Perturbation Analysis Method for the Impact of Satellite Orbit Errors on Positioning Accuracy (Postprint)

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Abstract

Satellite navigation system positioning accuracy is comprehensively influenced by multiple factors, including pseudorange measurement errors, atmospheric delay errors, satellite atomic clock biases, and satellite orbit errors. Traditionally, evaluation of positioning errors typically employs methods based on Dilution of Precision (DOP) and User Equivalent Range Error (UERE). However, the derivation of its accuracy characterization formula requires several assumptions regarding the coefficient matrix H of the measurement equation system and the error distribution of the User Equivalent Range Error, making it essentially an approximate evaluation formula. Furthermore, among various error sources, satellite orbit errors constitute three-dimensional errors that require coordinate transformation and empirical parameter models to be converted into User Equivalent Range Error. To address this issue, we propose utilizing matrix perturbation theory to investigate the influence of satellite orbit errors on the solution of the positioning equation system, employing the spectral norm condition number to characterize the system' s structure. Simulation results demonstrate that the proposed method can directly reflect the impact of satellite orbit errors on positioning accuracy without requiring conversion of orbit coordinates or User Equivalent Range Error, thereby enabling a more direct and accurate assessment of the influence of satellite orbit errors on positioning solution accuracy.

Full Text

Study on Perturbation Analysis Method of the Influence of Satellite Orbit Error on Positioning Accuracy

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Abstract

GNSS positioning accuracy is comprehensively affected by multiple factors including pseudo-range measurement error, atmospheric delay error, satellite atomic clock error, and satellite orbit error. Traditionally, the Dilution of Precision (DOP) and User Equivalent Range Error (UERE) parameters are employed to evaluate positioning error. However, the derivation of their precision representation formula requires several assumptions about the measurement equation coefficient matrix H and the error distribution of UERE, making it essentially an approximate evaluation formula. Furthermore, among various error sources, satellite orbit error is a three-dimensional error that requires coordinate transformation and empirical parameter models to convert into UERE. To address this, we propose employing matrix perturbation mathematical theory to study the influence of satellite orbit error on the solution of positioning equations, using the spectral norm condition number to characterize the equation system' s morphology. Simulation results demonstrate that this method can directly reflect the impact of satellite orbit error on positioning accuracy without requiring orbit coordinate conversion or UERE transformation, enabling more direct and accurate assessment of how satellite orbit error affects positioning solution precision.

Keywords: Satellite Orbit Error; Navigation and Positioning; User Equivalent Range Error (UERE); Norm; Condition Number

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GNSS positioning error is typically characterized by the Dilution of Precision (DOP) and User Equivalent Range Error (UERE), as shown in [1-3]: MATH_1 , (1). For a given satellite, UERE is regarded as the statistical sum of influences from various error sources associated with that satellite, including pseudo-range measurement error, atmospheric delay error, satellite atomic clock error, and satellite orbit error. Throughout the construction and development of various GNSS systems, this error evaluation methodology has played a crucial role. It serves not only as an important basis for constellation system design but also as a key indicator for users to perform constellation optimization and positioning accuracy prediction and analysis. Depending on the type of DOP parameter, users can also rapidly evaluate the solution accuracy for unknowns in different coordinate directions as well as receiver clock error.

However, for the sake of convenience, the DOP-UERE based precision characterization method has several theoretical limitations. First, the precision representation formula for DOP and UERE is derived based on two assumptions: (1) the observation equation coefficient matrix H has no random components, allowing H to be moved outside the expectation operator during derivation; and

(2) all UEREs have identical variance and are uncorrelated zero-mean values, enabling simplification of the expectation operator. However, the coefficient matrix H only becomes deterministic when all observed satellites are orthogonally distributed, while the composition of UERE is highly complex. In addition to random measurement errors following Gaussian white noise, it includes various atmospheric delays, biases caused by hardware equipment, and multipath effects at the receiver. Consequently, both assumptions are difficult to satisfy in actual positioning scenarios.

Second, among various positioning error sources, pseudo-range measurement error, atmospheric delay error, and satellite atomic clock error are one-dimensional vectors—errors along the satellite-to-user line-of-sight direction—that can be easily converted into UERE. However, satellite orbit error (generally considered as satellite broadcast ephemeris error) is a three-dimensional error quantity that is difficult to directly convert into UERE. To evaluate the impact of satellite orbit error on positioning accuracy, the satellite broadcast ephemeris error must typically be transformed from the Earth-Centered Earth-Fixed (ECEF) coordinate system to the orbital coordinate system (RTN frame) at that epoch, obtaining deviations in the radial (R), along-track (T), and cross-track (N) directions [4-8]. Based on this, it is assumed that radial orbit error has the greatest impact on user positioning accuracy, and then relevant empirical models and parameters are used to calculate the satellite URE value, which is incorporated into UERE for comprehensive consideration [9-10]. For hybrid-constellation GNSS such as China' s BeiDou Navigation Satellite System (BDS), different empirical parameters must be applied to different orbit types (MEO, GEO, and IGSO), increasing the difficulty and complexity of precision analysis and evaluation.

To address these issues, this paper proposes employing matrix perturbation mathematical theory to study and analyze the influence of satellite orbit error on the solution of positioning observation equations, using the spectral norm-based condition number to characterize the morphology of the measurement equation system (in fact, the mathematical essence of DOP is precisely the Frobenius norm of a matrix). Simulation results demonstrate that matrix perturbation theory can directly reflect the impact of satellite orbit error on positioning accuracy without requiring UERE conversion, thereby enabling more direct and accurate investigation of positioning equation system performance, solution errors, and the influence of satellite orbit error on positioning solution precision.

2 Observation Equations and Error Equations

The linear equation system for GNSS single-point positioning solution can be expressed in matrix form as MATH_2 , where H is the direction cosine matrix: MATH_3 , (2) MATH_4 , (3) where , , and are the direction cosines of the unit vector from the user estimated position to the i-th satellite.

is the correction vector between the approximate solution and true solution:

MATH_5 . is the difference between the geometric distance calculated from user estimated position and clock bias estimates and the observed pseudo-range: MATH_6 , (4). When n=4, equation (2) is a determined system. Its solution is: MATH_7 .

When n>4, equation (2) is an over-determined system. The redundant solution is obtained using the least squares method: MATH_8 . According to the derivation of the covariance matrix for least squares solutions in [1-3], the covariance of user coordinate position error can be obtained as MATH_9 , (6) MATH_10 , (8) where E is the expectation operator; is the mean square error of user position; is the pseudo-range observation error vector; and n is the number of observed satellites.

In the above derivation, the system matrix H was assumed to have no random components and could therefore be moved outside the expectation operator. Further assuming that all user equivalent range errors have identical variance and are uncorrelated zero-mean values, the expectation operator in equation (8) can be simplified to MATH_11 , (9) where I is the 4×4 identity matrix. Thus, the two equations in (8) can be simplified to MATH_12 , (10) where is the error amplification factor—Dilution of Precision—when converting user equivalent range error to user position error, which is inversely proportional to the solid volume outlined by the user-to-satellite unit vectors.

Depending on the user' s coordinate reference system, DOP can characterize position errors in different directions. When the user is in the user equivalent range error coordinate system, its actual meaning is MATH_13 , (11) where the square roots of the diagonal elements correspond to the error amplification factors—Dilution of Precision—for three-dimensional coordinate positions and receiver clock error.

Since the derivation of DOP assumed that the coefficient matrix H has no random components, the covariance matrix of the matrix in equation (11) is also considered zero. From this, it can be observed that the definition of DOP is consistent with the Frobenius norm (F-norm, the square root of the sum of squares of all matrix elements) in matrix norms, and its mathematical essence is precisely the matrix F-norm. This also indicates that matrix norm perturbation theory can be further employed to conduct in-depth research on positioning error solutions of measurement equation systems.

3 Orbit Error Perturbation Analysis Method

From the above error equation derivation, it is evident that the DOP-based analysis method makes conditional assumptions about the error statistics of measurements and the random component situation of coefficient matrix H (assuming measurement errors follow Gaussian distribution and are mutually independent, and that system matrix H has no random components). However, in actual satellite navigation positioning scenarios, it is difficult to satisfy these ideal conditions.

In contrast, perturbation analysis theory based on norm concepts can study the actual morphology of measurement equation systems and provide more direct and realistic assessment of satellite orbit error impact.

When investigating the influence of satellite orbit error on the positioning solution, we consider that perturbation error is introduced to the coefficient matrix H on the left side of the equation system, causing perturbation to the coordinate and clock bias unknown vector. That is: MATH_14 , (12). It can be proven that since matrix H is non-singular and the perturbation error caused by satellite orbit position error is extremely small, the matrix can maintain its non-singularity [12]. Therefore, subtracting equation (2) from equation (12) yields: MATH_15 .

Consequently, applying norm expressions to both sides of equation (12) gives: MATH_16 . Further derivation yields: MATH_17 , (13) MATH_18 , (14) MATH_19 . (15) Here, is the error amplification rate, meaning the relative error of the solution is times the relative error of the measurement data. is also called the matrix condition number. Therefore, equation (15) can be written as: MATH_20 , (16). Since is generally sufficiently small, equation (17) can usually be approximated as: MATH_21 , (17) MATH_22 . (18) This indicates that when H has perturbation, the resulting relative error of the solution does not exceed times the relative error of H [11-14].

From the above analysis, it can be seen that when the equation system has perturbation error in H, the solution error can be determined by the condition number, which acts as an error transmission amplification factor similar to DOP. The condition number is related to the chosen matrix norm, and mathematically, the spectral norm (i.e., 2-norm) is generally used for characterization: MATH_23 , (19). In this case, is also called the matrix spectral condition number [11-12,14].

The condition number simultaneously characterizes the morphology of the equation system, reflecting the sensitivity of the solution to errors. When is relatively large, the measurement equation system is called ill-conditioned; when is relatively small, it is called well-conditioned.

4 Simulation Experiments and Analysis

To analyze and verify the feasibility of the norm-based perturbation analysis method, a combined approach of real measurements and simulations was adopted. First, a GNSS receiver was used at a fixed location with known coordinates to measure GPS satellite orbit coordinate data over a period of epochs. To facilitate error isolation and control, corresponding GPS pseudo-range data were simulated with 1 =3m pseudo-range error. Second, to investigate the impact of satellite orbit error on positioning accuracy, 1 =3m orbit coordinate mean square error was generated and added separately in the X, Y, and Z coordinate directions of satellite positions (resulting in a three-dimensional orbit error of 1 =5.196m). By calculating positioning results before and after adding orbit error, the actual solution error caused by satellite orbit error was obtained and compared with the error estimated by the norm-based perturbation analysis method, with results shown in Figure 1 [Figure 1: see original paper].

Figure 1 Comparison between perturbation analysis method error estimates and actual solution errors. The results demonstrate that matrix norm perturbation analysis theory can directly evaluate and analyze the influence of satellite orbit error on GNSS positioning solutions without relying on error distribution assumptions or undergoing orbit coordinate decomposition and transformation. Moreover, the error upper bounds calculated by the norm perturbation analysis method are consistently positioned above the actual solution errors, with good agreement between them (differences remain within several meters), indicating that the norm perturbation analysis method can accurately and reliably characterize and evaluate GNSS positioning solution errors.

The DOP-UERE based precision characterization method has played an important role in constellation optimization, positioning accuracy prediction, and analysis. However, since the precision representation formula for DOP and UERE is derived based on several assumptions, it can only serve as an approximate evaluation method in the strict sense and remains theoretically inadequate. Additionally, converting all positioning error sources into the UERE concept for evaluation introduces certain difficulties and increases the complexity of assessing the impact of satellite orbit error on user positioning accuracy.

This paper proposes a new theoretical and practical method based on norm mathematical concepts and matrix perturbation analysis to study satellite orbit error effects, enriching and complementing the traditional DOP-UERE based precision characterization approach. When prior statistical information of satellite orbit error is known, this method can be used to predict and evaluate the degree of influence of satellite orbit error, or after calculating real satellite orbit error using post-processed precise ephemeris, to perform more accurate and convenient analysis of satellite system constellation performance. This method avoids conditional assumptions about measurement error statistics and the random component situation of the observation equation coefficient matrix, requires no satellite orbit error decomposition and coordinate transformation, and does not rely on statistical significance, enabling detailed description of error perturbation for a single positioning solution. Therefore, it can more comprehensively and directly evaluate and analyze the influence of satellite orbit error on user positioning accuracy. Simulation experiments and analysis demonstrate that the matrix norm-based perturbation analysis method features concise calculation processes, accurate and reliable evaluation results, and possesses both theoretical significance and practical value.

References

[1] Misra P, Enge P. Global Positioning System - signals, measurements, and performance[M]. Lincoln: Ganga-Jamuna Press, 2001.

[2] Parkinson B W, Spilker J J, Axelrad P, et al. Global Positioning System: theory and applications[M]. Washington: American Institute of Aeronautics and Astronautics, 1996.

[3] 谢钢. GPS 原理与接收机设计 [M]. 北京: 电子工业出版社, 2009.

[4] 施闯, 赵齐乐, 李敏, 等. 北斗卫星导航系统的精密定轨与定位研究 [J]. 中国科学: 地球科学, 2012, 42(6): 854-861.

Shi Chuang, Zhao Qile, Li Min, et al. Precise orbit determination of Beidou Satellites with precise positioning[J]. Scientia Sinica: Terrae, 2012, 42(6):854-861.

[5] 刘伟平, 郝金明, 李建文, 等. 一种北斗卫星精密定轨方法 [J]. 测绘科学技术学报, 2013, 30(3): 247-250.

Liu Weiping, Hao Jinming, Li Jianwen, et al. A method of precise orbit determination of BeiDou navigation satellite[J]. Journal of Geomatics Science and Technology, 2013, 30(3): 247-250.

[6] 李征航, 丁文武, 李昭. GPS 广播星历的轨道误差分析 [J]. 大地测量与地球动力学, 2008, 28(1): 50-54.

Li Zhenghang, Ding Wenwu, Li Zhao. Error analysis of orbit determined by GPS broadcast ephemeris[J]. Journal of Geodesy and Geodynamics, 2008, 28(1): 50-54.

[7] 李济生. 人造卫星精密轨道确定 [M]. 北京: 解放军出版社, 1995.

[8] 徐小钧, 艾国祥, 马利华, 等. 基于不同轨道高度 iHCO 通信卫星的 CAPS 星座优化研究 [J]. 天文研究与技术—国家天文台台刊, 2014, 11(4): 350-355.

Xu Xiaojun, Ai Guoxiang, Ma Lihua, et al. A study of optimization of a CAPS constellation of iHCO communication satellites of different orbit heights[J]. Astronomical Research & Technology—Publications of National Astronomical Observatories of China, 2014, 11(4): 350-355.

[9] 高为广, 苏牡丹, 郭树人, 等. 北斗系统空间信号精度测试与评估 [C]//第四届中国卫星导航学术年会电子文集. 2014.

[10] 罗璠, 李建文, 黄海, 等. BDS 广播星历的轨道误差分析 [C]//第五届中国卫星导航学术年会电子文集. 2015.

[11] 何旭初, 苏煜城, 包雪松. 计算数学简明教程 [M]. 北京: 人民教育出版社, 1980.

[12] 数学手册编写组. 数学手册 [M]. 北京: 高等教育出版, 1979.

[13] 施浒立, 孙希延, 李志刚. 转发式卫星导航原理 [M]. 北京: 科学出版社, 2009.

[14] 王世儒, 王金金, 冯有前, 等. 计算方法 [M]. 西安: 西安电子科技大学出版社, 2004.

Note: Figure translations are in progress. See original paper for figures.