Joint Detection and Decoding Technology for SCMA and Polar Codes (Postprint)

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Abstract

By combining polar codes with SCMA multiple access systems and addressing the issue of poor system performance in traditional independent detection and decoding (IDD) schemes due to the lack of reuse of decoding output information, a joint detection and decoding (JDD) scheme for SCMA and polar codes is proposed. This scheme utilizes the intrinsic information obtained from decoding at the receiver to assist in updating the initial prior information of the SCMA multi-user detector, enabling iterative feedback of soft information between the detector and decoder, thereby achieving more significant system performance gains. Simulation results demonstrate that the system performance with the JDD scheme achieves significant improvement compared to the IDD scheme, with the bit error rate improved by approximately 2 dB.

Full Text

Preamble

Joint Detection and Decoding Scheme for SCMA System with Polar Codes

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Abstract: This paper combines Sparse Code Multiple Access (SCMA) with Polar Codes and proposes a joint SCMA detection and Polar decoding (JDD) scheme for the system. The independent SCMA detection and Polar decoding (IDD) scheme is unable to take full advantage of decoding information, which leads to poor system performance. To address this problem, the proposed JDD scheme achieves significant performance gain by iteratively exchanging soft information between the detector and the decoder, using intrinsic information to update the initial priori probabilities in the SCMA detector. Simulation results show that the performance of the system with the JDD scheme has been significantly improved, obtaining about 2 dB performance gain compared with the IDD scheme.

Key words: Polar Codes; Sparse Code Multiple Access; joint detection and decoding; high performance gain

0 Introduction

In 2009, Arikan first elaborated on channel polarization phenomena and proposed Polar Codes encoding theory based on this principle. Polar Codes have explicit encoding schemes and are a class of channel coding theories proven to achieve Shannon capacity limits on binary discrete memoryless channels (B-DMC) with low encoding and decoding complexity. Due to their excellent capacity-achieving performance, Polar Codes were officially adopted by 3GPP in 2016 as the final coding scheme for control channels in 5G eMBB scenarios. Research on Polar Codes has been widely expanded, including the belief propagation (BP) decoding algorithm which enables parallel information iteration across factor graphs, achieving excellent performance in decoding delay and throughput.

Sparse Code Multiple Access (SCMA) is a promising 5G wireless air interface technology designed to support massive user connectivity and achieve much higher data transmission rates than 4G. SCMA evolved from LDS-CDMA technology, integrating high-dimensional modulation with sparse spreading by directly mapping encoded and interleaved bit streams to predefined complex-domain multi-dimensional codewords, enabling multi-user detection at the receiver for higher overloading rates. While maximum a posteriori probability (MAP) algorithms can achieve optimal detection performance, their prohibitive complexity limits practical application. Thanks to the sparse structure of SCMA codes, the message passing algorithm (MPA) can effectively reduce detection complexity with minimal performance loss.

As both SCMA and Polar Codes are excellent candidate technologies for 5G standards, this paper combines SCMA multiple access technology with Polar Codes encoding. To further improve system performance and achieve lower bit error rates, we propose a joint detection and decoding (JDD) scheme for the receiver. This scheme mathematically processes the soft information from decoder output and SCMA detector output to extract intrinsic information (prior information), which is then fed back to update the initial user prior probabilities in the SCMA detector. Through iterative updates, the system achieves significant performance gains, addressing the limitation of traditional independent detection and decoding (IDD) schemes where performance improvement is marginal due to lack of information reuse.

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1 System Model

[Figure 1: see original paper] illustrates a simplified uplink Polar-SCMA multiple access system with J users and F resource blocks. Each user simultaneously occupies different resource blocks, with each resource block superimposing data from different users to achieve overloading gain. For user j, K information bits are encoded by the Polar encoder to generate N coded bits, with code rate defined as $R = K/N$. To mitigate interference from burst errors, coded bits $\mathbf{c}_j$ pass through an interleaver to produce interleaved bits $\mathbf{b}_j$, which are then fed into the SCMA encoder. The interleaved bits $\mathbf{b}_j$ are grouped into sets of $\log_2(M)$ bits, each selecting a K-dimensional codeword from the SCMA codebook according to the mapping rule $\chi : \{0,1\}^{\log_2 M} \rightarrow \mathcal{X}_j$, where $\mathcal{X}_j \subset \mathbb{C}^K$ and $|\mathcal{X}_j| = M$. The received signal $\mathbf{y}$ is given by:

$$\mathbf{y} = \sum_{j=1}^{J} \text{diag}(\mathbf{h}_j)\mathbf{x}_j + \mathbf{z}$$

where $\mathbf{h}_j$ represents the channel vector for user j, $\mathbf{x}_j$ is the codeword symbol after bit mapping, and $\mathbf{z}$ is additive Gaussian noise.

Before user data enters the Polar encoder, code construction is performed through channel reliability estimation. This involves calculating and sorting subchannel capacities (higher capacity means higher reliability), placing user data on the most reliable subchannels while filling remaining subchannels with redundant data (frozen bits, typically set to 0). Current methods for channel reliability estimation include Monte Carlo methods, density evolution, and Gaussian approximation.

Polar Codes can be represented as $(N, K, \mathcal{A}, u_{\mathcal{A}^c})$, where N is the coded bit length, K is the information bit length, $\mathcal{A}$ denotes the set of subchannel positions for information bits, and $u_{\mathcal{A}^c}$ represents frozen bits (generally set to 0). The encoding process is:

$$\mathbf{b}_j = \mathbf{u}_j \mathbf{G}_N = \mathbf{u}_j \mathbf{B}_N \mathbf{F}^{\otimes n}$$

where $\mathbf{u}_j$ is the user data, $\mathbf{B}_N$ is the bit-reversal permutation matrix, $\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, and $\otimes$ denotes Kronecker product.

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2 Basic Algorithms

This section introduces the SCMA multi-user detection algorithm based on MPA and the Polar Codes BP decoding algorithm, which form the fundamental building blocks of the joint detection and decoding (JDD) scheme.

2.1 SCMA Detection Algorithm

The MPA-based SCMA detection operates on a factor graph with variable nodes (VNs) representing users and function nodes (FNs) representing resource blocks. The algorithm proceeds as follows:

Initialization: Due to lack of prior information about codewords, all codeword symbols are assumed equally probable. The message from VN j to FN f is initialized as:

$$V_{j\to f}^{(0)}(x_j) = \frac{1}{M}, \quad m = 1, 2, \dots, M$$

Message Update: Using the received signal $\mathbf{y}$ and channel information $\mathbf{h}_j$, the message from FN f to VN j is updated as:

$$U_{f\to j}^{(p)}(x_j) = \sum_{\mathbf{x}_{\partial f\setminus j}} \frac{1}{\pi N_0} \exp\left(-\frac{\|y_f - \sum_{j'\in\partial f} h_{j',f} x_{j',f}\|^2}{N_0}\right) \prod_{j'\in\partial f\setminus j} V_{j'\to f}^{(p-1)}(x_{j'})$$

where ∂f denotes the set of user nodes connected to FN f, and $\partial f \setminus j$ represents all users connected to FN f except user j.

VN Update: The variable node updates its message as:

$$V_{j\to f}^{(p)}(x_j) = \prod_{f'\in\partial j\setminus f} U_{f'\to j}^{(p)}(x_j)$$

To prevent numerical overflow, normalization is applied after each update. After several iterations, VN j outputs the symbol probabilities, which are converted to bit probabilities for the decoder:

$$P(x_j) = \prod_{f\in\partial j} U_{f\to j}^{(p)}(x_j)$$

2.2 Polar Codes Decoding Algorithm

The BP decoding algorithm for Polar Codes operates on the factor graph shown in [Figure 2: see original paper]. [Figure 3: see original paper] depicts the basic processing element consisting of four nodes indexed by (i, j), where $1 \le i \le N$ and $1 \le j \le n+1$. Each node contains two types of messages: left-to-right messages $L_{i,j}$ and right-to-left messages $R_{i,j}$.

Algorithm 1: Polar BP Decoding

1. **Initialization:** For all $i \in \{1, 2, \ldots, N\}$:
    - $R_{i,n+1} = \begin{cases} 0 & \text{if } i \in \mathcal{A} \\ \infty & \text{if } i \notin \mathcal{A} \end{cases}$
    - $L_{i,1} = \ln \frac{P(y_i|0)}{P(y_i|1)}$
2. **Iterative Decoding:** For iteration $t = 1$ to T:
    - **Left update:** For $j = 1$ to n, $i = 1$ to $N/2$:

      $$L_{i,j+1} = f(L_{i,j}, L_{i+N/2,j})$$
      $$L_{i+N/2,j+1} = f(R_{i,j}, L_{i+N/2,j}) + L_{i+N/2,j}$$

    - **Right update:** For $j = n$ down to 1, $i = 1$ to $N/2$:

      $$R_{i,j} = f(R_{i,j+1}, L_{i+N/2,j}) + R_{i+N/2,j}$$
      $$R_{i+N/2,j} = f(R_{i,j+1}, L_{i,j})$$

      where $f(x, y) = \ln \frac{1+e^{x+y}}{e^x+e^y}$ can be approximated as $f(x, y) \approx \text{sign}(x)\text{sign}(y) \min(|x|, |y|)$.
3. **Output:** After T iterations, output the LLRs: $L_i = L_{i,n+1}$ for $i = 1, \ldots, N$.

3 Joint Detection and Decoding Algorithm

Traditional independent detection and decoding (IDD) schemes fail to fully exploit decoding information, leaving significant performance gains unrealized. Joint detection and decoding algorithms have been applied to SCMA systems with various channel codes. This section applies the joint detection and decoding (JDD) algorithm to Polar-coded SCMA systems, achieving excellent performance gains by reusing decoder output information to update user prior probabilities in the detector.

3.1 Probability Conversion Module

The SCMA detector outputs symbol probabilities, while the decoder operates on bit probabilities. Therefore, conversion between these domains is necessary. The conversion uses log-likelihood ratios (LLRs) to simplify calculations.

Symbol-to-bit probability conversion:

$$L(c_{j,v}) = \ln \frac{P(c_{j,v} = 0|\mathbf{y})}{P(c_{j,v} = 1|\mathbf{y})} = \ln \frac{\sum_{\mathbf{x}_j \in \mathcal{X}_j^0(v)} P(\mathbf{x}_j|\mathbf{y})}{\sum_{\mathbf{x}_j \in \mathcal{X}_j^1(v)} P(\mathbf{x}_j|\mathbf{y})}$$

where $\mathcal{X}_j^b(v)$ is the set of codewords where the v-th bit equals b.

Bit-to-symbol probability conversion:

$$P(\mathbf{x}_j) = \prod_{v=1}^{\log_2 M} \frac{\exp((1 - 2c_{j,v})L(c_{j,v}))}{1 + \exp(L(c_{j,v}))}$$

3.2 Joint Detection and Decoding Structure

[Figure 4: see original paper] illustrates the JDD structure for Polar-SCMA systems. The process involves iterative exchange of extrinsic information between the SCMA detector and Polar decoder.

Algorithm 2: Joint Detection and Decoding

1. **Initialization:** Set iteration counter $t = 0$. Initialize SCMA detector with equal prior probabilities: $L_p^{(0)}(c_{j,v}) = 0$ for all users j and bits v.
2. **SCMA Detection:** Run MPA detection to obtain symbol probabilities $P^{(t)}(\mathbf{x}_j|\mathbf{y})$, then convert to bit LLRs $L^{(t)}(c_{j,v})$ using the symbol-to-bit conversion.
3. **Extrinsic Information Extraction:** Compute extrinsic information:

$$L_{ext}^{(t)}(c_{j,v}) = L^{(t)}(c_{j,v}) - L_p^{(t)}(c_{j,v})$$

4. **Deinterleaving and Decoding:** Deinterleave $L_{ext}^{(t)}(c_{j,v})$ and feed to Polar BP decoder to obtain updated LLRs $L_{dec}^{(t)}(c_{j,v})$.
5. **Prior Information Update:** Transform decoder output to symbol priors:

$$L_p^{(t+1)}(c_{j,v}) = \text{Transform}(L_{dec}^{(t)}(c_{j,v}))$$

6. **Iteration:** Increment t and repeat steps 2-5 until maximum outer iterations O is reached.
7. **Final Decision:** After final iteration, make hard decisions: $\hat{u}_j = 0$ if $L_{dec}^{(t)}(c_{j,v}) \geq 0$, else $\hat{u}_j = 1$.

The core innovation is the information backtracking iteration structure, where decoder-generated prior information $L_p(c_{j,v})$ continuously refines the SCMA detector’ s initial assumptions.

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4 System Simulation Analysis

This section validates the proposed JDD scheme through performance simulations and comparative analysis with traditional IDD schemes under various conditions.

4.1 System Parameter Settings

System parameters for IDD and JDD algorithms are summarized in . The SCMA sparse matrix is:

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Table 1: System Simulation Parameters

Parameter	Value
SCMA detection iterations	2, 3, 4
BP decoding iterations	50
Outer iterations (JDD)	1, 2, 3, 4
Polar code construction	Improved Gaussian approximation
Code rates	0.5
Modulation	SCMA codebook mapping

The SCMA detector output $P^{(t)}(\mathbf{x}_j|\mathbf{y})$ is converted to bit LLRs $L^{(t)}(c_{j,v})$ using:

$$L^{(t)}(c_{j,v}) = \ln \frac{P^{(t)}(c_{j,v}=0|\mathbf{y})}{P^{(t)}(c_{j,v}=1|\mathbf{y})} = L^{(t)}_{ext}(c_{j,v}) + L^{(t)}_{p}(c_{j,v})$$

where $L^{(t)}_{ext}(c_{j,v})$ is extrinsic information from the detector and $L^{(t)}_{p}(c_{j,v})$ is prior information from the previous decoding iteration. At initialization ($t = 0$), $L^{(0)}_{p}(c_{j,v}) = 0$ (equal probability).

4.2 Simulation Results Analysis

[Figure 5: see original paper] and [Figure 6: see original paper] compare the bit error rate (BER) performance for code lengths $N = 256$ and $N = 1024$ respectively, both with code rate $R = 0.5$.

Key observations: - The JDD scheme significantly outperforms IDD across all SNR ranges. - For $N = 256$, JDD provides approximately 1.5 dB gain over IDD at BER 10^{-4}. - For $N = 1024$, the gain increases to approximately 2 dB, demonstrating that JDD benefits more from longer code lengths. - Performance gains saturate after 3-4 outer iterations due to increasing correlation in backtracked information.

Polar Codes exhibit better performance with longer code lengths. [Figure 7: see original paper] and [Figure 8: see original paper] show subchannel reliability distributions for $N = 256$ and $N = 1024$. We define subchannel reliability $z_i \in [0, 1]$. According to polarization theory, as $N \to \infty$, subchannels become completely polarized ($z_i \to 0$ or 1). In practice, some subchannels have intermediate reliability. Simulation statistics show:

- For $N = 256$: 672 of 1024 subchannels have $0.1 < z_i < 0.9$ (65.6%)
- For $N = 1024$: 168 of 1024 subchannels have $0.1 < z_i < 0.9$ (16.4%)

Thus, $N = 1024$ shows better polarization and system performance, confirming that longer codes yield superior gains. The JDD scheme' s performance improvement is more pronounced with better-polarized codes.

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5 Conclusion

This paper proposes a joint detection and decoding (JDD) scheme for SCMA systems with Polar Codes. By reusing decoder output information to update user prior probabilities in the SCMA detector, the JDD scheme achieves significant performance gains through iterative soft information exchange. Simulation results demonstrate:

- Approximately 1.5 dB gain for $N = 256$ and 2 dB gain for $N = 1024$ compared to IDD.
- Performance improvements saturate after 3-4 outer iterations.
- Longer code lengths provide better polarization and greater JDD gains.

The JDD approach effectively addresses the limitation of independent detection and decoding by creating a feedback loop that leverages decoding confidence to refine multi-user detection, making it a promising solution for 5G systems requiring high reliability and massive connectivity.

References

[1] Arikan E. Channel polarization: a method for constructing capacity-achieving codes for symmetric binary-input memoryless channels [J]. IEEE Trans on Information Theory, 2009, 55(7): 3051-3073.

[2] Tal I, Vardy A. List decoding of Polar Codes [J]. IEEE Trans on Information Theory, 2015, 61(5): 2213-2226.

[3] Niu Kai, Chen Kai. CRC-aided decoding of polar codes [J]. IEEE Communications Letters, 2012, 16(10): 1668-1671.

[4] Arikan E. A performance comparison of polar codes and Reed-Muller codes [J]. IEEE Communications Letters, 2008, 12(6): 447-449.

[5] Nikopour H, Baligh H. Sparse code multiple access [C]//Proc of International Symposium on Personal Indoor and Mobile Radio Communications. 2013: 332-336.

[6] Hoshyar R, Wathan F P, Tafazolli R. Novel low-density signature for synchronous CDMA systems over AWGN channel [J]. IEEE Trans on Signal Processing, 2008, 56(4): 1616-1626.

[7] Kschischang F R, Frey B J, Loeliger H A. Factor graphs and the sum-product algorithm [J]. IEEE Trans on Information Theory, 2001, 47(2): 498-519.

[8] Arikan E. Channel Polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels [J]. IEEE Trans on Information Theory, 2009, 55(7): 3051-3073.

[9] Mori R, Tanaka T. Performance of polar codes with the construction using density evolution [J]. IEEE Communications Letters, 2009, 13(7): 519-521.

[10] Chung S Y, Richardson T J, Urbanke R L. Analysis of sum-product decoding of low-density parity-check codes using a Gaussian approximation [J]. IEEE Trans on Information Theory, 2001, 47(2): 657-670.

[11] Yang Junmei, Zhang Chuan, Zhou Huayi, et al. Pipelined belief propagation polar decoders [C]//Proc of IEEE International Symposium on Circuits and Systems. 2016: 413-416.

[12] Han Kaining, Zhang Zhenbing, Hu Jianhao, et al. A factor graph based joint detection and decoding scheme for LDPC coded SCMA system [J]. Journal of University of Electronic Science and Technology of China, 2017, 46(5): 685-691.

[13] Wu Yiqun, Zhang Shunqing, Chen Yan. Iterative multiuser receiver in sparse code multiple access systems [C]//Proc of IEEE International Conference on Communications. 2015: 2918-2923.

[14] Zhang Zhenbing, Han Kaining, Hu Jianhao, et al. Joint detection and decoding schemes for turbo coded SCMA systems [C]//Proc of GLOBECOM Workshops. 2017: 1-6.

[15] Xiao Baisen, Xiao Kexin, Zhang Shutian, et al. Iterative detection and decoding for SCMA systems with LDPC codes [C]//Proc of International Conference on Wireless Communications & Signal Processing. 2015: 1-5.

[16] Dai Jincheng, Niu Kai, Si Zhongwei, et al. Evaluation and Optimization of Gaussian Approximation for Polar Codes [J]. Proceedings of the American Society for Information Science & Technology, 2016, 51(1): 1-2.

Note: Figure translations are in progress. See original paper for figures.