

Adaptive Stopping Criterion for Iterative Deblurring Algorithms (Postprint)

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Abstract

Due to the lack of an effective iterative stopping criterion (ISC), most existing deblurring algorithms simply adopt a fixed number of iterations, suffering from issues such as low computational efficiency and unsatisfactory deblurring performance. To this end, we propose a deblurring measure (DM) based on residual images (the difference between the blurred image and the convolution of the intermediate estimated image obtained during the iteration process with the blur kernel), and design an adaptive iterative stopping criterion (adaptive ISC, AISC) based on this DM. The proposed AISC iterative stopping criterion is applied to the classical NCSR (nonlocally centralized sparse representation) iterative deblurring algorithm. Extensive experimental data under three typical blur distortion types—uniform blur, Gaussian blur, and motion blur—demonstrate that, compared with the original NCSR algorithm employing a fixed number of iterations, the NCSR algorithm with the adaptive iterative condition achieves significant improvement in computational efficiency, while the restored images exhibit minimal difference in PSNR, SSIM, and FSIM metric values from the original algorithm.

Full Text

Preamble

Adaptive Iterative Stopping Criterion for Deblurring Algorithms

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Abstract: Currently, due to lack of effective iterative stopping criterion (ISC), most deblurring algorithms are usually implemented with fixed iteration steps, leading to low efficiency and suboptimal deblurring results. Therefore, this

paper proposes a deblurring measure (DM) based on the residual image (the difference between the intermediate estimated image convolved with blur kernel and the blurred image during each iterative step). Based on the proposed DM metric, it designs and applies an adaptive iterative stopping criterion (AISC) to the classic nonlocally centralized sparse representation (NCSR). Extensive experiments on uniform blur, Gaussian blur and motion blur show that, compared with the original NCSR algorithm using fixed iteration steps, the NCSR algorithm adopting AISC has obvious advantages in terms of efficiency, and the reconstructed images have similar quality in terms of peak signal-to-noise ratio (PSNR), structural similarity (SSIM) and feature similarity (FSIM).

Keywords: image deblurring; execution efficiency; iterative stopping criterion; residual image; deblurring measure

0 Introduction

Image deblurring is a typical ill-posed inverse problem that has attracted considerable attention from researchers, leading to numerous proposed algorithms [1–6]. Among these, regularization-based methods represent a common and effective class of solutions. In regularization-based deblurring algorithms, prior knowledge of the original image is utilized to construct appropriate regularization terms that constrain both the restored image and the degradation function. The mathematical model can be formally expressed as:

where g and f represent the blurred image (i.e., observed image) and original image, respectively; h denotes the point spread function (PSF), commonly referred to as the degradation function; and $*$ is the convolution operator. Image deblurring is fundamentally an inverse problem that aims to obtain the optimal estimate of the original undistorted image by solving an optimization problem involving an objective function based on the observed blurred image. Since image deblurring is a typical ill-posed problem, regularization techniques are employed to obtain stable solutions. The general form of the regularization-based model is:

where the first term is the data fidelity term; $\Phi(x)$ is the regularization term representing image prior knowledge; and λ is the regularization parameter. Among methods for solving the optimization problem defined in Equation (2), total variation (TV) regularization [5,6] has been widely applied due to its excellent edge-preserving and noise-suppression capabilities. However, TV methods often produce staircase effects in flat image regions because they ignore detailed structural characteristics. Consequently, many improved TV deblurring algorithms have been proposed.

In recent years, inspired by non-local means (NLM) denoising algorithms [7], numerous researchers have incorporated non-local self-similarity into regularization methods, proposing non-local regularization-based deblurring algorithms

that address the limitations of TV regularization [8,9]. These algorithms extend the traditional concept of local spatial neighborhoods to a more generalized sense, leveraging the similarity of local structures within images to achieve significant improvements in texture and structure preservation. For example, Kheradmand et al. [9] utilized normalized graph Laplacian as a regularization term to fully exploit spatial self-similarity characteristics. Their algorithm implements an inner-outer iterative process: the outer iteration updates similarity weights based on previous estimates, while the inner iteration employs conjugate gradient methods to minimize the objective function.

More recently, researchers have combined non-local self-similarity with sparsity, proposing numerous novel image restoration algorithms [10~13]. Zhang et al. [10] introduced group sparse representation (GSP) for image restoration to address the inefficiency and inaccurate sparse coefficient estimation of traditional patch-based sparse representation. This algorithm successfully integrates image sparsity with non-local self-similarity and uses split Bregman iteration (SBI) to solve for the optimal image estimate. Similarly, Wang et al. [11] proposed an iterative incremental GSP-based regularization deblurring algorithm for remote sensing images, utilizing non-negative and sparse constraints to iteratively update the original image estimate. Liu et al. [12] proposed a joint sparse representation (JSR) algorithm in the 3D transform domain, which introduces non-local regularization terms to effectively characterize non-local statistical features, providing more accurate results than existing sparse representation methods. Liu's algorithm employs SBI within a Bayesian framework to obtain the optimal solution for the original undistorted image. Dong et al. [13] proposed the nonlocally centralized sparse representation (NCSR) algorithm, which combines sparse prior knowledge with non-local similarity. NCSR first estimates sparse coefficients of noisy patches using the repetitive characteristics of images, then implements image restoration through iterative shrinkage of patch sparse coefficients, achieving superior restoration quality compared to most algorithms.

Currently, most regularization-based deblurring algorithms solve for the optimal value of the objective function through iterative methods [14]. However, due to the lack of effective criteria for determining iteration termination, these algorithms often rely on empirical fixed iteration counts. This implementation suffers from low execution efficiency and potentially suboptimal deblurring results. Using the NCSR algorithm [13] as an example, which employs inner and outer loops with 6 outer iterations and 120 inner iterations (totaling 720 iterations), experimental analysis reveals two key issues:

- a) **Low execution efficiency.** In many cases, the algorithm achieves relatively high image quality values after only a small number of iterations. In subsequent iterations, the image quality value rises only slowly, with minimal improvement (visually imperceptible) even after completing all 720 iterations. In other words, the fixed iteration count approach wastes considerable execution time without significantly improving deblurring qual-

ity.

- b) **Deblurred image quality may not be optimal.** In some instances, the intermediate estimated image produced after fewer iteration steps reaches optimal quality. As iteration count increases further, image quality values may actually degrade, resulting in over-iteration that not only incurs substantial time overhead but also produces progressively lower-quality restored images.

Therefore, these observations indicate the need for an iterative stopping condition for iterative deblurring algorithms like NCSR, enabling them to achieve optimal deblurring effects with minimal computational cost. A straightforward approach to address this issue in NCSR would be to employ no-reference image quality assessment (NR-IQA) algorithms to evaluate each intermediate estimate during iteration, terminating when quality values peak. However, NR-IQA algorithms are not specifically designed for deblurring and generally have high computational complexity. Currently, effective NR-IQA algorithms require considerable execution time, typically on the order of seconds and sometimes even tens of seconds [15~17]. Frequent invocation of NR-IQA during NCSR iteration would drastically reduce the deblurring algorithm's efficiency.

To overcome this limitation, this paper proposes an adaptive iterative stopping criterion (AISC) based on the difference image between the intermediate estimate convolved with the blur kernel and the blurred image—referred to as the residual image—rather than evaluating intermediate estimates directly. Specifically, we define a deblurring measure (DM) based on brightness differences between pixels in the residual image, then design AISC based on this DM metric. To validate AISC's effectiveness, we apply it to the classic NCSR algorithm. Extensive experiments under uniform blur, Gaussian blur, and motion blur demonstrate that compared with the original fixed-iteration NCSR, the AISC-enhanced version significantly improves execution efficiency while maintaining comparable image quality in terms of PSNR, SSIM [18], and FSIM [19] metrics.

1 Related Work

1.1 NCSR Algorithm Overview

In the classic NCSR algorithm [13], a dictionary is constructed online from the given blurred image itself. The blurred image is decomposed into patches, each of which is sparsely coded using the learned dictionary. After processing sparse coefficients through shrinkage, the image is reconstructed via the inverse process to achieve deblurring. Sparse coding is the core technique, tasked with calculating sparse coefficient values for patches given the dictionary.

For an image $g \in \mathbb{R}^N$, let $R_i g$ denote a patch of size $n \times n$ extracted at pixel location i , where R_i is the matrix operation for patch extraction from g at

position i . Given dictionary $\phi \in \mathbb{R}^{n \times m}$ (where $m \leq n$), a patch extracted from image g can be sparsely represented as:

The image deblurring problem is solved through the following optimization:

where P is the total number of patches extracted from image g ; α_i represents the sparse coefficients of patch $R_i g$ with respect to dictionary ϕ ; and λ is the regularization parameter. Image g can be represented through the sparse coefficient set $\{\alpha_i, g\}$. Reconstructing g from set $\{\alpha_i, g\}$ is an over-determined system problem solvable via least squares:

For convenience, Equation (5) can be rewritten as:

where α represents the concatenation of all α_i . Thus, Equation (4) can be written as:

In NCSR, the optimal estimate of f from g is obtained by solving:

where λ balances fidelity and sparsity terms; β_i estimates unknown sparse coefficients α_i . NCSR performs image deblurring primarily through iterative solution of Equation (8). Each iteration produces an estimate $\hat{f}^{(j)}$ of the original image f :

where δ is a predefined constant; g is the blurred image; h is the blur kernel; $\hat{f}^{(j)}$ is the estimated image at iteration j ; and $\hat{f}^{(j+1/2)}$ represents an intermediate result used to obtain the next sparse coding. Deblurring relies on shrinkage to compute new α_i^{j+1} , defined as:

where $S_\tau(\cdot)$ is the classic soft-thresholding operator and τ is the soft-thresholding parameter. Once α_i^{j+1} is obtained in each inner iteration, the intermediate deblurred image can be reconstructed as:

Note that NCSR fixedly uses $\hat{f}^{(720)}$ as the final estimate of the original image (i.e., the deblurred result).

1.2 Existing Problems

NCSR handles iteration count selection through fixed values. However, since image content and blur distortion types vary, the fixed 720 iterations cannot guarantee optimal deblurring for different images and distortions. To analyze problems arising from fixed iteration counts, we use the commonly tested Barbara image (256×256) to examine quality metric trends across 720 iterations under uniform blur, Gaussian blur, and motion blur. Experimental parameters ensure comprehensive coverage of blur types and distortion levels: for uniform blur, kernel sizes $K=[5\ 5], [15\ 15], [25\ 25]$; for Gaussian blur, kernel sizes $K=[5\ 5], [9\ 9], [15\ 15]$ with standard deviation Sigma (S)=1 and 3; for motion blur, distance Length (L)=10, 20, 40 at angle Angel (A)=0.

Table 1 records image quality values at key iteration steps under three blur types. The data reveals two trends: (a) **Case A**, where image quality peaks after relatively few iterations and subsequently declines; (b) **Case B**, where

image quality improves rapidly early then rises very slowly in later iterations. For Case A, over-iteration occurs—excessive iterations waste time and degrade quality. For Case B, early iterations produce significant quality gains while later iterations yield minimal improvement (visually imperceptible). Both cases necessitate adaptive iteration termination to balance optimal deblurring quality with satisfactory efficiency.

To visually illustrate restoration trends across iterations, Figure 1 [Figure 1: see original paper] shows deblurring results for Barbara under two scenarios. Figures 1(a)(b) reflect Case A (motion blur $A=0$, $L=10$), where the 720-iteration result (left) is noticeably blurrier than the best-quality image (right). Figures 1(c)(d) reflect Case B (Gaussian blur $K=[15\ 15]$, $S=3$), where the 720-iteration result (left) differs minimally from the best image (obtained at iteration 200, right), with differences indistinguishable to the human eye. This demonstrates NCSR's need for an iteration termination condition to balance efficiency and quality.

2 Adaptive Iterative Stopping Condition

2.1 Residual Image

The residual image is defined as the difference between the intermediate estimated image convolved with the blur kernel and the blurred image during iteration:

Theoretically, during initial iterations when the original image estimate is inaccurate, residual image r shows large pixel differences (especially in challenging edge details). As iterations progress and $\hat{f}$ becomes more accurate, $\hat{f} * h$ increasingly approximates degraded image g , causing residual image pixel brightness values to shrink and fluctuate near zero, with inter-pixel differences decreasing. Using a 256×256 Barbara image as an example, residual images at iterations 1, 10, 20, 50, and 720 are shown in Figure 2 [Figure 2: see original paper] (brightness normalized to $[0, 255]$ for visual observation; only motion blur case $A=0$, $L=10$ shown due to space limitations; uniform and Gaussian blur cases are similar).

Figure 2 reveals that at iteration 1, the residual image clearly shows key edge information reflecting image content. By iteration 20, structural information becomes barely visible. As iteration count increases, brightness differences between residual image pixels shrink further. At iteration 50, image content information in the residual image becomes visually undetectable, indicating near-optimal deblurring. Figure 3 [Figure 3: see original paper] shows histograms of residual image pixel brightness values at these five iteration steps, demonstrating that brightness value ranges narrow with increasing iterations, confirming that intermediate estimates progressively approach the original image.

2.2 Iteration Termination Metric

Analysis of residual image pixel brightness trends reveals that as iteration count increases, the deblurred estimate approaches the original image, causing residual image pixel brightness values and their differences to decrease. Based on this observation, we define a deblurring measure (DM) to evaluate deblurring effectiveness.

First, the residual image is computed via Equation (11) and normalized to zero mean and unit variance:

where r represents the normalized residual image; $\text{mean}(r)$ and $\text{var}(r)$ are the mean and variance of the residual image, respectively. Then, for a pixel at coordinates (i, j) , we compute the product of its brightness value with that of a pixel at offset (m, n) . Repeating this operation for all pixels in r' and summing yields:

where L is the neighborhood radius. Table 2 shows DM values for a 256×256 Barbara image under uniform blur ($K=[5 \ 5]$), Gaussian blur ($K=[15 \ 15]$, $S=3$), and motion blur ($A=0$, $L=10$). The data demonstrates:

- a) DM values change systematically: they decrease rapidly during initial iterations, indicating fast quality improvement; as iteration count increases, DM decreases more slowly, indicating stabilizing image quality.
- b) For different blur types, distortion levels, and image contents, it's difficult to determine termination based on absolute DM values alone. For instance, DM stabilizes at 0.38, 0.05, and 0.59 in Table 2, making direct thresholding impractical.

2.3 Adaptive Iterative Stopping Condition

Given DM's variation trend, termination cannot be determined from absolute values. Therefore, we propose AISC based on DM's change trend: termination occurs when the absolute change in DM over a consecutive Step range falls below a preset threshold ϵ . When AISC is satisfied, the deblurring iteration terminates. Thus, consecutive step count Step and threshold ϵ are two critical parameters determined based on the specific deblurring algorithm's convergence speed and extensive experimental data.

To set reasonable Step and ϵ values, we conduct experiments on 10 images from the TID2013 database [20] (shown in Figure 3 [Figure 3: see original paper]). Using Gaussian blur ($K=[9 \ 9]$, $S=3$) with Step values of 15, 20, 25 and threshold ϵ values of 0.01, 0.02, 0.03, Table 5 shows average differences between image quality values at termination and at 720 iterations for 10 test images. Positive values indicate AISC produces better quality than the original fixed 720-step NCSR, while negative values indicate worse quality. Bold underlined values represent optimal PSNR, SSIM, and FSIM under different settings. Overall, only when $\epsilon = 0.01$ does FSIM become positive, with Step=20 yielding the

best deblurring effect at $e = 0.01$. Therefore, we select Step=20 [Figure 20: see original paper] and $e = 0.01$ as AISC parameters for all subsequent experiments.

3 Experimental Results and Analysis

3.1 Experimental Platform and Environment

To validate effectiveness and stability, we test on two representative image sets: (1) widely used standard test images including Barbara, Boat, Cameraman, Couple, Elk, Goldhill, House, Lena, Man, and Peppers (Figure 4 [Figure 4: see original paper]); (2) a challenging set combining 10 TID2013 images [20] and 90 images from reference [17] (partially shown in Figure 5 [Figure 5: see original paper]). This challenging set, with complex content and rich texture features, is ideal for verifying AISC's robustness. The hardware platform is an Intel Core(TM) i7-6700 3.40 GHz CPU with 8 GB RAM, running Windows 7 and MATLAB 2014a. PSNR, SSIM [18], and FSIM [19] metrics evaluate deblurring quality.

3.2 Common Image Set

AISC effectiveness is first validated on the common image set, with results in Tables 6 ~8. The data shows: (a) termination steps distribute within 200 iterations, far fewer than NCSR's fixed 720; (b) image quality at termination steps differs minimally from step 720 quality (original NCSR), making visual distinction difficult. Two specific cases are illustrated: Figure 6 [Figure 6: see original paper] shows Elk under Gaussian blur ($K=[15\ 15]$, $S=3$), where the 720-step result has slightly better quality than adaptive termination but is visually indistinguishable; Figure 7 [Figure 7: see original paper] shows Lena under uniform blur ($K=[5\ 5]$), where the 720-step result is slightly worse but also visually indistinguishable. Thus, AISC enables timely termination, dramatically improving computational efficiency while sacrificing little image quality. Though quality may occasionally improve or degrade slightly, the differences are visually negligible.

3.3 Challenging Image Set

To further verify AISC's robustness, experiments are conducted on the challenging image set. Tables 9 ~11 show results across three blur types, exhibiting similar patterns to the common image set. Overall, AISC terminates iterations appropriately while incurring minimal quality loss (and sometimes improvement), effectively enhancing computational efficiency.

3.4 Execution Time

To measure the runtime overhead of the DM metric, we record per-iteration time for NCSR's inner loop and DM computation time for a 256×256 Lena image,

shown in Table 12. The proposed DM metric adds negligible time (on the order of milliseconds) because it is specifically designed for deblurring algorithms, with convolution between intermediate estimates and blur kernels being the primary computational load—an operation with many mature acceleration techniques. In contrast, existing NR-IQA algorithms often require seconds (sometimes tens of seconds) [15–17]. Since AISC typically terminates within 200 iterations, execution time is substantially reduced. The savings come not only from reduced inner iterations but also from significant reductions in dictionary learning time in outer loops. Therefore, AISC considerably improves NCSR’s computational performance while often improving deblurring quality. Even when quality is slightly lower than original NCSR, visual differences remain minimal.

4 Conclusion

Existing iterative deblurring algorithms using fixed iteration counts cannot guarantee optimal deblurring quality and suffer from low time efficiency. This paper analyzes residual image characteristics and defines a deblurring measure based on pixel brightness differences in residual images. Based on this DM, we design an adaptive iterative stopping criterion (AISC). The AISC-enhanced NCSR algorithm adaptively terminates iterations while maintaining image quality, significantly improving the original algorithm’s time-consuming nature. Notably, AISC is not limited to NCSR but is applicable to all iterative deblurring algorithms, though its two key parameters must be set appropriately based on the specific algorithm’s convergence characteristics.

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