Multi-Attribute Decision-Making Method Based on Ideal Point and Vector Cosine Projection (Postprint)

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Date: 2018-05-24T00:00:00+00:00

Abstract

To address the shortcomings of existing multi-attribute decision-making processes that mostly fail to reflect decision-makers' personal multi-level preferences for products and lack an objective and reasonable multi-attribute evaluation benchmark, a multi-attribute preference decision-making method based on vector cosine projection is proposed. First, the preference degree of decision-makers for each attribute is determined through a multi-level pairwise comparison method; second, the ideal values of each attribute for the ideal evaluation object are determined based on the ideal point; finally, the projection intensity of a general multi-attribute problem vector in the direction of the ideal multi-attribute problem vector is calculated using the vector cosine projection method, and the priority ranking results of each alternative are then determined according to the magnitude of the intensity. The effectiveness and feasibility of the proposed method are verified through a numerical example.

Full Text

Preamble

Multiple Attribute Decision Making Based on Ideal Point-Vector Cosine Projection Method

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Abstract: Existing multiple attribute decision-making methods suffer from two major deficiencies: they cannot reflect decision-makers' individual multi-grade preferences for alternatives, and they lack an objective and reasonable multiple attribute evaluation benchmark. To address these issues, this paper proposes a multiple attribute preference decision-making method based on vector cosine projection. First, the multi-grade pairwise comparison method is used to determine the decision-maker' s preference degree for each attribute. Second, the ideal point method is employed to determine the ideal values of each attribute for the ideal evaluation object. Finally, the vector cosine projection method calculates the projection intensity of a general multiple attribute problem vector in the direction of the ideal multiple attribute problem vector, and the ranking of alternatives is determined based on these projection intensities. The feasibility and effectiveness of the proposed method are demonstrated through an illustrative example.

Keywords: multiple attribute decision making; individual multi-grade preference; ideal point method; vector cosine projection method

0 Introduction

As research on decision-making problems deepens, the number of attributes considered in the decision-making process continues to increase, making multi-attribute decision-making increasingly challenging. Multi-attribute decision-making problems have become a focal point of attention in the decision-making field. Due to the inherent complexity of decision problems, the methods for handling multi-attribute decision-making problems are relatively limited. Multi-attribute decision-making problems are characterized by subjectivity, ambiguity, and uncertainty, which means there is no unified standard for their study, and scholars have approached them from various perspectives.

A multi-attribute decision-making problem involves comprehensively evaluating decision alternatives using scientific decision-making methods under the premise of considering multiple attributes, and then ranking or selecting alternatives based on the evaluation results. Such problems widely exist in numerous real-world domains, including engineering, healthcare, economics, and management. Both the objectivity of the process and the decision-maker's personal preferences significantly influence attribute weights in multi-attribute decision-making, necessitating comprehensive consideration of both subjective and objective factors when calculating attribute weights. Since attribute weight values determine the scientific validity and rationality of the entire decision, unreasonable weight values may lead to decision failure, making attribute weight calculation a critical research focus.

Current methods for determining multi-attribute weights primarily include subjective weighting methods, objective weighting methods, and integrated subjective-objective weighting methods. Subjective weighting methods determine weights based on decision-makers' subjective emphasis on each attribute, introducing considerable arbitrariness and subjectivity. Objective weighting methods automatically calculate weights for each attribute according to specific rules, ensuring process objectivity but failing to reflect decision-makers' personal preferences. Integrated subjective-objective weighting methods combine both approaches, preserving personal preferences while maintaining objectivity and rationality, and have thus been favored by many scholars.

For instance, Chen Wei et al. incorporated expert group preferences into attribute weight calculations by establishing ratio scales to obtain subjective weights and then employing combination weighting to derive final attribute weights. He Dayi et al. proposed using the minimum cross-entropy criterion to construct optimization models for determining subjective and objective weight matrices separately. Song Dongmei et al. developed a new subjective-objective weighting method to address the limitations of traditional approaches where subjective methods exhibit excessive decision-maker bias while objective methods rely too heavily on measured data. However, most of these methods employ simple weighting techniques such as AHP (Analytic Hierarchy Process) or ANP (Analytic Network Process), which can only construct single-grade comparison relationships and cannot simultaneously express multiple different preference relationships.

Existing research on multi-attribute decision-making commonly employs interval numbers, intuitionistic fuzzy sets, and hesitant fuzzy numbers, but these methods often involve complex calculations and have limited applicability, making them difficult to generalize. Therefore, this paper proposes a multi-attribute preference decision-making method based on vector cosine projection. The method first uses multi-grade pairwise comparison to determine decision-makers' preference degrees for each attribute, then applies the ideal point method to determine ideal attribute values, and finally employs vector cosine projection to calculate the projection intensity of general multi-attribute problem vectors onto ideal multi-attribute problem vectors for alternative ranking.

1.1 Multi-Grade Pairwise Comparison Method

The multi-grade pairwise comparison method was originally applied in multi-attribute weight calculation. Since the weights obtained through this method overcome the limitation of traditional methods that can only achieve single-grade pairwise comparison, it has gained favor among many scholars. Given its excellent performance in multi-attribute weight calculation, this method is introduced in this paper to calculate preference degrees. To fully express decision-makers' multi-grade preferences for product attributes, a distributed preference relationship must be defined. The distributed preference relationship is defined on a multi-grade symmetric framework that can simultaneously describe four relationships between paired objects: superiority, inferiority, indifference, and uncertainty. This is also referred to as a distributed preference.

The recognition framework used in classical distributed preference theory is $\Omega = \{H_{-2}, H_{-1}, H_0, H_1, H_2\}$. The grades H_{-2} and H_{-1} represent weak to strong degrees of inferiority between paired objects, H_1 and H_2 represent weak to strong degrees of superiority, and H_0 represents indifference between objects. The framework is symmetric around H_0.

1.2 Ideal Point Method

The ideal point method involves decision-makers using available information to construct an ideal point that satisfies all objectives. This method is often associated with multi-index problems and has broad applicability. The ideal point method classifies indicators into positive indicators (where larger values are better) and negative indicators (where smaller values are better). The definitions of ideal and anti-ideal points are as follows:

For positive indicators:

$$\begin{cases} f_i(x^*) = \max\{f_i(x)\}, & i = 1, 2, \dots, n \\ f_i(x^*) = \min\{f_i(x)\}, & i = 1, 2, \dots, n \end{cases}$$

For negative indicators:

$$\begin{cases} f_i(x^*) = \min\{f_i(x)\}, & i = 1, 2, \dots, n \\ f_i(x^*) = \max\{f_i(x)\}, & i = 1, 2, \dots, n \end{cases}$$

1.3 Vector Cosine Projection Method

Vector cosine projection is widely used in multi-dimensional fields due to its simplicity and ease of use. The schematic diagram of the vector cosine projection method is shown in [Figure 1: see original paper]. As illustrated, the cosine projection intensity of vector $\vec{a}$ in the horizontal direction is $\cos\theta = \frac{\vec{a}\cdot\vec{R}}{|\vec{a}|\cdot|\vec{R}|}$.

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2 Vector Cosine Projection-Based Multi-Attribute Preference Decision Making

2.1 Problem Description

Assume a multi-attribute decision-making problem includes m alternatives and n evaluation attributes. Let $Y = \{y_1, y_2, \dots, y_m\}$ be the set of alternatives and $X = \{x_1, x_2, \dots, x_n\}$ be the set of attributes. Due to decision-makers' personal preferences, their degrees of preference for attributes in the set are inconsistent. For a specific real-world situation, decision-makers must select the most suitable alternative from the alternative set.

2.2 Vectorization of Multi-Attribute Decision Schemes

For multi-attribute decision schemes, the attributes are independent of each other with no correlation, which aligns with the theoretical meaning of multi-dimensional vectors. Therefore, this paper introduces the concept of multi-dimensional vectors, treating each attribute as a dimension and converting each multi-attribute decision scheme into a multi-dimensional vector. When a multi-attribute scheme contains two attributes, it can be represented as a two-dimensional vector in a two-dimensional coordinate system, as shown in Figure 1, where A1 and A2 represent two-attribute decision problems. When a multi-attribute scheme contains three attributes, it can be represented as a three-dimensional vector in a three-dimensional coordinate system, as shown in [Figure 2: see original paper], where B1 and B2 represent three-attribute decision problems. This can be extended to higher dimensions. While coordinate systems beyond three dimensions cannot be perceived or drawn, they exist conceptually.

2.3 Calculation of Multi-Attribute Preference Degrees

The multi-grade pairwise comparison method is used to calculate decision-makers' preference degrees for each attribute. The main steps for solving decision-maker attribute preference degrees are as follows:

a) Establish the symmetric framework Ω and grade score values H, and calculate the score matrix S using formula (3):

$$S = \begin{bmatrix} 0 & [s_{12}^-, s_{12}^+] & \cdots & [s_{1M}^-, s_{1M}^+] \\ [s_{21}^-, s_{21}^+] & 0 & \cdots & [s_{2M}^-, s_{2M}^+] \\ \vdots & \vdots & \ddots & \vdots \\ [s_{M1}^-, s_{M1}^+] & [s_{M2}^-, s_{M2}^+] & \cdots & 0 \end{bmatrix}$$

b) Decision-makers consider each object in the object set from different perspectives and provide the distributed preference relationship on the object set.

c) Based on the S matrix obtained in the previous steps, calculate the normalized score matrix D:

$$D = \begin{bmatrix} 0.5 & [s_{12}^-, s_{12}^+] & \cdots & [s_{1M}^-, s_{1M}^+] \\ [s_{21}^-, s_{21}^+] & 0.5 & \cdots & [s_{2M}^-, s_{2M}^+] \\ \vdots & \vdots & \ddots & \vdots \\ [s_{M1}^-, s_{M1}^+] & [s_{M2}^-, s_{M2}^+] & \cdots & 0.5 \end{bmatrix}$$

d) On the basis of the normalized score matrix D, construct an optimization model for each object' s priority using formula (4), then solve for w_i^- and w_i^+ using formula (5):

$$w_i^- = \frac{1}{M-1}\sum_{j\neq i}\frac{s_{ij}^-}{s_{ij}^- + s_{ij}^+}, \quad w_i^+ = \frac{1}{M-1}\sum_{j\neq i}\frac{s_{ij}^+}{s_{ij}^- + s_{ij}^+}$$

e) Calculate p_{ij} using formula (6) to form the possibility degree matrix P:

$$p_{ij} = \max\left\{0, \frac{w_i^+ - w_j^-}{l(w_i) + l(w_j)}\right\}, \quad \text{where } l(w_i) = w_i^+ - w_i^-$$

f) Combine the possibility degree matrix P and calculate each object's utility value u_i using formula (7):

$$u_i = \frac{1}{M(M-1)}\left(\sum_{j=1}^{M} p_{ij} + \frac{M}{2} - 1\right)$$

The utility value magnitude reflects each object' s importance, and the obtained utility values represent the corresponding object preference degrees.

2.4 Vectorization and Vector Projection Intensity Calculation

According to formula (8), the decision-maker' s personal preferences and the evaluation values of each scheme' s attributes are combined to determine each scheme' s preference multi-attribute vector:

$$\vec{A}_i = (u_1 x_{i1}, u_2 x_{i2}, \dots, u_n x_{in})$$

where i' s maximum value is the number of multi-attribute schemes, and j' s maximum value is the number of attributes in the multi-attribute scheme. Based on the previously mentioned ideal point calculation formula (6), the ideal preference multi-attribute decision scheme vector is determined:

$$\vec{A}^* = (\max_i\{u_1 x_{i1}\}, \max_i\{u_2 x_{i2}\}, \dots, \max_i\{u_n x_{in}\})$$

The previous section introduced the vectorization process for multi-attribute decision problems. Formula (8) successfully transforms a multi-attribute problem into a multi-dimensional vector, and formula (9) determines the ideal preference multi-attribute decision scheme. The critical challenge is how to integrate the multiple attributes in a multi-attribute problem to obtain a numerical value for evaluating alternative schemes. While weighted averaging is the most common approach, weight determination often involves significant subjectivity. Therefore, this paper adopts the vector cosine projection method for multi-attribute integration, which overcomes the difficulty of controlling subjectivity in linear weighted averaging and ensures calculation objectivity. The schematic diagram of multi-attribute vector cosine projection is shown in [Figure 4: see original paper].

Formula (10) calculates the projection intensity of a general multi-attribute scheme vector onto the ideal multi-attribute decision scheme vector, and alternatives are ranked based on the obtained projection intensities q_i:

$$q_i = \frac{\vec{A}_i \cdot \vec{A}^*}{|\vec{A}^*|} = \frac{u_1 x_{i1} \cdot \max_i\{u_1 x_{i1}\} + u_2 x_{i2} \cdot \max_i\{u_2 x_{i2}\} + \cdots + u_n x_{in} \cdot \max_i\{u_n x_{in}\}}{\sqrt{(\max_i\{u_1 x_{i1}\})^2 + (\max_i\{u_2 x_{i2}\})^2 + \cdots + (\max_i\{u_n x_{in}\})^2}}$$

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3 Decision Steps

The calculation steps for the vector cosine projection-based multi-attribute preference decision-making method are summarized as follows:

a) Calculate the decision-maker' s preference degree for each attribute using the multi-grade pairwise comparison method.

b) Substitute the results from step (a) into formula (8) to obtain the multi-attribute preference scheme vectors.

c) Vectorize the multi-attribute preference schemes and determine the ideal multi-attribute preference scheme vector using formula (9).

d) Calculate the projection intensity of general multi-attribute preference scheme vectors onto the ideal multi-attribute preference scheme vector using formula (10).

e) Rank all scheme vectors based on their projection intensities and select the most satisfactory scheme.

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4 Example Analysis

A company plans to organize an employee trip with four alternative destinations: Hangzhou, Beijing, Kunming, and Harbin. The destination selection is primarily influenced by four factors: scenery, cost, food, and accommodation. The company leader must choose the final destination from these four alternatives. The company collected employee evaluations of destination information through questionnaires and determined the score for each attribute based on the optimistic decision criterion (majority rule). The original decision matrix for this case is presented in .

Table 1 Information on Alternative Travel Destinations

Destination	Scenery (B_1)	Cost (B_2)	Food (B_3)	Accommodation (B_4)
Hangzhou (C_1)	9	7	8	9
Beijing (C_2)	9	8	7	8
Kunming (C_3)	10	7	9	8
Harbin (C_4)	8	6	8	8

Note: Matrix values represent decision-makers' evaluation scores for each alternative on corresponding attributes, with scores ranging from 1-10. Attributes B_1, B_3, and B_4 are positive indicators (higher scores are better), while B_2 is a negative indicator (lower scores are better).

This is clearly a multi-attribute decision-making problem where scenery, cost, food, and accommodation represent the four attributes B_1, B_2, B_3, B_4, and Hangzhou, Beijing, Kunming, and Harbin are represented by C_1, C_2, C_3, C_4. To demonstrate the effectiveness of the proposed method, both AHP (Analytic Hierarchy Process) and the method proposed in this paper are applied for decision analysis.

4.1 AHP-Based Decision Analysis

Based on the case analysis, the hierarchical structure shown in [Figure 5: see original paper] is obtained. The judgment matrices obtained through expert scoring are as follows:

Matrix A (Criteria layer relative to target layer):

$$A = \begin{bmatrix} 1 & 2 & 1/2 & 2 \\ 1/2 & 1 & 1/3 & 1 \\ 2 & 3 & 1 & 3 \\ 1/2 & 1 & 1/3 & 1 \end{bmatrix}$$

Matrices B_1 through B_4 (Scheme layer relative to criteria layer):

$$B_1 = \begin{bmatrix} 1 & 2 & 1/2 & 1 \\ 1/2 & 1 & 1/2 & 1 \\ 2 & 2 & 1 & 2 \\ 1 & 1 & 1/2 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 1/3 & 1/2 & 1/4 \\ 3 & 1 & 2 & 1/2 \\ 2 & 1/2 & 1 & 1/2 \\ 4 & 2 & 2 & 1 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 1 & 2 & 1/2 & 2 \\ 1/2 & 1 & 1/2 & 1 \\ 2 & 2 & 1 & 2 \\ 1/2 & 1 & 1/2 & 1 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 1 & 2 & 1/2 & 1 \\ 1/2 & 1 & 1/2 & 1/2 \\ 2 & 2 & 1 & 2 \\ 1 & 2 & 1/2 & 1 \end{bmatrix}$$

Using MATLAB programming, the weight ranking results for the scheme layer are:

$$W_A = [0.2628, 0.1409, 0.4554, 0.1409]$$

$$W_{B1} = [0.4236, 0.2270, 0.2270, 0.1223], \quad W_{B2} = [0.1428, 0.0874, 0.3849, 0.3849]$$

$$W_{B3} = [0.3333, 0.1667, 0.3333, 0.1667], \quad W_{B4} = [0.2270, 0.2270, 0.4236, 0.1223]$$

The final weights for the four travel destinations are:

$$W_{C1} = 0.3152, \quad W_{C2} = 0.1799, \quad W_{C3} = 0.3254, \quad W_{C4} = 0.1795$$

The priority ranking of the four travel destinations is Kunming, Hangzhou, Beijing, and Harbin.

4.2 Ideal Point-Vector Cosine Projection-Based Decision Analysis

a) This paper divides decision-maker preferences into five grades: $H_{-2}, H_{-1}, H_0, H_1, H_2$, assigned values of -1, -0.5, 0, 0.5, and 1, respectively. The distributed preference relationships among the four attributes B_1, B_2, B_3, B_4 are shown in .

Table 2 Distributed Preference Relationships Among Attributes

Preference	B_1	B_2	B_3	B_4
B_1	$H_0(1)$	$H_1(0.4)$	$H_{-1}(0.4)$	$H_2(0.2)$
B_2	$H_{-1}(0.4)$	$H_0(1)$	$H_{-1}(0.5)$	$H_{-1}(0.2)$
B_3	$H_1(0.4)$	$H_1(0.5)$	$H_0(1)$	$H_1(0.3)$
B_4	$H_{-2}(0.2)$	$H_1(0.2)$	$H_{-1}(0.3)$	$H_0(1)$

Based on the distributed preference relationships shown in Table 2, the score matrix S and normalized score matrix D obtained using formulas (3) and (4) are:

$$S = \begin{bmatrix} 0 & [0.2, 0.2] & [0.4, 0.25] & [0.2, 0.35] \\ [0.2, 0.2] & 0 & [0.3, 0.3] & [0.1, 0.1] \\ [0.25, 0.4] & [0.3, 0.3] & 0 & [0.2, 0.1] \\ [0.35, 0.2] & [0.1, 0.1] & [0.1, 0.2] & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.5 & [0.6, 0.6] & [0.3, 0.325] & [0.6, 0.675] \\ [0.4, 0.4] & 0.5 & [0.35, 0.35] & [0.45, 0.55] \\ [0.625, 0.7] & [0.65, 0.65] & 0.5 & [0.4, 0.55] \\ [0.325, 0.4] & [0.45, 0.55] & [0.45, 0.6] & 0.5 \end{bmatrix}$$

Using formulas (5) through (7), the decision-maker' s preference degrees for the four attributes B_1, B_2, B_3, B_4 are:

$$u_1 = 0.258, \quad u_2 = 0.167, \quad u_3 = 0.375, \quad u_4 = 0.200$$

b) Substituting the results from step (a) into formula (8) yields the multi-attribute preference scheme vectors:

$$\vec{S}_1 = (2.322, 1.503, 3.375, 2.000), \quad \vec{S}_2 = (2.322, 1.670, 3.000, 1.800)$$

$$\vec{S}_3 = (2.580, 1.503, 3.375, 1.800), \quad \vec{S}_4 = (2.064, 1.336, 3.000, 1.800)$$

c) The ideal preference multi-attribute decision scheme vector is determined as:

$$\vec{S}^* = (2.580, 1.336, 3.375, 2.000)$$

d) Using formula (10), the projection intensities of the general multi-attribute preference scheme vectors onto the ideal multi-attribute preference scheme vector are calculated as:

$$q_1 = 4.790, \quad q_2 = 4.183, \quad q_3 = 4.882, \quad q_4 = 4.268$$

After normalization:

$$q_1' = 0.264, \quad q_2' = 0.231, \quad q_3' = 0.269, \quad q_4' = 0.236$$

e) The final ranking obtained by sorting the step (d) results in descending order is $S_3 > S_1 > S_4 > S_2$, giving the priority ranking as Kunming, Hangzhou, Harbin, and Beijing.

Comparing the results from both methods shows that the ranking outcomes are relatively close, demonstrating the effectiveness and feasibility of the proposed method.

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5 Conclusion

This paper identifies deficiencies in existing multi-attribute decision-making methods, such as excessive subjectivity in attribute weight calculation, limited application conditions, and relatively complex processes that hinder generalization. To improve methodological rationality, a vector cosine projection-based multi-attribute preference decision-making method is proposed. The method employs multi-grade pairwise comparison to calculate decision-makers' preference degrees for each attribute, incorporating preferences into the decision-making process and overcoming the limitation of traditional single-grade comparison while reducing subjective arbitrariness. Additionally, the ideal point method is used to determine the ideal multi-attribute preference scheme vector, establishing a more scientific and reasonable evaluation benchmark. Finally, the vector cosine method calculates the projection intensity of constructed general multi-attribute preference scheme vectors onto the ideal vector. This method is simple to operate, has a concise calculation process, and offers good generalizability, such as in industrial product innovation design, which can generate substantial economic benefits for enterprises. The example analysis demonstrates the method' s effectiveness and feasibility in handling multi-attribute decision-making problems.

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Note: Figure translations are in progress. See original paper for figures.