Postprint: Percolation Density and Coverage Node Density in Three-Dimensional Wireless Sensor Networks Based on Percolation Theory

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Abstract

To ensure coverage and connectivity in wireless sensor networks, we explore the impact of the ratio between communication radius and sensing radius on coverage and connectivity in three-dimensional wireless sensor networks. Based on percolation theory, we derive the overlapping volume function for cooperative transmission paths in three-dimensional wireless sensor networks. Considering the relationship between the ratio of communication radius to sensing radius and the overlapping volume function, we obtain the influence of the overlapping volume function and the ratio of communication radius to sensing radius on the percolation density of three-dimensional wireless sensor networks. On this basis, when the communication radius equals the sensing radius, we combine the obtained percolation density with the coverage node density to derive the relationship between the specified coverage ratio and the coverage node density. Simulation experiments demonstrate that in three-dimensional wireless sensor networks, the overlapping volume is directly proportional to the percolation density; the ratio of communication radius to sensing radius is inversely proportional to the percolation density; and as the specified coverage ratio increases, the obtained coverage node density also increases.

Full Text

Preamble

Study on Seepage Density and Coverage Node Density of Three-Dimensional Wireless Sensor Networks Based on Seepage Theory

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Abstract: To ensure coverage and connectivity in wireless sensor networks, this paper explores the impact of the ratio between communication radius and sensing radius on three-dimensional wireless sensor network coverage and connectivity. Based on percolation theory, we derive the overlapping volume function for cooperative transmission paths in three-dimensional wireless sensor networks. Considering the relationship between the ratio of communication radius to sensing radius and the overlapping volume function, we analyze their combined influence on the seepage density of three-dimensional wireless sensor networks. Building upon this analysis, when the communication radius equals the sensing radius, we establish the relationship between specified coverage and coverage node density by integrating seepage density with node density. Simulation experiments demonstrate that in three-dimensional wireless sensor networks, overlapping volume is directly proportional to seepage density, while the ratio of communication radius to sensing radius is inversely proportional to seepage density. Furthermore, as specified coverage increases, the derived coverage node density also increases.

Keywords: wireless sensor networks; three-dimensional coverage; seepage density; overlapping volume function

0 Introduction

Wireless sensor networks possess sensing, computing, and communication capabilities, with broad applications in military reconnaissance, environmental monitoring, infrastructure management, disaster search and rescue, equipment diagnostics, and other industrial domains. In these networks, nodes (also called sensors) are typically powered by low-capacity batteries. To extend network lifetime, reducing node energy consumption is a critical consideration. A fundamental research objective in wireless sensor networks is to achieve maximum coverage area with the minimum number of nodes.

Coverage and connectivity provide higher-quality services for wireless sensor networks while conserving energy more effectively. Numerous models have been proposed for coverage and connectivity research. Alam systematically addressed three-dimensional random coverage and connectivity problems in practical communication scenarios, ensuring 100% coverage with minimal node deployment. Wang investigated local coverage issues in three-dimensional lattice models for wireless sensor networks, while Ammari examined node coverage problems using spherical models for three-dimensional wireless sensor networks. This study adopts the spherical model approach. Wang proposed a strip-based node deployment pattern to achieve regional coverage, while Wang utilized node provisioning rates to represent deployment efficiency in random three-dimensional wireless sensor networks. Sui introduced differential evolution algorithms to optimize sensor node distribution, and Dang improved node deployment efficiency through scheduling algorithms and neighbor node classification scheduling algorithms. Gupta addressed random coverage and connectivity problems in three-dimensional wireless sensor networks using heterogeneous directional sensor models, while Yan proposed an improved deployment algorithm for three-dimensional wireless sensor networks. Broadbent applied percolation theory to solve connectivity problems in wireless sensor networks.

Various factors influence network coverage. Pompili investigated the impact of several fundamental performance metrics on coverage and connectivity in three-dimensional wireless sensor networks. Building on this work, Adlakha studied the relationship between coverage/connectivity and the ratio of communication radius to sensing radius. This paper also considers these two factors and provides a detailed analysis.

This research focuses on the overlapping coverage volume function and the ratio of communication radius to sensing radius as key factors affecting coverage and connectivity performance in three-dimensional wireless sensor networks. Using geometric principles, we derive the actual overlapping volume function for two spheres in a cooperative transmission path and establish the relationship between overlapping volume function and seepage density. We demonstrate that the seepage density under the actual overlapping volume function is smaller than that under approximate overlapping volume functions, making the actual function more advantageous. Furthermore, we derive the relationship between the actual overlapping volume function and the ratio of communication radius to sensing radius, showing that when this ratio increases within the interval, seepage density gradually decreases. The ratio is inversely proportional to seepage density, and for a fixed ratio, smaller overlapping volume functions yield smaller seepage densities, providing greater advantages. Finally, by combining the overlapping volume function with coverage node density, we establish the relationship between specified coverage and coverage node density, demonstrating that coverage node density increases with specified coverage.

1 Definitions and Theorems

Definition 1 (Spatial Poisson Distribution). Let X_λ be a homogeneous Poisson process with density λ in three-dimensional space, where ξ_i represents oriented sensors centered at ξ_i.

Assumption 1 (Unit Sphere Model). The sensing range of sensor s_i is a sphere with radius r, defined by the set of points $\{B_i^r = \{\xi \in \mathbb{R}^3 : \|\xi - \xi_i\| \leq r\}\}$. The communication range of sensor s_i is a sphere with radius R, defined by the set of points $\{B_i^R = \{\xi \in \mathbb{R}^3 : \|\xi - \xi_i\| \leq R\}\}$, where $\|\xi - \xi_i\|$ denotes the Euclidean distance between ξ and ξ_i.

Assumption 2 (Uniform Sensing Model). All deployed sensors have identical sensing radius r and identical communication radius R.

Definition 2 (Collaborative and Communication Sensors). Two sensors s_i and s_j are collaborative if and only if the Euclidean distance between their sensing disk centers satisfies $\|\xi_i - \xi_j\| \leq 2r$, meaning the two sensing disks are tangent or overlapping (as shown in Figure 1: see original paper). A set of collaborative sensors is denoted as $\mathrm{Col}(s_i)$.

Two sensors s_i and s_j can communicate if and only if the Euclidean distance between their sensing disk centers satisfies $\|\xi_i - \xi_j\| \leq R$ (as shown in Figure 1: see original paper). A set of communicating sensors is denoted as $\mathrm{Com}(s_i)$.

Definition 3 (Coverage and Connectivity Sensors). Two sensors s_i and s_j achieve both coverage and connectivity only when they simultaneously satisfy both collaborative and communication conditions, as illustrated in [Figure 2: see original paper].

Definition 4 (Collaborative and Transmission Paths). A collaborative path between two sensors s_i and s_j is a sequence of sensors $\{s_i, s_{i+1}, \dots, s_{j-1}, s_j\}$ where any pair of adjacent sensors can collaborate, with $l \leq i < j$ (as shown in Figure 3: see original paper).

A transmission path between two sensors s_i and s_j is a sequence of sensors $\{s_i, s_{i+1}, \dots, s_{j-1}, s_j\}$ where any pair of adjacent sensors can communicate, with $l \leq i < j$ (as shown in Figure 3: see original paper).

Definition 5 (Boolean Model). The Boolean model consists of two components: a point process X_λ and a connection function h. Here, X_λ is a homogeneous Poisson process with density λ in three-dimensional Euclidean space $\mathbb{R}^3$. The principle of X_λ is to cover certain regions with oriented sensors. The connection function h is defined as $h(\xi_i, \xi_j) = 1$ if the Euclidean distance between adjacent points ξ_i and ξ_j satisfies $\|\xi_i - \xi_j\| \leq d$, and $h(\xi_i, \xi_j) = 0$ if $\|\xi_i - \xi_j\| > d$.

Theorem 1 (Critical Seepage Density for Coverage and Connectivity). In three-dimensional wireless sensor networks, the critical seepage density can be expressed as:

$$\lambda_{cs} = 0.955 \cdot \frac{1}{\omega_s} \cdot \frac{1}{\alpha^3}, \quad 1 \leq \alpha \leq 2$$

where $\alpha = R/r$, $\omega_s = V_{\min}/V_s$ is the overlapping volume fraction, $V_{\min}$ is the minimum overlapping volume, and V_s is the volume of a sphere.

Theorem 2 (Coverage Node Density). In a three-dimensional monitoring region with side length L, the coverage node density, denoted by n, represents the number of nodes covered by a node' s sensing area under a specified coverage rate ϕ. This is an important metric for evaluating node coverage area and can be expressed as:

$$n = \frac{\lg(1-\phi)}{\lg(1-\omega_s)}$$

where ϕ is the specified coverage rate with range $[0, 1]$, and $\omega_s = V_{\min}/V_s$ is the overlapping volume fraction.

2 Actual Overlapping Volume Function

In practical scenarios, a node' s sensing range exists in three-dimensional space. Considering a node' s connectivity region as a sphere, the distance between two nodes is $d \leq R$ under coverage and connectivity conditions, where R is the node' s connectivity radius. For a node pair, placing one node at the origin of a three-dimensional coordinate system and the other at position $(R, 0, 0)$ on the x-axis, the two sphere functions can be expressed as:

$$\begin{cases} x^2 + y^2 + z^2 = R^2 \\ (x - R)^2 + y^2 + z^2 = R^2 \end{cases}$$

From these equations, we obtain:

$$x = \frac{R}{2}$$

The intersection plane of the two spheres is parallel to the yz-plane at $x = R/2$. Substituting into the sphere equation yields:

$$y^2 + z^2 = \frac{3}{4}R^2$$

The overlapping volume of two spheres can be divided into two parts: (a) two cones (with base radius $\sqrt{3}R/2$ and height $R/2$); and (b) the remaining portion of the sphere after subtracting two spherical frustums. The sphere radius is R, the spherical frustum has an upper base radius of $\sqrt{3}R/2$, lower base radius of R, and height of $R/2$.

[Figure 4: see original paper] shows the planar view of two overlapping spheres. The first part consists of two cones as shown in [Figure 5: see original paper], and the second part is calculated as shown in [Figure 6: see original paper].

The volume of the two cones is:

$$V_1 = 2 \times \frac{1}{3} \times \pi \times \left(\frac{\sqrt{3}}{2} R \right)^2 \times \frac{R}{2} = \frac{\pi R^3}{4}$$

The volume of the spherical frustum is calculated as follows. The upper base area is $S_{\text{upper}} = \pi \times (\sqrt{3}R/2)^2 = \frac{3\pi R^2}{4}$, and the lower base area is $S_{\text{lower}} = \pi R^2$. The frustum volume is:

$$V_{\text{frustum}} = \frac{h}{3}(S_{\text{upper}} + S_{\text{lower}} + \sqrt{S_{\text{upper}} S_{\text{lower}}}) = \frac{7\pi R^3}{24}$$

The sphere volume is $V_{\text{sphere}} = \frac{4\pi R^3}{3}$. The volume of the six small regions after removing the two frustums is:

$$V_{\text{six small}} = V_{\text{sphere}} - 2V_{\text{frustum}} = \frac{4\pi R^3}{3} - 2 \times \frac{7\pi R^3}{24} = \frac{9\pi R^3}{12}$$

The volume of one small region is:

$$V_{\text{one small}} = \frac{V_{\text{six small}}}{6} = \frac{3\pi R^3}{24}$$

The volume of four small regions is:

$$V_{\text{four small}} = 4 \times V_{\text{one small}} = \frac{\pi R^3}{2}$$

Therefore, the minimum overlapping volume is:

$$V_{\min} = V_{\text{four small}} + V_1 = \frac{\pi R^3}{2} + \frac{\pi R^3}{4} = \frac{3\pi R^3}{4}$$

From Theorem 1, we have:

$$\omega_s = \frac{V_{\min}}{V_s} = \frac{3\pi R^3/4}{4\pi R^3/3} = \frac{9}{16}$$

The range of the unknown variable in the equation is:

$$\frac{1}{2} \leq \kappa \leq 1$$

2.1 Effect of R/r Ratio on Critical Seepage Density

Setting $R = r$ in the equations yields the minimum actual overlapping volume under connectivity conditions:

$$V_{\min} = \frac{3\pi R^3}{4}$$

The corresponding actual seepage density is:

$$\lambda_{cs} = 0.955 \cdot \frac{1}{\omega_s} = 0.955 \cdot \frac{1}{9/16} = 1.698$$

The coverage node density using the approximate overlapping volume can be expressed as:

$$n_{\text{approx}} = \frac{\lg(1-\phi)}{\lg(1-\omega_s^{\text{approx}})}$$

The approximate seepage density is:

$$\lambda_{cs}^{\text{approx}} = 0.955 \cdot \frac{1}{\omega_s^{\text{approx}}}$$

[Figure 7: see original paper] compares the actual seepage density derived in this paper with previous approximate seepage densities. In the figure, the left point in the seepage density function represents the actual overlapping volume value. The actual seepage density (y2) begins to exist from the left blue line, while the approximate seepage density (y1) begins from the right blue line, indicating that the minimum actual overlapping volume is smaller than the minimum approximate overlapping volume, resulting in a smaller actual seepage density.

[Figure 8: see original paper] shows that as the overlapping volume fraction increases, seepage density decreases. Among curves y1, y2, y3, and y4, the overlapping volume fractions increase gradually while seepage density decreases. As the ratio R/r increases, seepage density also decreases. In the figure, y1 represents the actual overlapping volume fraction, y2 the approximate overlapping volume fraction, y3 an overlapping volume fraction of 0.5, and y4 an overlapping volume fraction of 0.75. The actual value yields the smallest density compared to approximate and other values, demonstrating its advantage.

2.2 Coverage Node Density

The coverage node density under actual minimum overlapping volume is:

$$n = \frac{\lg(1-\phi)}{\lg(1-9/16)} = \frac{\lg(1-\phi)}{\lg(7/16)}$$

In literature [9], the distance between two nodes in a cooperative transmission path is divided into two cases: $d = r/2$ and $d = 3r/2$. The coverage node densities for these cases are:

When $d = r/2$, the overlapping volume is:

$$V_{\min} = \frac{7\pi r^3}{12}$$

From Theorem 1:

$$\omega_s = \frac{V_{\min}}{V_s} = \frac{7}{16}$$

When $d = 3r/2$, the overlapping volume is:

$$V_{\min} = \frac{11\pi r^3}{96}$$

From Theorem 1:

$$\omega_s = \frac{V_{\min}}{V_s} = \frac{11}{128}$$

[Figure 9: see original paper] shows the simulation results from equations (29) and (30). In the figure, y1 represents the relationship between specified coverage and coverage node density using the derived actual minimum overlapping volume, while y2 represents the relationship using approximate overlapping volume. When specified coverage increases within $[0, 1]$, coverage node density also increases. y1 grows faster than y2, meaning that for the same specified coverage, y1 yields higher coverage node density, making the method more advantageous.

[Figure 10: see original paper] presents simulation results from equations (29), (33), and (36). In the figure, y1 represents the coverage node density derived in this paper, y2 represents the coverage node density when inter-node distance is $r/2$, and y3 represents the coverage node density when inter-node distance is $3r/2$. When specified coverage increases within $[0, 1]$, coverage node density increases in all three cases. However, y1 grows faster than y2 and y3, yielding the highest coverage node density for the same specified coverage, thus demonstrating superior performance.

Numerical comparisons in [Figure 9: see original paper] and [Figure 10: see original paper] confirm that the derived coverage node density is maximal under the same specified coverage, providing the greatest advantage.

3 Conclusion

This paper investigates coverage and connectivity issues in three-dimensional wireless sensor networks, considering the influencing factors of seepage density: overlapping volume function and the ratio of communication radius to sensing radius. Through calculations of the actual overlapping volume function and comparison with approximate overlapping volumes, we demonstrate that the actual overlapping volume is smaller than approximate values, resulting in smaller seepage density and greater advantages. When the ratio R/r increases, seepage density gradually decreases, showing an inverse proportional relationship. Additionally, as specified coverage increases, coverage node density also increases. Under the derived actual overlapping volume function, the minimum number of nodes required to achieve coverage and connectivity in a target region can be determined.

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