Postprint of Multi-objective Particle Swarm Optimization Algorithm Based on Rotating Basis Technique

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Date: 2018-05-24T00:00:00+00:00

Abstract

To address the problem of solving multi-objective optimization, a multi-objective particle swarm optimization algorithm based on rotating basis technique (rt-MOPSO) is proposed. First, the rotating basis visualization technique is improved and the Pareto front is mapped onto an improved rotating basis sector plane, with a differential entropy indicator employed to monitor the population evolution state; second, to balance the convergence and diversity of the archive set, two novel concepts of angle dominance and angle dominance strength are proposed and a new sorting method for the archive set is designed; finally, based on the fusion of rotating basis angle and distance concepts, an improved selection strategy for global guide particles is proposed. The improved algorithm adopts two categories of test functions and conducts comparative experiments with five multi-objective optimization algorithms. Experimental results demonstrate that the improved algorithm exhibits significant advantages in terms of convergence and diversity.

Full Text

Preamble

Title: Multi-objective Particle Swarm Algorithm Based on Rotation Basis

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Abstract: This paper proposes a multi-objective particle swarm algorithm based on rotation basis (rtMOPSO) to solve multi-objective optimization problems. Firstly, the rotation basis visualization technique is improved and the Pareto front is mapped onto an enhanced rotation basis sectorial plane, with differential entropy indicators used to monitor population evolution status. Secondly, to balance the convergence and diversity of the archive, two novel concepts—angle dominance and strength of angle dominance—are proposed, and a new sorting method for the archive is designed. Finally, an improved global guide particle selection strategy is proposed based on the fusion of rotation basis angle and distance concepts. The improved algorithm employs two categories of test functions and conducts comparative experiments with five multi-objective optimization algorithms. Experimental results demonstrate that the improved algorithm exhibits significant advantages in both convergence and diversity.

Keywords: particle swarm; multi-objective optimization; rotation basis technique

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0 Introduction

In recent years, multi-objective optimization problems have received widespread attention in academia and industry, yielding numerous research achievements. However, when the dimensionality of multi-objective optimization problems exceeds three, accurately presenting the distribution of the Pareto front to implement corresponding control strategies for obtaining optimal Pareto solution sets becomes extremely challenging. Visualization technology serves as an important means for solving multi-objective optimization problems, primarily employing dimensionality reduction algorithms to map the Pareto front. These methods can be categorized into lossless and lossy dimensionality reduction. Lossless methods, which do not damage data information during dimensionality reduction, mainly include parallel coordinates and rotation basis coordinates. Lossy methods first extract features and relationships from high-dimensional data before mapping them to low-dimensional space, primarily including principal component analysis, Gaussian stochastic process models, and clustering techniques.

While particle swarm optimization can achieve rapid convergence, it is prone to premature convergence and local optima. Although measures can be taken to avoid population precocity, they inevitably affect the diversity of the population in later stages. Therefore, effectively balancing local convergence and population diversity requires urgent solutions. To this end, further improvements in archive management and maintenance, global guide particle selection, and population evolution state monitoring have become critical for enhancing particle swarm convergence and diversity.

Currently, in archive management, scholars have proposed various maintenance strategies that fall into two main categories. The first category involves pruning the archive based on population density, using methods such as adaptive grids, crowding distance, and -dominance. The second category organizes the archive according to the distance between archive particles and reference points or regions, including preference algorithms based on reference regions and reference points. In global guide particle selection, numerous high-performance algorithms and improvements have been proposed, including sigma methods and their variants, grey relational analysis, information entropy methods, differential algorithms, dynamic weighting, and roulette wheel selection. These algorithms can obtain globally optimal particles, thereby ensuring the correct flight direction of the population and improving convergence and distribution. Additionally, various balance algorithms adjust convergence and diversity according to population evolution status, such as dynamically adjusting inertia weights and learning factors, adaptive population size algorithms, population crossover and mutation algorithms, and multi-level information interaction algorithms. However, these algorithms cannot accurately monitor population evolution status, thus failing to clearly distinguish evolutionary states and effectively execute evolution strategies.

To monitor population evolution status, researchers have proposed concepts of state entropy and differential entropy, analyzing and identifying changes in archive state entropy to determine boundary thresholds for particle swarm evolution states. However, these approaches use density-based sorting methods to maintain the archive, which cannot simultaneously satisfy both convergence and diversity maintenance requirements. Comprehensive analysis reveals that existing methods have limitations in archive maintenance and global guide particle selection. This paper combines parallel coordinates with the rotation basis technique to improve rotation basis mapping strategies for high-dimensional to two-dimensional space mapping, enabling accurate analysis and decision-making for Pareto solution sets. Furthermore, novel concepts of rotation basis angle dominance and angle dominance strength are proposed for designing a dual-sorting algorithm for the archive. Additionally, rotation basis angle and distance are incorporated as selection indicators for global optimal particles, enabling the acquisition of particles with the strongest comprehensive performance as global optima. Finally, the Pareto entropy-based state monitoring method is improved by monitoring population status to select different archive sorting methods, achieving comprehensive dynamic management of the archive and improving population convergence and distribution.

1.1 Multi-objective Optimization Problem

A multi-objective problem can be described as follows:

$$\min f(X) = \{f_1(X), f_2(X), \dots, f_m(X)\}$$

subject to:

$$\begin{cases} g_i(x) \geq 0, & i = 1, 2, \dots, p \\ h_j(x) = 0, & j = 1, 2, \dots, q \end{cases}$$

where X is the decision variable in $\mathbb{R}^n$ space; $g_i(x)$ and $h_j(x)$ represent inequality and equality constraints, respectively.

Multi-objective optimization problems generally employ Pareto solution sets as optimization results. A Pareto solution set typically contains multiple non-dominated solutions, from which appropriate solutions can be selected as optimal solutions based on decision-making requirements. The ideal state for a Pareto solution set is uniform distribution along the Pareto front and infinite proximity to true values.

1.2 Basic Principles of Particle Swarm Algorithm

Particle swarm optimization is a population-based bionic evolutionary algorithm. Let x represent particle position and v represent particle velocity, with their relationship described as:

$$\begin{cases} v_i^{t+1} = wv_i^t + c_1 r_1 (p_i - x_i^t) + c_2 r_2 (g - x_i^t) \\ x_i^{t+1} = x_i^t + v_i^{t+1} \end{cases}$$

where p_i and g represent individual best values and global best values, respectively; w is the inertia weight; c_1 and c_2 are learning factors; r_1 and r_2 are random numbers between [0, 1]; t denotes the iteration number; and D represents particle dimensionality.

2.1 Improvement of Rotation Basis Technique

Multidimensional data visualization belongs to the research field of information visualization, primarily applied in data mining, networks, social media, and text visualization. Rotation basis technology is an important visualization technique that can present multidimensional data directly to users or decision-makers to assist in decision-making. However, this algorithm is not directly suitable for monitoring population evolution status. Therefore, this paper improves rotation basis and applies it to multi-objective particle swarm optimization.

The fundamental principle of rotation basis visualization involves using the rotation angle of basis vectors to represent Pareto solution values across each objective dimension, then superimposing basis vectors from each objective to form a polyline on a 90° sector plane. The rotation angle of plotted points indicates solution performance, while the length or distance from the endpoint vector to the origin represents performance fluctuation. The standardized rotation angle formula is:

$$\theta_{ki} = \frac{f_{ki} - f_m}{f_M - f_m} \times \frac{\pi}{2}$$

where i represents the number of objectives, $i = 1, 2, \dots, M$; k denotes particle index in the population, $k = 1, 2, \dots, K$; and f_m, f_M are the minimum and maximum values of the fitness function.

The rotation basis vector connection process proceeds as follows: First, establish a set of fixed-length path vector bases $\{w_i, i = 1, 2, \dots, M\}$, where the number of vectors equals the number of objective dimensions. Second, starting from the origin and moving horizontally, rotate counterclockwise by angle θ_{k1} to obtain the mapping coordinates for the first objective of the Pareto solution. Then, using this point as the starting coordinate, draw the mapping coordinates for the next objective in the same manner. This process continues iteratively to complete mapping coordinate drawing for the M-th objective.

Analysis reveals that rotation basis visualization technology can represent the quality of Pareto solutions but does not employ metrics like density to display the distribution status of all particles on the Pareto front. Therefore, this paper combines rotation basis with parallel coordinates to improve the rotation basis technique. Specifically: according to the number of population particles, the rotation basis 90° sector region is divided into K sectors, each with a central angle of $\pi/2K$. Then, adopting the parallel coordinates concept, M vertical axes are established for objective dimensions, with value ranges on each axis uniformly distributed from the minimum to maximum of the objective fitness function. To integrate rotation basis into parallel coordinates, the M vertical axes of parallel coordinates are transformed into M concentric circular arcs within the rotation basis 90° sector, forming $K \times M$ sector blocks. The rotation basis plane mapped through parallel coordinates is called the rotation basis sector plane, with the sector plane divided by concentric circular arcs referred to as rotation basis sector blocks. For example, with $K = 3$ and $M = 4$, a total of 12 sector blocks are formed, as shown in [Figure 1: see original paper].

Thus, the high-dimensional Pareto front is mapped onto the rotation basis sector plane, with population particles simultaneously mapped onto rotation basis sector blocks. By analyzing the number of particles within sector blocks, the distribution status of Pareto front particles can be obtained. To calculate the sector block location for each particle' s objective component, the following formula is used. For the i-th objective component of the k-th particle mapped to sector block number H_{ki} in the rotation basis sector plane:

$$H_{ki} = \left\lceil \frac{K \cdot \theta_{ki}}{\pi/2} \right\rceil$$

where $\lceil \cdot \rceil$ represents the ceiling function; H_{ki} is an integer in [1, K]; if θ_{ki} is 0, H_{ki} can be set to 1. For example, assuming a multi-objective optimization problem with 4 objectives and 3 particles, each particle index can be calculated according to formula (4). The distribution is illustrated in [Figure 2: see original paper].

2.2.1 State Entropy

This paper employs Pareto entropy and differential entropy to monitor the evolution status of the Pareto front. Pareto entropy and differential entropy can accurately describe the distribution of population particles on the rotation basis sector plane.

1) State entropy formula:

$$E(t) = -\sum_{i=1}^{M}\sum_{H=1}^{K}\frac{\mathrm{Sector}_H^i(t)}{KM}\log\frac{\mathrm{Sector}_H^i(t)}{KM}$$

where $\mathrm{Sector}_H^i(t)$ represents the number of coordinate components falling within the sector block at row H and column i.

2) Differential entropy formula:

$$\Delta E(t) = E(t) - E(t-1)$$

2.2.2 Determination of Evolution State Thresholds

During particle swarm evolution, the population experiences three states: convergence state, diversity state, and stagnation state. The diversity state serves as an intermediate state between convergence and stagnation. Therefore, by determining only the maximum boundary threshold α and minimum threshold β for the intermediate state' s differential entropy, the evolution states can be identified. To this end, an extreme case of population distribution is established: the Pareto optimal solution set contains 3 particles with 3 objective components, where particle k_3 has two components in each of its sectors, as shown in [Figure 3: see original paper].

When the Pareto front iterates and updates, the first extreme case occurs when old particle k_3 is replaced by updated particle k_3 and all components fall into 3 empty sector blocks, as shown in [Figure 4: see original paper]. Another case occurs when the updated particle k_3 falls into only 1 empty sector block. For particles in the archive that have been updated, their differential entropy changes, while other unchanged particles maintain constant differential entropy values. Therefore, by calculating the entropy difference before and after updates for changed particles, the archive' s differential entropy change can be computed, as shown in equations (7) and (8).

Let Q represent the current number of particles in archive A. Therefore, when $|\Delta E| > \alpha$ or $Q(t) \neq Q(t-1)$, the population is in a convergence state; when $A = K$ and $\beta < |\Delta E| < \alpha$, it is in a diversity state; when $A = K$ and $|\Delta E| < \beta$, the population is in a stagnation state.

3.1 Archive Maintenance and Management

All non-dominated solutions of the particle swarm are generally stored in the archive. An efficient archive maintenance strategy ensures the convergence and diversity of the archive. Currently, most archives employ single management strategies that struggle to balance both convergence and diversity metrics. For example, archives often use particle density as a sorting metric, which focuses on controlling selection pressure, while adaptive grid strategies emphasize distribution control. In reality, archive maintenance strategies should adaptively switch according to population evolution status: when the population is in a convergence state, convergence-strong particles should be retained to maintain convergence speed; when in a diversity state, high-density particles should be removed to maintain archive diversity. Therefore, this paper adopts a dual-sorting maintenance strategy that selects different sorting strategies at different evolution stages to balance selection pressure and distribution. This paper proposes convergence degree indicators and distribution degree indicators as the basis for dual-sorting.

3.1.1 Convergence Degree Indicator

To measure population convergence degree, this paper proposes concepts of angle dominance and angle dominance strength, using angle dominance strength as the convergence degree indicator.

1) Angle dominance concept: For any two solutions x and y in the archive, with rotation basis angles $\theta_{x,i}$ and $\theta_{y,i}$ respectively, x is said to angle-dominate y if and only if:

$$\forall i = 1, 2, \dots, M : \theta_{x,i} \leq \theta_{y,i} \quad \text{and} \quad \exists j = 1, 2, \dots, M : \theta_{x,j} < \theta_{y,j}$$

2) Angle dominance strength concept: For any solution $x \in A$ in the archive, its dominance strength S_i is the number of solutions it angle-dominates:

$$S_i = \{u \mid x \in A, x \prec_{\text{angle}} u\}$$

Angle dominance strength indicates the degree to which particles in the archive converge toward the Pareto front. Greater angle dominance strength suggests the solution is closer to the true Pareto front.

3.1.2 Distribution Degree Indicator

The distribution degree indicator F_i reflects the uniformity of the non-dominated solution set' s corresponding front. A smaller value indicates better distribution performance:

$$F_i = \frac{1}{K}\sum_{i=1}^{K}\left|d_i - \bar{d}\right|$$

where K is the total number of solutions in the Pareto solution set; d_i represents the distance from a non-dominated solution' s objective vector to the optimal solution' s objective vector; and $\bar{d}$ is the average of d_i.

3.1.3 Rotation Basis Polyline Distance

When sorting archive particles, if particles have identical S_i and F_i indicators, rotation basis mapping polyline distance is used as an auxiliary judgment metric. The rotation basis polyline distance formula is:

$$L_k = \frac{1}{M}\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M}\sqrt{1+(\cos\varphi)^2 - 2\cos\varphi\cos(\theta_{ki}-\theta_{kj})}$$

where:

$$\varphi = \arctan\left(\frac{\sum_{i=1}^{M}\sin\theta_{ki}}{\sum_{i=1}^{M}\cos\theta_{ki}}\right)$$

A larger rotation basis polyline distance indicates smaller performance fluctuation across objectives and more balanced performance.

3.1.4 Archive Sorting and Maintenance

Based on evolution status: if the population is in the convergence phase, particles are first sorted by S_i, F_i, and L_k in sequence, with S_i in ascending order and F_i and L_k in descending order; if in the diversity phase, archive particles are sorted by F_i, S_i, and L_k in sequence, with S_i in ascending order and F_i and L_k in descending order. Finally, according to evolution status and archive size, excess particles with lower rankings are removed from the archive.

3.2 Global Optimal Particle Selection

The particle swarm' s global optimal particle determines the population' s evolution direction and influences selection pressure and diversity. Therefore, to select the particle with strongest comprehensive performance as the global optimal particle, an effective and reasonable selection strategy must be established. This paper leverages the advantages of rotation basis visualization technology, using rotation angle and rotation basis polyline distance as the basis for global optimal particle selection. The smaller the rotation angle and the larger the rotation basis polyline distance, the better the particle' s multi-objective comprehensive performance:

$$\text{GlobalBest} = \arg\min_{k\in A} (\theta_k, -L_k)$$

The general selection process for global optimal particles is as follows: first, sort by rotation angle and select the particle with the smallest rotation angle as the global optimal particle; if multiple particles have identical rotation angles, compare their rotation basis polyline distances and select the particle with the longest distance as the global optimal particle.

4.1 Algorithm Flow

a) Population parameter initialization. Main parameters include archive size K, population size n, maximum iterations $t_{\max}$, weight w, and learning factors c_1, $c_2. Archive particle count k and iteration variable t are initialized to zero.

b) Evolution status judgment. Map archive particles to the rotation basis sector plane using equations (3) and (4); calculate state entropy and differential entropy using equations (7) and (8); finally, determine evolution status according to the thresholds described in Section 2.2.

c) Archive management and maintenance. Based on evolution status, sort archive particles according to the rules in Section 3.1 and prune excess particles.

d) Update global optimal particle using the strategy in Section 3.2.

e) Update particle swarm population using equation (2).

f) If maximum iterations are reached, stop and output the optimal solution set; otherwise, return to step b).

4.2 Algorithm Time Complexity Analysis

In the rtMOPSO algorithm, for computational convenience, particle swarm population and archive sizes are both set to N, the number of objectives is M, and particle dimensionality is D. Based on the algorithm flow, the time complexity of major steps is analyzed as follows:

- Step 1: Initialization complexity is $O(ND)$.

- Step 2: Rotation basis sector mapping is a linear operation; state entropy and differential entropy computation is $O(NM)$.
- Step 3: First, selecting effective particles from the population into the archive has complexity $O(MN^2)$. Second, executing the maintenance strategy requires calculating convergence degree, distribution degree, and auxiliary judgment metrics with complexity $O(NM^2)$. Multi-indicator sorting complexity is $O(N \log N)$.
- Step 5: Particle position and velocity updates are linear operations.

In summary, the maximum time complexity of the proposed algorithm is $O(MN^2)$. Therefore, this algorithm has the same time complexity as the algorithm in reference [8] and other multi-objective particle swarm optimization algorithms primarily using Pareto dominance and NSGA fast sorting methods.

5.1 Test Algorithms and Comparison Algorithm Selection

To evaluate the performance of the improved rtMOPSO algorithm, this paper selects six functions from the ZDT and DTLZ series as test suites, including ZDT1, ZDT2, ZDT3 and DTLZ1, DTLZ2, DTLZ6. The ZDT series tests the algorithm' s ability to optimize convex, non-convex, and discontinuous two-dimensional Pareto fronts, while the DTLZ series tests multi-objective, multi-variable function optimization capabilities. Comparison algorithms include two multi-objective particle swarm algorithms (dMOPSO and SMPSO) and three typical evolutionary algorithms (NSGA-II, SPEA2, and MOEA/D).

In comparative experiments, to ensure objectivity and fairness, all test functions use consistent parameter settings: particle population size of 100, archive size of 100, maximum evaluation count of 30,000; objective dimensionality of 2 for ZDT series and 3 for DTLZ series; crossover rate of 0.9 and mutation probability of 0.1 for evolutionary algorithms.

5.2 Evaluation Methods

To evaluate the convergence and diversity of comparison algorithms, three metrics are selected:

a) **Generational Distance (GD)** measures how close the obtained Pareto solution set is to the true front. Smaller values indicate better proximity to the true front.

b) **Spread** metric primarily indicates the uniformity of solution set distribution. Smaller values indicate better distribution.

c) **Hypervolume (HV)** measures the objective space region covered by the solution set and simultaneously evaluates both convergence and diversity performance. Larger HV values indicate better comprehensive performance.

5.3 Experimental Results and Analysis

Experimental results are presented in tables for clarity, as shown in Tables 2 and 3. Table 2 shows results from 30 runs of rtMOPSO and various comparison algorithms. According to Table 2, rtMOPSO outperforms other comparison algorithms in both GD and Spread metrics for two-dimensional objective tests, demonstrating excellent convergence and diversity for 2D problems. For three-dimensional objective problems, the proposed algorithm generally shows outstanding convergence, with only MOEA/D approaching its performance on DTLZ1 and DTLZ2, while its performance on DTLZ6 is comparable to SPEA2. Experimental data indicate that while most algorithms show slight deficiencies in DTLZ1 tests, the proposed algorithm has no obvious flaws in 2D and 3D objective tests.

To further demonstrate effectiveness, the HV metric is used for comprehensive performance comparison. As shown in Table 2, although SPEA2 and MOEA/D are relatively excellent algorithms with good comprehensive performance, rtMOPSO performs better in HV, indicating the proposed algorithm can better cover the Pareto front and obtain optimal solution sets.

To visually and clearly illustrate rtMOPSO' s superiority, Table 3 presents simulation curves of rtMOPSO and partial comparison algorithms. According to Table 3, other algorithms exhibit varying degrees of deficiency across the three performance metrics. For instance, dMOPSO and SMPSO show slightly insufficient performance on 3D objectives, particularly poor distribution on DTLZ1. The NSGA-II algorithm performs poorly on ZDT3 and DTLZ2 tests. The solution sets obtained by the proposed algorithm are very close to true solutions on both 2D ZDT true front curves and 3D DTLZ true front surfaces. Therefore, comprehensive consideration demonstrates that the proposed algorithm outperforms the five comparison algorithms.

The primary reasons for rtMOPSO's significant advantages are: (1) the rotation basis visualization technology facilitates decision-makers in selecting optimal particles, improving archive management quality and ensuring selection of global optimal particles; (2) the evolution state monitoring strategy enables reasonable archive sorting and search strategies based on evolution status, maintaining both good convergence and diversity in the population.

6 Conclusion

This paper improves rotation basis visualization technology to clearly present the distribution and evolution status of archive particles on the Pareto front, facilitating decision-makers in selecting appropriate strategies for archive maintenance and global optimal particle selection. Additionally, by employing Pareto entropy and differential entropy concepts to monitor population evolution and automatically selecting different evolution management strategies based on evolution status, the imbalance between rapid convergence and diversity during particle swarm evolution is resolved. The paper designs a dual-sorting archive maintenance strategy using rotation basis visualization technology, improving traditional single-sorting methods based on density to simultaneously satisfy both convergence and diversity requirements. Finally, a global optimal particle selection strategy is proposed using rotation basis angle and distance. This strategy integrates the advantages of rotation basis visualization technology to facilitate selection of particles with the strongest comprehensive performance, guiding the population toward the Pareto front.

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Note: Figure translations are in progress. See original paper for figures.