Signal Carrier Frequency Estimation Method Based on Compressed Sensing and Cyclic Spectrum Theory (Postprint)

Authors: Xie Yao, Li Simin, Tang Zhiling

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Abstract

To achieve parameter estimation of carrier frequency under low signal-to-noise ratio conditions, an algorithm is proposed. First, the cyclic spectrum of the signal is calculated based on cyclic spectrum theory. Then, the compressed sensing method is employed to reconstruct the cyclic spectrum profile of the signal. Finally, the carrier frequency is calculated using an averaging method based on the relationship between the discrete spectral line positions in the signal' s cyclic spectrum profile and the signal' s carrier frequency. Numerical simulations show that under conditions of high signal sparsity, the estimation accuracy of the carrier frequency can be controlled below 10^3 Hz. Under low signal-to-noise ratio conditions, this algorithm can effectively achieve estimation of the signal' s carrier frequency and possesses certain practical value in engineering applications.

Full Text

Signal Carrier Frequency Estimation Based on Compressed Sensing and Cyclic Spectrum Theory

Xie Yao[1], Li Simin[2], Tang Zhiling[1]† 1. School of Electronic Engineering and Automation, Guilin University of Electronic Technology, Guilin, Guangxi 541004, China 2. School of Electrical and Information Engineering, Guangxi University of Science and Technology, Liuzhou, Guangxi 545006, China

Abstract: To achieve parameter estimation of carrier frequency under low SNR conditions, this paper proposes an algorithm. First, the cyclic spectrum of the signal is calculated based on cyclic spectrum theory. Then, the cyclic spectrum section of the signal is reconstructed using compressed sensing. Finally, the carrier frequency is calculated using the averaging method according to the relationship between the positions of discrete spectral lines in the signal' s cyclic spectrum section and the signal's carrier frequency. Numerical simulations show that under conditions of high signal sparsity, the estimation accuracy of carrier frequency can be controlled below 10^3 Hz. Under low SNR conditions, the algorithm can effectively estimate the signal' s carrier frequency, demonstrating certain practical value in engineering applications.

Keywords: cyclostationarity; parameter estimation; compressed sensing; signal reconstruction
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0 Introduction

In recent decades, with the rapid development of modern communication technology, modulation recognition and signal parameter estimation have played crucial roles in both civil and military applications, such as disaster prevention, geological and ocean exploration, electronic countermeasures, and intelligence reconnaissance. The estimation of signal modulation parameters, including carrier frequency, symbol rate, and signal bandwidth, serves as an important prerequisite for successful modulation identification.

Cyclic spectrum theory, extensively researched and developed by Gardner et al. [?], offers advantages including high resolution, strong anti-interference capability, and rich dual-frequency domain information that is easy to extract, making it suitable for analyzing cyclostationary signals. In communications, radar, telemetry, underwater acoustics, and electronic countermeasures, many signals exhibit cyclostationarity due to certain intentional or unintentional modulations. Consequently, cyclic spectrum theory has been increasingly studied in modern signal processing domains such as signal detection, parameter estimation, system identification, and modulation recognition. Among these applications, carrier frequency and symbol rate are critical for demodulation and represent necessary conditions for signal decoding in non-cooperative communication environments like military reconnaissance, residential radio monitoring, and software-defined radio receivers. Therefore, these parameters have received growing attention in research over the past decades.

Numerous methods exist for carrier frequency estimation. In frequency-domain analysis, the periodogram method from [?] and frequency centering algorithm from [?] are suitable for signals with strong carrier power and symmetric power spectral density. In time-domain analysis, the zero-crossing algorithm from [?] and the Strip Spectral Correlation Algorithm (SSCA) from [?] offer computational simplicity and low complexity but are sensitive to noise. Wavelet-based methods introduced in [?, ?], including wavelet transform and negative entropy maximization algorithms, provide good denoising performance but suffer from high computational complexity. Reference [?] proposed computing cyclic spectrum sections for dimensionality reduction, significantly decreasing computational load but with poor adaptability. References [?, ?] improved cyclic spectrum algorithms by analyzing the cyclic spectrum matrix to both enhance computational methods for easier eigenvalue extraction and reduce dimensionality, thereby substantially lowering computational complexity. References [?] suggested segmenting signal samples with overlap between consecutive segments, computing the cyclic spectrum for each segment, and averaging to obtain the final cyclic spectrum, which reduces noise effects and improves the SNR threshold. While this approach enhances algorithm effectiveness for limited data, the number of segments requires manual adjustment, leading to poor stability and increased computational load. References [?, ?] combined high-order cumulants with cyclic spectrum for precise carrier frequency estimation, leveraging the anti-noise advantages of high-order cumulants to extract signal features. This method offers high accuracy and strong noise immunity but entails substantial computation.

Since cyclic spectrum exhibits sparsity in the frequency domain, [?] proposed reconstructing cyclic spectrum using compressed sensing to eliminate redundant information and extract effective signal components. However, this approach requires numerous measurements and yields relatively low reconstruction accuracy.

To address these existing issues, this paper proposes an algorithm based on Stagewise Orthogonal Matching Pursuit (STOMP). During iteration, the algorithm can select multiple elements at once, significantly reducing signal reconstruction time. Additionally, it employs a threshold-based approach to determine elements, eliminating the need for a fixed element count and avoiding information redundancy or loss. The algorithm does not depend on signal sparsity, offering unique advantages for signal reconstruction.

1 Cyclic Spectrum Calculation Method

Cyclic spectrum is a common method for signal stationary analysis, enabling identification of signals with different parameters through variations in their cyclic spectra. Since stationary noise lacks cyclostationary characteristics, cyclic spectrum analysis can effectively separate signals from noise, offering significant advantages in signal analysis [?, ?].

Let x(t) be a generalized cyclostationary process if its first-order statistical characteristic (mean) and second-order statistical characteristic (autocorrelation function) exhibit periodicity with period T. The mean and autocorrelation function are respectively [?]:

$$\bar{x}(t) = \bar{x}(t + T_m), \quad R_x(t, \tau) = R_x(t + T_m, \tau)$$

where $T = K/\alpha$, K is an integer, and α is the cycle frequency. Expanding the periodic function $R_x(t,\tau)$ as a Fourier series yields:

$$R_x(t,\tau) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t}$$

where $R_x^{\alpha}(\tau)$ is the cyclic autocorrelation function, a function of cycle frequency α and time delay τ:

$$R_x^{\alpha}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} R_x(t,\tau) e^{-j2\pi\alpha t} dt$$

The Fourier transform of the cyclic correlation function $R_x^{\alpha}(\tau)$ is:

$$S_x^{\alpha}(f) = \int_{-\infty}^{\infty} R_x^{\alpha}(\tau) e^{-j2\pi f\tau} d\tau$$

where $S_x^{\alpha}(f)$ is called the cyclic spectral density function.

For a BPSK signal model, assuming the received signal is:

$$X(t) = S(t) + n(t)$$

where $S(t)$ is the BPSK signal and $n(t)$ is Gaussian white noise with zero mean and variance σ^2. The mathematical expression for $S(t)$ is:

$$S(t) = A\exp\{j2\pi f_c T_t + \theta_0 + \phi(T_t)\}$$

where $t = 1, 2, \cdots, N$, $\phi(T_t) = \sum_{i=1}^{N_s} \alpha_i \prod(T_t - iT_b)$, $\alpha_i \in \{\pm 1\}$, A is signal amplitude, N is sample count, T is time interval, N_s is symbol count, T_b is symbol width, $\prod(\cdot)$ is the pulse shaping function, f_c is carrier frequency, and θ_0 is initial phase.

Assuming $Q_{rc}(f) = \frac{\sin(\pi f T_0)}{\pi f T_0}$ and $\alpha(f) = Q_{rc}(f+\alpha/2) Q_{rc}^{*}(f-\alpha/2)$, the cyclic spectrum of the BPSK signal can be calculated from expressions (3)-(5) as [?]:

$$S_x^{\alpha}(f) = S_c^{\alpha}(f+f_c) + S_c^{\alpha}(f-f_c) + S_c^{\alpha+2f_c}(f) e^{-j2\theta_0} + S_c^{\alpha-2f_c}(f) e^{j2\theta_0}$$

where $\alpha = \pm 2f_c + k/T_b$, $\alpha = k/T_b$, and k is an integer. For stationary white noise, its spectral density concentrates in the $\alpha = 0$ region, with minimal noise impact when $\alpha \neq 0$. This demonstrates that parameter estimation based on cyclic spectrum can effectively suppress noise.

2 Compressed Sensing

The Nyquist sampling theorem has long served as the bridge between analog and digital signals, playing a central role in signal processing. Signal sampling is a prerequisite for digital signal modulation classification and recognition. However, as Nyquist sampling rate requirements increase, the massive discrete data generated after signal discretization creates enormous pressure on data storage and transmission. Candes, Donoho, and Tao [**?**, **?**] proposed compressed sensing theory (CS), providing a new solution: for sparse signals, accurate reconstruction can be achieved through optimization algorithms, breaking the Nyquist limit and effectively reducing sampling rates.

For ultra-wideband signals, finding their sparse domain enables distortion-free reconstruction from small amounts of compressed data, eliminating dependence on front-end sampling equipment [**?**]. When a signal is sparse in some domain or inherently sparse, a measurement matrix satisfying certain conditions can project the signal to maintain sparsity while preserving sufficient information for reconstruction. Thus, in compressed sensing, the sampling frequency is independent of both the signal’ s Nyquist rate and its highest frequency component, depending only on the signal’ s intrinsic properties. Furthermore, signal discretization can bypass the Nyquist theorem by projecting signals into a low-dimensional space through matrix operations [**?**].

2.1 Signal Sparse Representation

Signal sparsity, also called compressibility, means that in a certain transform basis, only a few coefficients are non-zero while most are near-zero. Sparse analysis represents data effectively using pre-constructed basis functions under specific prior conditions. For signal $x \in \mathbb{R}^N$, a linear combination of orthogonal bases can represent it as:

$$x = \sum_{i=1}^{N} \varphi_i \theta_i = \Psi\Theta$$

where $\Psi = [\varphi_1, \varphi_2, \cdots, \varphi_N]$ is the orthogonal basis matrix with column vectors φ_i as basis functions, and $\Theta = [\theta_1, \theta_2, \cdots, \theta_N]^T$ is the coefficient vector. If Θ has only K large coefficients, then x is compressible. Signal sparsity determines the optimal compression effect—retaining only the K largest coefficients during compression processing without causing severe signal quality degradation.

2.2 Observation Matrix

For signal $x \in \mathbb{R}^N$, if an orthogonal matrix Ψ exists making x K-sparse, observation matrix $\Phi \in \mathbb{R}^{M\times N}$ observes signal x to obtain $y \in \mathbb{R}^M$, where $K < M < N$:

$$y = \Phi x$$

In 2006, Candes, Romberg, and Tao proposed the Restricted Isometry Property (RIP) theory, which defines conditions for observation matrix design and serves as a foundational theory ensuring compressed sensing accuracy. Assuming signal x' s projected sparse vector Θ has length N and sparsity K, for sensing matrix $A = \Phi\Psi \in \mathbb{R}^{M\times N}$, if there exists a constant $\delta \in (0, 1)$ satisfying:

$$(1-\delta)\|\Theta\|_2^2 \leq \|A\Theta\|_2^2 \leq (1+\delta)\|\Theta\|_2^2$$

then sensing matrix A is said to satisfy the K-order restricted isometry property [?, ?].

2.3 Signal Reconstruction

Successful compressed sampling depends on accurate original signal reconstruction. In compressed sensing, since the number of observations is smaller than unknowns, signal reconstruction is an underdetermined problem. Given observation y and sensing matrix A, the sparse solution or its approximation $\hat{\Theta}$ is sought under constraint $y = A\Theta$, with infinite possible solutions:

$$\hat{\Theta} = \arg\min \|\Theta\|_0 \quad \text{s.t.} \quad y = A\Theta$$

The ℓ_0-norm counts non-zero elements in Θ, representing a non-convex optimization problem that is difficult to solve directly. Relaxation techniques typically convert this to a convex optimization problem. The transformed equation becomes:

$$\hat{\Theta} = \arg\min \|\Theta\|_1 \quad \text{s.t.} \quad y = A\Theta$$

Signal reconstruction algorithms 主要分为 optimization algorithms and greedy algorithms. Optimization algorithms include Basis Pursuit (BP) and convex optimization methods. Greedy algorithms include Orthogonal Matching Pursuit (OMP), Regularized Orthogonal Matching Pursuit (ROMP), and Stagewise Orthogonal Matching Pursuit (STOMP). The reconstruction process uses optimal search methods to find frequency bands containing effective spectral information and their corresponding dictionary positions to recover the signal. Since search methods have high complexity, high-precision approaches require many considerations and conditions, limiting their practical engineering advantages. Practical applications favor greedy algorithms, which have lower computational complexity but higher hardware requirements. Through minimal iterations, greedy algorithms identify the most representative vectors for weighted representation of sparse multi-band signals, then reconstruct using weighted expressions [?].

To improve iteration speed, the Stagewise Orthogonal Matching Pursuit (STOMP) algorithm incorporates a stagewise search approach into OMP. During each iteration, a threshold is set—dictionary column vectors with correlation to the residual exceeding this threshold are considered correct spectral support region query results. Based on residual correlation characteristics, threshold selection typically ranges between 2-3 [**?**, **?**]. The STOMP algorithm steps are:

a) **Initialization:** Sparse solution $\Theta = 0$, sensing matrix $A = \Phi\Psi$, output matrix y, residual vector $e = y$. Support set is empty: $I_0 = \emptyset$. Set threshold TH.

b) **Residual back-projection:** $u_k = A^T e_{k-1}$. Under the dictionary randomness assumption, most computed values are independent Gaussian vectors with zero mean. Compare u_k with threshold TH to obtain the column index set of A.

c) **Support set update:** During iteration, update sensing matrix A as $A_t = A_{t-1} \cup a_t$ and support set I as $I_t = I_{t-1} \cup J_t$.

d) **Update sparse solution and residual:** $\Theta_t = (A_t^T A_t)^{-1} A_t^T y$. After obtaining updated Θ_t, compute new residual $e_t = y - A_t \Theta_t$.

STOMP requires fewer iterations than OMP, effectively simplifying the algorithm. Each iteration selects multiple column vectors based on threshold magnitude, choosing several basis vectors per iteration for faster convergence. Additionally, the stagewise threshold selection yields smaller signal reconstruction errors.

3 Carrier Frequency Estimation

Due to the shaping filter characteristics, $Q_{rc}(f)$ reaches its maximum at $f = 0$. When $\alpha = 0$, the carrier frequency can be calculated in the frequency domain. First, compute the signal’ s cyclic spectrum:

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau) e^{-j2\pi f\tau} d\tau$$

Then reconstruct the cyclic spectrum section $S_x^0(f)$ using greedy reconstruction algorithms. The most common greedy algorithm, Orthogonal Matching Pursuit (OMP), aims to represent the sparse multi-band signal’ s frequency-domain signal using the minimum number of column vectors as basis vectors through multiple iterations, ultimately reconstructing the signal through weighted functions of these basis vectors. The process queries required information and its position during each iteration, then updates the residual until meeting termination conditions. While convex optimization algorithms offer high robustness despite large computational load and complexity, OMP provides significant speed improvements but lower robustness. Combining their advantages led to the Regularized Orthogonal Matching Pursuit (ROMP) algorithm. Unlike OMP, ROMP arbitrarily selects multiple column vectors per iteration. To avoid excessive misselection, ROMP introduces regularization to constrain candidate matrices, reducing error rates, iterations, and improving recovery accuracy [**?**].

Like OMP, ROMP requires knowledge of signal sparsity to determine maximum iteration counts. However, ROMP has defects: when the sensing output matrix contains noise, iteration may fail to terminate, requiring prior sparsity knowledge to control iterations.

To further improve iteration rate, STOMP adds stagewise search to OMP' s framework. Each iteration sets a threshold—dictionary column vectors with residual correlation exceeding this threshold are considered correct spectral support region queries. Based on residual correlation characteristics, threshold selection typically ranges 2-3 [**?**, **?**]. The STOMP implementation steps are shown above.

STOMP iterations are fewer than OMP' s, simplifying the algorithm. Each iteration selects multiple column vectors based on threshold magnitude, choosing several basis vectors for faster convergence. Additionally, stagewise threshold selection yields smaller reconstruction errors.

Finally, search for peaks in the reconstructed cyclic spectrum section $S_x^0(f)$ and estimate carrier frequency using frequency averaging:

$$\hat{f}_c = \frac{\sum_{i=1}^{M} |f_i|}{M}$$

The complete carrier frequency estimation algorithm flow is shown in Figure 1 [Figure 1: see original paper].

For MPSK signals, the received signal model is:

$$X(t) = S(t) + n(t)$$

where $S(t)$ is the MPSK signal and $n(t)$ is zero-mean Gaussian white noise with variance σ^2. The mathematical expression for $S(t)$ is:

$$S(t) = A \exp\{j2\pi f_c T_t + \theta_0 + \phi(T_t)\}$$

For BPSK signals, cyclic spectrum appears only in sections where $\alpha = \pm 2f_c + k/T_b$ and $\alpha = k/T_b$, with f_c as carrier frequency and T_b as symbol period. For real signals, cyclic spectrum algorithms exhibit symmetry—knowing the region where $\alpha \in (0, +\infty)$ and $f \in (0, +\infty)$ reveals the complete cyclic spectrum.

4 Simulation Results

Signal reconstruction is key to compressed sensing. The sparser the signal in a particular domain, the higher the reconstruction probability. For the STOMP algorithm described herein, threshold parameter selection and observation quantity critically affect reconstruction quality. With signal count $N = 256$, the relationship between threshold values, sparsity, and observation count is shown in Figures 2-4 [FIGURE:2-4].

These figures demonstrate that better signal sparsity in a domain requires more observations for successful reconstruction. Larger thresholds extract less information from the original signal, requiring more observations for good reconstruction. Smaller thresholds extract redundant information, also requiring more observations. A threshold of 2.6 yields slightly better reconstruction than thresholds of 2 or 3. When observation quantity reaches a certain level, signals can be completely reconstructed.

Since stationary noise and interference do not exhibit spectral correlation at $\alpha \neq 0$, modulation signal detection and estimation using spectral correlation functions at $\alpha \neq 0$ can completely eliminate background noise effects. For BPSK-modulated signals with noise, the double carrier frequency can be easily detected on the $f = 0$ cyclic spectrum section, completing signal detection. This experiment uses BPSK signals with simulation parameters: sampling frequency $f_s = 1600$ kHz, carrier frequency $f_c = 200$ kHz, code rate $f_b = 10$ kbps, SNR = -3 dB, and data length $N = 1000$. MATLAB simulations of BPSK cyclic spectrum and its $f = 0$ and $\alpha = 0$ sections are shown in Figures 5-7 [FIGURE:5-7].

Reconstructing the cyclic spectrum section signal via STOMP yields the result shown in Figure 8 [Figure 8: see original paper]. According to the theory, the BPSK signal’ s $f = 0$ cyclic spectrum section contains spectral signals only at double frequencies. Searching for the two largest peaks in the reconstructed diagram and calculating their corresponding frequency values yields the carrier frequency. The measured peaks are 406 kHz and -394 kHz, with a 6 kHz error from theoretical cyclic spectrum values. Using Equation (16), the estimated carrier frequency is $(|406| + |-394|)/2 = 200$ kHz, matching the experimental result. The results show that under -3 dB SNR, the estimated carrier frequency matches the provided value with accuracy below 10^3 Hz.

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5 Conclusion

Cyclostationary theory primarily utilizes signal statistical parameters such as mean and correlation functions to study non-stationary signals, finding wide application in signal detection algorithms. This paper analyzes the feasibility of using cyclic spectrum theory for signal detection and parameter extraction, proposing a signal reconstruction approach for cyclic spectrum sections based on compressed sensing principles. The STOMP algorithm employed here adds stagewise search to OMP’ s framework, using a threshold during each iteration to select atoms, ensuring more accurate reconstruction of original signals. Simulation results show that with a threshold of 2.6, fewer observations are needed for signal reconstruction, reducing computational load. Under -3 dB SNR, the estimated carrier frequency matches experimental values with accuracy below 10^3 Hz, demonstrating practical engineering value.

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