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Abstract

Utilizing precise ephemeris to solve for satellite coordinates constitutes the fundamental prerequisite for high-precision GPS differential positioning. Constrained by the sampling time interval of GPS precise ephemeris, interpolation and extrapolation of GPS precise ephemeris data are necessary to obtain satellite coordinates at arbitrary epochs. Following the analysis of temporal variation patterns of satellite positions, the generalized extension method is employed to construct interpolation and extrapolation models with time as the independent variable for a segment of the satellite' s orbit, thereby simplifying the computational process for satellite coordinates without introducing substantial position errors. Comparative experiments were conducted with the Lagrange interpolation method. The results demonstrate that the interpolation error using the generalized extension method is less than 5 cm, and accuracy can be maintained for extrapolation periods up to 30 minutes, significantly outperforming the Lagrange interpolation method.

Full Text

Application of the Generalized Continuation Method in GPS Precise Ephemeris Interpolation and Extrapolation

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Abstract: Utilizing precise ephemeris to calculate satellite coordinates is the fundamental prerequisite for achieving high-precision GPS differential positioning. Limited by the sampling interval of GPS precise ephemeris, interpolation and extrapolation of global positioning system precise ephemeris data are necessary to obtain satellite coordinates at arbitrary epochs. After analyzing the temporal variation 规律 of satellite positions, this study employs the generalized continuation method to construct time-coordinate-based interpolation and extrapolation models for a segment of satellite orbit. This approach simplifies the satellite coordinate calculation process without introducing significant position errors, and comparative experiments are conducted against the Lagrange interpolation method. The results demonstrate that the generalized continuation method achieves interpolation errors of less than 5 cm and maintains accuracy for up to 30 minutes of extrapolation, significantly outperforming the Lagrange interpolation method.

Keywords: Precise ephemeris; interpolation; extrapolation; generalized continuation; GPS

The International GNSS Service (IGS) provides precise ephemeris commonly used in GPS precise point positioning and other data processing applications. However, the sampling interval of IGS precise ephemeris is 15 minutes, while receiver sampling rates in GPS precise positioning are generally much smaller. Consequently, high-precision interpolation of precise ephemeris is required. Furthermore, since IGS precise ephemeris only provides data for the time period 00:00:00–23:45:00 UTC each day, extrapolation is also necessary to obtain satellite coordinates for the period 23:45:00–24:00:00 using only that day' s data.

To achieve smooth and stable interpolation results, the interpolation polynomial and its derivatives must be continuous and smooth. Common interpolation methods include Chebyshev polynomial interpolation, Newton polynomial interpolation, and Lagrange interpolation [1-3]. Among these, Lagrange interpolation is widely used in current data processing. Reference [4] notes that Lagrange interpolation is essentially equivalent to Neville interpolation. Reference [5] improved the Lagrange method by employing a sliding-window Lagrange interpolation to obtain GPS precise ephemeris. Reference [6] compared the interpolation effects of Lagrange and Newton methods and analyzed the results. In these studies, various interpolation methods require combinations of high and low orders to achieve optimal results within certain time periods, and extrapolation performance is generally unsatisfactory. Since the interpolation function constructed by the generalized continuation approximation method has the minimum least-squares approximation error among all interpolation functions of the same order, this paper applies the generalized continuation principle to interpolate and extrapolate GPS precise ephemeris, using the obtained approximation function to calculate precise coordinates of GPS satellites at arbitrary epochs.

1 Generalized Continuation Interpolation and Extrapolation Model

The generalized continuation approximation method satisfies interpolation conditions at piecewise boundary points, ensures coordinated variation between pieces, and fully utilizes surrounding node information in the interpolation region to achieve optimal fitting within the piecewise region. This approach organically combines the advantages of interpolation and fitting methods, achieving high-precision approximation in data processing [7-9].

First, domain partitioning is performed to identify approximation functions and basis functions within element domains. The definition domain is divided into non-overlapping subdomains: each element domain contains nodes with corresponding coordinates that satisfy the definition domain. The element domain is then extended to become an extension domain, with node coordinates satisfying the extension domain, as shown in [Figure 1: see original paper].

Fig. 1 Extension domains and their corresponding function values

Assume there are nodes in the extension domain. Using the extension domain containing more nodes, the approximation within the element domain is constructed as follows: where represents a set of basis functions; denotes undetermined coefficients; is the number of approximation function terms, with. When solving for the undetermined coefficients of the function, its applicable range is expanded from the element domain to the extension domain. Thus, the generalized continuation approximation interpolation model is established in the extension domain as shown in equation (3), where are prior information sampling points and represents the error between the approximation function and prior values:

The generalized continuation approximation method nests the element domain within the extension domain, absorbing node information from the extension domain outside the element domain to construct a fitting approximation function within the element domain, while constraining boundary nodes of the element domain to achieve coordination and continuity between adjacent element approximation functions, thereby attaining optimal fitting effects within the element domain. The approximation function constructed using generalized continuation can then be used for interpolation of target variables.

The generalized continuation interpolation model can serve not only as an interpolation model but also as an extrapolation model for extension applications. For a continuously growing data sequence, with known previous data values, the value at time can be predicted based on the variation 规律 and trends of prior data. The data extrapolation schematic is shown in [Figure 2: see original paper].

Fig. 2 Data extrapolation schematic

Following the design concept of generalized continuation interpolation and extrapolation, let be the latest time epoch, and employ the extrapolation algorithm to obtain the value at the next time epoch. The generalized continuation extrapolation model is established as:

In this model, since the interpolation point is the value of the latest sampling point, to overcome errors caused by this single-point value 突变, an improved generalized continuation model is considered:

That is, the average of the latest sampling points is selected as the constraint. The solution method for the improved model is essentially the same as that for the original model, but the data 平稳性 is better, the interpolation fitting curve is smoother, and the extrapolation accuracy is higher.

2 GPS Precise Ephemeris Interpolation and Extrapolation Method

2.1 GPS Precise Ephemeris Calculation Method

Considering that the actual satellite orbit deviates from the unperturbed orbit, GPS ephemeris parameters typically adopt a set of extended Keplerian orbital parameters consisting of 16 parameters: 6 Keplerian elements, 9 perturbation parameters, and the ephemeris reference time.

Based on the calculation algorithm provided in reference [10], the satellite positioning calculation process can be briefly described as follows: where,, and are the satellite coordinates to be solved for each epoch. After coordinate transformation and perturbation correction, the coordinate calculation matrix for the satellite in the WGS-84 Earth-centered Earth-fixed coordinate system at time is finally obtained:

where represents the satellite position coordinates in the orbital plane at signal transmission time; is the right ascension of the ascending node at signal transmission time; and is the orbital inclination, all of which can be calculated from ephemeris parameters.

From the above solution process, it is evident that calculating the spatial position of a GPS satellite at a certain epoch using ephemeris parameters requires accurate ephemeris data and involves substantial computational effort.

2.2 GPS Precise Ephemeris Interpolation Feasibility Analysis

The three curves in [Figure 3: see original paper] respectively show the variation of a satellite' s position components X, Y, and Z in the WGS-84 Earth-centered Earth-fixed coordinate system over time, with an epoch interval of 15 minutes, demonstrating periodic variation in satellite orbital position. As shown in [Figure 4: see original paper], each component of the satellite position varies smoothly over a short time period, exhibiting nearly linear 变化.

Fig. 3 Space position of satellite over a long period of time
Fig. 4 Space position of satellite in short time

Therefore, a segment of satellite orbit can be expressed using a time-domain interpolation polynomial [11]. If this approximation function is properly constructed, the interpolation method will not introduce significant satellite position errors, avoiding the complex calculation method described in Section 2.1

and substantially reducing computational effort.

2.3 GPS Precise Ephemeris Interpolation and Extrapolation Method Based on Generalized Continuation

Using the generalized continuation interpolation principle, generalized continuation models are established separately for the X, Y, and Z components of satellite position and velocity in the WGS-84 Earth-centered Earth-fixed coordinate system. Within the time period from to, taking the X-direction component of satellite position as an example, if the two endpoints of the interval are selected as prior constraint points, the generalized continuation interpolation is:

where represents the X-direction component of satellite position in the WGS-84 Earth-centered Earth-fixed coordinate system at the left endpoint time of the interpolation interval; represents the X-direction component at the right endpoint time; are coefficients to be determined; and are the precise ephemeris update time points.

Solving the above model yields the normal equation, where in equation (9), is the Lagrange multiplier, with and,. Solving this normal equation and obtaining the coefficients yields the generalized continuation interpolation function for the X-direction component of satellite position varying with time. Substituting the required time epoch enables rapid solution.

Next, the generalized continuation extrapolation model is constructed from equation (5):

where the variable meanings are as shown in equation (8), with representing the average of the latest satellite position X-components selected as the constraint for the extrapolation model. The solution method for this model is as follows:

The coefficient solution matrix is obtained: After solving for the undetermined coefficients, the extrapolation formula is obtained: Similarly, extrapolation formulas for the other components of satellite position can be derived.

3 Example Analysis

To verify the computational effectiveness of the generalized continuation interpolation and extrapolation model, precise ephemeris data with a 15-minute sampling interval provided by IGS on June 6, 2017 (GPS Week 1952) are used, with satellite PRN15 selected. The first four sampling data points are taken as prior sampling points, with the Lagrange interpolation method used as a reference and compared against the standard curve fitted from precise ephemeris data. The approximation effects of the two precise ephemeris interpolation methods are shown in [Figure 5: see original paper].

Fig. 5 Comparison of interpolation and extrapolation effects of two precise ephemeris interpolation methods

As seen in [Figure 5: see original paper], when one hour of precise ephemeris data is used as prior points to fit the approximation function, the approximation functions constructed by both the Lagrange interpolation method and the generalized continuation method basically coincide with the fitted curve of precise ephemeris within a 1.5-hour period. However, beyond 1.5 hours, the extrapolation effect of the generalized continuation method is significantly better than that of the Lagrange interpolation method. To more intuitively observe the accuracy of the model constructed by the generalized continuation method, the errors in the X, Y, and Z components are integrated into position error e, yielding the position error variation over time as shown in [Figure 6: see original paper].

Fig. 6 Chart of satellite position versus time

[Figure 6: see original paper] shows that the curve fitted from the first four comparison points represents the interpolation effect of generalized continuation, with interpolation errors in satellite position less than 5 cm. Starting from the fifth comparison point represents the extrapolation effect of generalized continuation, demonstrating that the satellite position error is less than 10 cm for approximately one hour of extrapolation and less than 20 cm for two hours of extrapolation, after which the satellite position error begins to increase rapidly.

Since the precision of IGS-provided precise ephemeris is 5 cm, the generalized continuation method for GPS precise ephemeris interpolation meets accuracy requirements. Moreover, when using generalized continuation for extrapolation, accuracy can be maintained within 30 minutes without introducing significant position errors over longer periods.

The generalized continuation method demonstrates good performance in interpolating GPS precise ephemeris for satellite positions, enabling a segment of satellite orbit to be expressed using a time-coordinate-based model. This allows satellite position calculation to be performed without complex and lengthy procedures while maintaining precision, substantially reducing computational effort. Additionally, case analysis shows that the generalized continuation extrapolation method can ensure satisfactory accuracy for one hour of extrapolation, while the generalized continuation interpolation method can obtain precise ephemeris data for GPS at any epoch.

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Note: Figure translations are in progress. See original paper for figures.