Postprint of Fractional-Order PI^ Controller Parameter Tuning Based on Artificial Fish Swarm Algorithm

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Abstract

For the parameter tuning of fractional-order #1; controllers, a method combining the graphical method and artificial fish swarm optimization algorithm is proposed for fractional-order PI controllers. First-order and second-order systems are respectively employed to model typical speed servo systems, based on which fractional-order PI controllers are designed. First, in the frequency domain, equations are derived according to conditions such as system relative stability and robustness to gain variations; then, the parameters of the fractional-order #1; controller are solved using the graphical method. Using the solved parameters as the center, the optimization range is specified, and the artificial fish swarm algorithm is subsequently utilized to perform optimization in the surrounding region. Finally, simulation studies demonstrate that the controller obtained through the artificial fish swarm algorithm enables the system to achieve superior dynamic response characteristics compared to the fractional-order PI controller obtained by the graphical method alone, while satisfying the robustness condition to gain variations.

Full Text

Preamble

Parameter Tuning of Fractional-Order PI Controller Based on Artificial Fish Swarm Algorithm

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Abstract: For parameter tuning of fractional-order PI controllers, this paper proposes a method combining the image method with artificial fish swarm optimization. First-order and second-order systems are used to model typical speed servo systems, serving as the controlled plant for fractional-order PI controller design. In the frequency domain, equations are derived based on system relative stability and robustness to gain variations. The image method is then used to solve for the controller parameters, which serve as the center point for specifying the search range. The artificial fish swarm algorithm subsequently performs optimization in this vicinity. Simulation studies demonstrate that the controller obtained through the artificial fish swarm algorithm yields superior dynamic response characteristics compared to the image method alone, while satisfying the robustness conditions for gain variations.

Keywords: artificial fish swarm algorithm; image method; fractional-order controller; parameter tuning

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0 Introduction

Fractional calculus has recently attracted increasing attention from scholars both domestically and internationally, particularly in system theory and control domains, as it enables the construction of more accurate mathematical models and enhances system control performance. The concept of fractional calculus has been applied to numerous engineering fields, leading to the development of various fractional-order controllers. Oustaloup developed the CRONE controller based on the idea of controlling dynamic systems and demonstrated its advantages over traditional integer-order PID controllers. Matignon et al. investigated the stability, controllability, and observability of fractional-order systems. Subsequently, Podlubny proposed the fractional-order PID controller in 1994, which offers better robustness and accuracy than traditional integer-order PID while adding two adjustable parameters and that provide greater flexibility.

Traditional integer-order PID tuning methods have been extensively studied, and many scholars have proposed fractional-order PID controller tuning approaches based on these foundations. Hu Haibo et al. combined bacterial foraging algorithm with particle swarm optimization to design fractional-order PID controllers, improving computational efficiency while achieving better dynamic characteristics than integer-order controllers. Qin Junqin et al. applied fractional-order PID controllers to greenhouse temperature regulation, demonstrating improved adaptability and robustness compared to conventional integer-order PID and fuzzy PID controllers. Mohanty et al. designed fractional-order PID controllers using the imperialist competitive algorithm for reactive power compensation in standalone microgrids, verifying the controller' s advantages through input parameter and load condition variations. Jauregui et al. tuned fractional-order controllers using an improved particle swarm algorithm. Altintas et al. optimized controller parameters for magnetic levitation systems using genetic algorithms, obtaining both fractional-order PID (FOPID) and integer-order PID (IOPID) controllers and validating the superior flexibility and dynamic response of FOPID. Singh designed fractional-order fuzzy PID controllers based on ant colony optimization. Jain et al. applied particle swarm optimization to design fractional-order PID controllers for DC motor speed control.

This paper builds upon the image method employed in reference [15] and utilizes the artificial fish swarm algorithm to optimize the three parameters of the fractional-order PI controller. The artificial fish swarm algorithm, proposed by Li Xiaolei in 2002, is an intelligent optimization method based on animal behavior. It imposes no special requirements on the search space or problem characteristics, relies on comparing objective function values rather than derivative information, and offers strong global optimization capability, parallel search capability, and fast convergence. Due to its insensitivity to initial values, the algorithm avoids local optima during optimization.

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1 Implementation of the PI Image Method

The fractional-order PI controller offers an additional adjustable parameter compared to integer-order PI controllers, enabling performance tuning over a wider range and achieving better control performance. The design criteria for fractional-order PI controllers must consider both stability and gain robustness requirements.

Using a typical first-order system P(s) as the controlled plant, as shown in equation (2), the fractional-order PI controller is designed via the image method. The open-loop transfer function of a typical first-order inertial system based on the fractional-order PI controller is given by equation (3), where C(s) is the typical fractional-order PI controller from equation (1). According to the system’ s phase-frequency and amplitude-frequency characteristics, the cutoff frequency c is first specified, and the amplitude condition at this frequency is given by equation (4).

To ensure system stability, the phase margin m is specified, leading to equation (5). Based on the robustness condition for control system gain variations, the gain robustness criterion serves as a design constraint, as shown in equation (6). This criterion specifies that at the gain crossover frequency c, the derivative of the system’ s phase-frequency characteristic curve is zero (i.e., the phase curve is flat). When the system gain experiences small perturbations, the phase margin remains essentially unchanged. The overshoot observed in the step response should remain nearly constant to satisfy the gain robustness condition.

Simplifying equations (4)-(6) yields the following formulas. Solving the system of equations (7), (8), and (9) gives equations (10) and (11). From equation (11), we obtain the relationship for .

Given the cutoff frequency c = 10 rad/s, phase margin m = 70°, and time constant T = 0.6 s, the function graphs of equations (10) and (12) with respect to are plotted, as shown in [Figure 1: see original paper]. The intersection point of the red curve (equation 12) and blue curve (equation 10) yields the desired parameters, giving (, ki) = (0.72, 14.75). Substituting into equation (7) yields kp = 2.868, resulting in the fractional-order PI controller: C(s) = 2.868(1 + 14.75/s^0.72).

From equation (3), the system' s open-loop transfer function G1(s) can be derived, and subsequently the closed-loop transfer function. Step response simulation of this closed-loop transfer function produces the system step response shown in [Figure 2: see original paper]. The step response curve yields system performance metrics: rise time tr = 0.13 s, settling time ts = 0.82 s, and overshoot % = 19%. Rise time reflects system response speed, while settling time (to within 2% of steady-state value, here taken as 0.998) measures regulation speed. Overshoot, defined by equation (14), reflects relative stability.

The image method solves for and ki by specifying c, then determines kp, but cannot arbitrarily assign c to achieve true optimality. Therefore, intelligent optimization algorithms are employed to optimize the controller parameters based on system gain robustness, allowing c and to combine freely and 筛选出最优组合。

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2 Artificial Fish Swarm Algorithm

2.1 Characteristics of Artificial Fish Swarm Algorithm

The artificial fish swarm algorithm is a group intelligence optimization method based on animal behavior, derived from fish foraging patterns. Due to their autonomy, fish can independently find or follow other fish to locations with maximum food concentration. Thus, areas with high fish density correspond to regions of abundant food. These characteristics are modeled to construct the artificial fish swarm algorithm, which simulates fish behaviors including foraging, clustering, following, and random movement to achieve optimization.

Three typical behaviors are defined:

Foraging Behavior: When fish move freely in water and discover food-rich areas, they autonomously swim toward these regions.

Clustering Behavior: For survival and protection, fish naturally form groups following three rules: (a) separation—avoid excessive crowding with neighbors; (b) alignment—match the average direction of nearby partners; and (c) cohesion—move toward the center of nearby partners.

Following Behavior: When a fish in the swarm discovers a food-rich area, other fish follow toward that location.

In the algorithm, food concentration in the water represents the objective function, and each fish' s state represents the variables to be optimized. Through these behaviors, optimal solutions can be found. Foraging behavior involves random movement based on food concentration, representing an extremal optimization process and self-learning mechanism that ensures population diversity. Clustering and following behaviors represent interactions between artificial fish and their environment, preventing excessive crowding while guiding the swarm toward food-rich areas to find optimal partners. The algorithm achieves population balance through individual and environmental information, ultimately locating the position with maximum food concentration.

These three modes can be nested: first attempt following behavior, then foraging if no improvement occurs, then clustering if still no progress, and finally random behavior if necessary. The algorithm exhibits several key features: (a) parallelism—multiple artificial fish search simultaneously; (b) simplicity—only objective function values are used; (c) globality—strong ability to escape local optima due to randomness in foraging; (d) rapidity—despite random factors, the search progressively moves toward optimality; and (e) tracking capability—ability to quickly track changes when optimal points drift due to varying conditions.

2.2 Construction of Artificial Fish Swarm Model

The artificial fish swarm algorithm employs a bottom-up design approach. Let X = (x1, x2, ···, xn) represent the variables to be optimized, where xi (i = 1, 2, ···, n) are the state values. The food concentration at the current position of an artificial fish is FC = F(X), where FC is the objective function. The distance between artificial fish i and j is dij = ||Xi - Xj||. Visual represents the perception distance, step the maximum movement step length, the crowding factor, fishnum the population size, gen the iteration count, and try_number the attempt limit.

A larger fishnum facilitates local optima avoidance and fast convergence but increases iteration count. For maximization problems, $0 < \ < 1$; larger values reduce allowable crowding, improving global convergence but reducing accuracy. For minimization problems, $\ > 1$; smaller values similarly reduce crowding and improve convergence while decreasing accuracy. Smaller visual ranges enhance foraging and random movement, while larger visual ranges strengthen following behavior. Larger step sizes benefit early optimization but may cause oscillation in later stages, affecting convergence speed, whereas smaller step sizes improve local search but slow global optimization.

2.3 Algorithm Design and Process

For optimizing the fractional-order PI controller, an objective function (food concentration) must be constructed. This study employs the Integral of Time-weighted Absolute Error (ITAE) as the objective function, which offers excellent engineering practicality and selectivity for control system performance evaluation. During optimization, the ITAE value is minimized. In this paper, the reciprocal of ITAE serves as the objective function, as shown in equation (19), with the artificial fish swarm algorithm seeking its maximum.

After determining the objective function, the search range must be defined to improve accuracy and reduce search time. In this study, each artificial fish' s position can be represented as Xi = (c_i, _i). The distance between artificial fish is given by equation (20). By limiting the ranges of c and , search time is reduced and accuracy increased. The values obtained from the image method serve as center points, with search ranges defined by equation (21), where 0 a 1.

The algorithm flowchart is shown in [Figure 3: see original paper]. The detailed optimization steps for the fractional-order PI controller using the artificial fish swarm algorithm are as follows:

a) Initialize the fish swarm: Set fishnum = 100, maximum iterations MAXGEN = 50, visual = 1, step = 0.01, = 0.618, and try_number = 100. Randomly generate fishnum artificial fish states Xi within the range of equation (21). Calculate coefficients using equations (7)-(10), then compute FC = 1/ITAE, with search ranges $0 <$ c < 20 and $0 <$ < 1.42. Initialize the bulletin board.

b) Individuals search for positions with maximum food concentration through foraging, following, and clustering behaviors to obtain new individuals Xi and FC.

c) Compare the three behaviors to identify the optimal individual, then compare with the bulletin board. If the new position' s food concentration FC exceeds the bulletin board value, replace the bulletin board' s position and FC; otherwise, retain the current values.

d) If gen reaches MAXGEN, the algorithm terminates and the bulletin board value is optimal; otherwise, increment gen and return to step b).

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3 Simulation Verification

A typical speed loop control model is shown in [Figure 4: see original paper]. P(s) represents a typical first-order speed model, and C(s) is the fractional-order PI controller. After optimization using the artificial fish swarm algorithm, c = 16.43 and = 0.58 are obtained. Substituting into equation (7) yields kp = 3.487, giving the fractional-order PI controller: C2(s) = 3.487(1 + 29/s^0.58).

The artificial fish swarm iteration process is shown in [Figure 5: see original paper]. The objective function essentially stabilizes after approximately 20 iterations, demonstrating rapid convergence.

First, controllers obtained by both methods are compared. The corresponding closed-loop transfer functions are derived and their step responses simulated, as shown in [Figure 6: see original paper]. The performance metrics are compared in . The optimized fractional-order PI controller shows reduced settling time, rise time, and peak value, improving response speed and regulation capability. The system reaches steady state faster with reduced overshoot (16.6% vs. 19%), enhancing stability. The introduction of integral order provides greater flexibility. The optimized = 0.58 is lower than the image method' s = 0.72, and this reduction decreases settling time and increases response speed.

To verify that the optimized result is truly optimal, values around c = 16.43 and = 0.58 are tested, as nearby values should exhibit better dynamic characteristics than distant ones. The selected values are: - X1: c = 14, = 0.58 - X2: c = 18, = 0.58 - X3: c = 16.43, = 0.4 - X4: c = 16.43, = 0.7

The corresponding controllers are: - C3(s) = 2.76(1 + 31.2473/s^0.58) - C4(s) = 2.88(1 + 28.3234/s^0.58) - C5(s) = 9.78(1 + 0.075/s^0.4) - C6(s) = 5.03(1 + 0.3376/s^0.7)

The corresponding open-loop transfer functions G3-G6 are derived, and their closed-loop step responses are simulated, as shown in [Figure 7: see original paper]. Performance metrics are summarized in . The results show C2 has superior settling time and peak value compared to the other controllers. Although C6 shows minimal oscillation, it fails to reach the desired steady-state value. Considering all metrics, the C2 controller is optimal.

To verify gain robustness, kp values of 3.1383, 3.8357, and 3.487 (90%, 110%, and 100% of the nominal 3.487) are tested. The corresponding dynamic response curves are shown in [Figure 8: see original paper]. The curves are nearly identical, demonstrating that overshoot remains essentially unchanged when kp varies by ±10%, indicating consistent phase margin and satisfying the gain robustness condition.

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4 Tuning of Fractional-Order Controllers for Second-Order Systems

For second-order systems, a typical second-order servo system serves as the controlled plant. The block diagram follows [Figure 4: see original paper], with P(s) as a second-order transfer function given by equation (22).

Based on robustness and stability requirements, equations (4)-(6) yield equations (23)-(27). Given c = 5 rad/s, m = 60°, and T = 0.5 s, the graphs of equations (26) and (12) with respect to are plotted, as shown in [Figure 9: see original paper]. The intersection (, ki) = (0.593, 0.2425) yields kp = 2.5484 from equation (23), giving the fractional-order PI controller: C7(s) = 2.5484(1 + 0.593/s^0.2425).

The search ranges are 0 < c < 10 and 0 < < 1.186. Applying the artificial fish swarm algorithm from Section 2 yields c = 6.04 and = 0.41, with ki = 0.0335 and kp = 3.1314, resulting in controller: C8(s) = 3.1314(1 + 0.41/s^0.0335).

The corresponding second-order open-loop transfer functions G7 and G8 are derived, and their closed-loop step responses are simulated, as shown in [Figure 10: see original paper]. Performance metrics are compared in . The optimized controller again shows improved settling time, rise time, and peak value, with reduced overshoot (26.5% vs. 31.2%), demonstrating faster response and enhanced stability.

To verify optimality, values around c = 6.04 and = 0.41 are tested: - X5: c = 5.04, = 0.41 - X6: c = 7.04, = 0.41 - X7: c = 6.04, = 0.5 - X8: c = 6.04, = 0.3

The corresponding controllers are: - C9(s) = 2.5339(1 + 0.41/s^0.242) - C10(s) = 3.78(1 + 0.41/s^0.117) - C11(s) = 3.15(1 + 0.5/s^0.035) - C12(s) = 3.143(1 + 0.3/s^0.0386)

The corresponding open-loop transfer functions G9-G12 are simulated, as shown in [Figure 11: see original paper]. The results indicate G8, G11, and G12 are essentially coincident, while G10 shows better dynamic response but fails to reach steady state, making the system unstable. Considering all factors, the optimized result yields the optimal controller for this second-order system.

Gain robustness is verified by testing kp values of 2.818, 3.44, and 3.1314 (90%, 110%, and 100% of nominal 3.1314). The simulation results in [Figure 12: see original paper] show overshoot values of 26.3%, 27.6%, and 23.9% respectively. When kp varies, overshoot changes only slightly with relatively small phase margin variation, essentially satisfying robustness requirements.

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5 Conclusion

This paper first obtains fractional-order PI controller parameters using the image method, then specifies ranges for cutoff frequency c and fractional-order coefficient . The artificial fish swarm algorithm optimizes these parameters to obtain optimal c and values. To verify the optimality of the algorithm-derived controller, simulations compare it with the image method controller and with controllers formed by nearby parameter values. The results demonstrate that the artificial fish swarm algorithm yields controllers with optimal dynamic characteristics. Gain robustness verification through ±10% kp variation shows minimal response curve changes for first-order systems, satisfying gain robustness conditions. Second-order systems exhibit slightly less robustness than first-order systems. Overall, the combination of artificial fish swarm algorithm and image method significantly improves system dynamic performance metrics.

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Note: Figure translations are in progress. See original paper for figures.