. Under the same rules and conditions, comparison with contemporary methods from EOP PCC is shown in Figure 9, using identical data time spans, EOP 05 C04 series as reference, and MAE as the statistical measure. A list of participants and prediction methods can be found in [**?**].

The comparison reveals that for 1–10 day predictions, the developed method is among the most accurate for forecasting polar motion, namely LS extrapolation of harmonic model and AR prediction (Figures 9(a) and 9(b)). For short-term prediction (up to 30 days), the method' s accuracy is comparable with or slightly worse than the three most accurate techniques (Figures 9(c) and 9(d)). For long-term forecast (up to one year), predictions are inferior to the most accurate results, particularly for y_p data, but significantly more accurate than predictions from other participants (Figures 9(e) and 9(f)).

[Figure 5: see original paper]
[Figure 6: see original paper]
[Figure 7: see original paper]
[Figure 8: see original paper]
[Figure 9: see original paper]

4. Summary and Conclusion

Our investigations show that pole coordinate time-series decompose into a small number of IMFs representing different time-scale fluctuations such as seasonal, inter-annual, and inter-decadal variations. Therefore, EMD can be considered a filter for extracting specific oscillatory signals in polar motion. For separating different components in time-series prediction, EMD is used as a filtering method to separate low- and high-frequency components of pole variations. Results indicate that both shorter-period high-frequency variations and low-frequency oscillations—including long-term trend, Chandler and annual wobbles—can be extracted through reconstruction of partial IMFs. Based on these findings, LS extrapolation of low-order polynomial and harmonic models and AR technology are recommended for predicting low-frequency and high-frequency components, respectively. Polar motion predictions are generated by combining LS extrapolation and AR prediction.

To evaluate the proposed strategy' s accuracy, we calculated EMD+LS+AR and LS+AR predictions year-by-year at different starting epochs between January 7, 2011 and December 31, 2015, comparing them with IERS official forecasts. MAE figures and absolute differences between pole coordinate data and predictions illustrate that the hybrid approach provides more accurate predictions than conventional LS+AR, implying that LS+AR performance can be enhanced by involving EMD. Our results are also better than IERS predictions, except for individual days. Additionally, comparison with EOP PCC forecasts confirms that EMD+LS+AR is a powerful tool for predicting pole coordinate data.

The main conclusion is that prediction error decreases when EMD is incorporated into the LS+AR combination. Furthermore, separation treatment of different time-scale oscillations may enhance polar motion predictions, suggesting such processing is necessary in time-series prediction.

References

[1] Gambis D, Luzum B. Earth rotation monitoring, UT1 determination and prediction [J]. Metrologia, 2011, 48(4): 65-170.
[2] Xu X Q, Zhou Y H. EOP prediction using least square fitting and autoregressive filter over optimized data intervals [J]. Advances in Space Research, 2015, 56(10): 2248-2253.
[3] Lei Yu, Cai Hongbing. Enhancing the prediction accuracy of the length of day change by eliminating the edge-effect of least squares fitting [J]. Astronomy Research and Technology, 2016, 13(4): 441-445.
[4] Liao D C, Wang Q J, Zhou Y H, et al. Long-term prediction of the Earth orientation parameters by the artificial neural network technique[J]. Journal of Geodynamics, 2012, 62: 87-92.
[5] Lei Yu, Cai Hongbing, Zhao Danning. Effects of training patterns on predictions of variations of length of day using an extreme learning machine neural network [J]. Astronomy Research and Technology, 2015, 12(3): 299-305.
[6] Kalarus M, Schuh H, Kosek W, et al. Achievements of the Earth orientation parameters prediction comparison campaign [J]. Journal of Geodynamics, 2010, 84(10): 587-596.
[7] Kosek W. Future improvements in EOP prediction[J]. Geodesy for Plant Earth, International Association of Geodesy Symposia, 2011, 136: 513-520.
[8] Huang N E, Wu Z H. A review on Hibert-Huang transform: method and its application to geophysical studies [J]. Reviews of Geophysics, 2018, 46(2): 1-23.
[9] Akaike H. Autoregressive model fitting for control [J]. Annals of the Institute of Statistical Mathematics, 1971, 23: 163-180.
[10] Bizouard C, Gambis D. The combined solution C04 for Earth orientation parameters consistent with international terrestrial reference frame 2005[J]. International Association of Geodesy Symposia, 2009, 134: 265-270.

Note: Figure translations are in progress. See original paper for figures.