# Postprint of Underdetermined Mixing Matrix Estimation Based on GASA-FCM Hybrid Clustering and Hough Transform

**Authors:** Li Hu, Xu Yan

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## Abstract

To address the low accuracy and poor robustness of the fuzzy C-means clustering (FCM) algorithm in underdetermined mixing matrix estimation, an underdetermined mixing matrix estimation algorithm based on Genetic Algorithm Simulated Annealing optimized FCM (GASA-FCM) hybrid clustering and Hough transform is proposed. The algorithm first combines the advantages of global search and high precision of the simulated annealing algorithm (SA) with the powerful spatial search capability of the genetic algorithm (GA), assigning the cluster centers obtained by the genetic simulated annealing algorithm to FCM to avoid the randomness in initial value selection. Subsequently, the Hough transform is employed to refine the centers of each class obtained from clustering, thereby improving the estimation accuracy of the mixing matrix. Experimental results demonstrate that the proposed algorithm significantly enhances the stability of the algorithm and the estimation accuracy of the mixing matrix, demonstrating its effectiveness and feasibility.

## Full Text

## Preamble

### Underdetermined Mixing Matrix Estimation Using GASA-FCM Hybrid Clustering and Hough Transform

*Li Hu, Xu Yan*
(School of Electronic & Information Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China)

**Abstract:** The fuzzy C-means (FCM) clustering algorithm suffers from low precision and poor robustness in underdetermined mixing matrix estimation. To address these shortcomings, this paper proposes an underdetermined mixing matrix estimation algorithm based on genetic simulated annealing optimized FCM (GASA-FCM) hybrid clustering and Hough transform. The algorithm first leverages the global search capability and high precision of the simulated annealing algorithm (SA) combined with the powerful spatial search ability of the genetic algorithm (GA) to obtain cluster centers that are then assigned to FCM, thereby avoiding the randomness of initial value selection. Subsequently, Hough transform is applied to refine the center of each data cluster, improving the estimation accuracy of the mixing matrix. Experimental results demonstrate that the proposed algorithm significantly enhances both the stability of the algorithm and the accuracy of mixing matrix estimation, confirming its validity and feasibility.

**Keywords:** fuzzy C-means clustering algorithm; genetic algorithm; simulated annealing algorithm; Hough transform; mixing matrix estimation

## 0 Introduction

Underdetermined blind source separation based on sparse signal analysis theory proposes a “two-step” approach where the core problem is mixing matrix estimation, as the accuracy of mixing matrix estimation directly determines the precision of source signal separation. For underdetermined mixing matrix estimation, researchers have exploited the linear clustering characteristics of observation data in the time or time-frequency domain to develop a series of clustering algorithms that compute line vectors from clustering results to estimate the mixing matrix. The most widely used method is the fuzzy C-means clustering algorithm, which represents a natural extension of hard C-means clustering and offers computational simplicity and convenient implementation. However, FCM is sensitive to initial cluster centers, requires manual determination of the number of clusters, easily falls into local optima, and is affected by noise points and outliers in the clustering results, leading to low stability and accuracy in mixing matrix estimation.

To address the limitations of FCM, reference [3] proposes a GA-optimized FCM that utilizes the global search performance of genetic algorithms to obtain initial cluster centers, avoiding the impact of improper cluster center selection on FCM results. Reference [4] introduces a differential evolution approach to optimize fuzzy C-means clustering, overcoming FCM’ s dependence on initial values and its tendency to fall into local optima. This paper proposes an underdetermined mixing matrix estimation algorithm based on GASA-FCM hybrid clustering and Hough transform. By integrating the strong global search capability of genetic algorithms with the robust local search capability of simulated annealing algorithms, the proposed method enhances both global and local search efficiency and effectiveness. While slightly increasing convergence time, this approach effectively and rapidly resolves FCM’ s excessive dependence on initial cluster centers and its susceptibility to local optima, thereby improving the stability of mixing matrix estimation results. However, this method alone cannot overcome the influence of isolated points on cluster centers. Therefore, this paper further employs Hough transform to refine the clustering results obtained from GASA-FCM, effectively improving mixing matrix estimation accuracy.

## 1 Underdetermined Mixing Matrix Estimation Based on Signal Sparsity

The general mathematical model for linear underdetermined blind source separation can be written as:

$$x(t) = As(t)$$

where $x(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T$ represents $M$ observed signals, $s(t) = [s_1(t), s_2(t), \dots, s_N(t)]^T$ represents $N$ source signals, and $A \in \mathbb{R}^{M\times N}$ is the mixing matrix. When source signals are sparse, the signal amplitude is zero or close to zero at most sampling points. At any given sampling instant, only one source signal has a non-zero value, yielding:

$$x(t) = a_i s_i(t) \quad (i = 1, 2, \dots, N)$$

However, most signals are not sufficiently sparse in the time domain in practical applications, necessitating alternative transformation methods to enhance signal sparsity. This paper employs the short-time Fourier transform (STFT), giving the STFT expression of equation (1) as:

$$X(t,\omega) = AS(t,\omega)$$

where $X(t,\omega)$ and $S(t,\omega)$ denote the STFT of $x(t)$ and $s(t)$, respectively. After STFT transformation, equation (2) can be rewritten as:

$$X(\omega) = AS(\omega)$$

## 2.1 Fuzzy C-Means Clustering

Fuzzy C-means clustering belongs to the category of objective function-based fuzzy clustering algorithms. FCM is an optimization algorithm that achieves data point classification by optimizing an objective function to determine each data point’ s membership degree. The algorithm is simple and offers high precision, making it a classic clustering algorithm for mixing matrix estimation.

Assuming a sample set $X = \{x_1, x_2, \dots, x_n\}$ to be divided into $c$ fuzzy groups, the cluster center of each group is $C = \{c_1, c_2, \dots, c_i\}$, and the objective function is minimized. FCM has a normalization constraint that the sum of membership degrees across the entire dataset equals 1. The FCM objective function is:

$$J(U, c_1, \dots, c_c) = \sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^m d_{ij}^2$$

where $u_{ij}$ represents the membership degree of sample $j$ belonging to class $i$, and $d_{ij} = \|c_i - x_j\|$ is the Euclidean distance between cluster center $i$ and data point $j$.

To constrain the objective function, Lagrange multiplier $\lambda$ is introduced to construct a new objective function. Taking partial derivatives yields the minimum value of the objective function and the cluster centers:

$$c_i = \frac{\sum_{j=1}^{n} u_{ij}^m x_j}{\sum_{j=1}^{n} u_{ij}^m}$$

$$u_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{kj}}\right)^{\frac{2}{m-1}}}$$

The FCM process is shown in [Figure 1: see original paper].

## 2.2 Genetic Simulated Annealing Algorithm

The genetic simulated annealing algorithm is a hybrid optimization algorithm based on combinatorial thinking. By integrating the strong local search capability of simulated annealing into the genetic algorithm, which possesses robust global search capability, the algorithm enhances both global and local search abilities and efficiency. These two algorithms complement each other in search capability, resulting in a superior search algorithm with improved performance and reduced parameter sensitivity.

The overall flow of the genetic simulated annealing algorithm is similar to the basic genetic algorithm. It begins by generating random initial solutions and performing global search using genetic algorithms. New individuals are then produced through crossover and mutation operations, followed by independent simulated annealing operations on each generated individual to produce the next generation population. This process repeats until termination conditions are satisfied. The basic procedure is as follows:

a) Initialize parameters (maximum iteration count, genetic algorithm crossover rate, simulated annealing initial temperature, etc.);

b) Randomly generate the initial population;

c) Calculate the fitness of all individuals in the initial population using the fitness function;

d) Select individuals from the population for crossover and mutation operations to produce a new population;

e) Apply simulated annealing operations to each individual in the new population to generate new individuals;

f) Calculate individual fitness values and evaluate them;

g) Check termination conditions. If satisfied, output the optimal solution and terminate; otherwise, return to step d).

## 2.3 SAGA-FCM-Based Hybrid Clustering Algorithm

To effectively improve the local convergence of FCM, this paper adopts the SAGA-FCM hybrid clustering algorithm, which enhances the robustness of clustering results through the strong local search capability of simulated annealing algorithms and the powerful global search capability of genetic algorithms. The process is illustrated in [Figure 3: see original paper].

Due to the sparsity of speech signals, observation data points in underdetermined conditions are primarily concentrated along $n$ lines with direction vectors $l_i$:

$$l_i = \begin{bmatrix} \sin\theta_i & \sin\theta_i & \cdots & \sin\theta_i \\ -\cos\theta_i & -\cos\theta_i & \cdots & -\cos\theta_i \end{bmatrix}^T$$

where $\theta_i \in [0, \pi)$ for $i = 1, 2, \ldots, M-1$. The line transformation to parameter space is given by:

$$\rho = x\cos\theta + y\sin\theta$$

## 3 Hough Transform for Cluster Center Refinement

Applying Hough transform to refine cluster centers obtained from clustering compensates for the impact of isolated points on estimation accuracy. The proposed underdetermined mixing matrix estimation algorithm based on GASA-FCM hybrid clustering and Hough transform first applies GASA-FCM clustering to time-frequency domain observation signals to obtain initial mixing matrix estimates, then uses Hough transform to refine the cluster center of each data class, yielding the final mixing matrix estimate.

The specific steps of the proposed algorithm are as follows:

a) Apply short-time Fourier transform to observation signals and examine the scatter plot in time and frequency domains;

b) If isolated points exist along line directions in the frequency domain scatter plot, employ the single-source time-frequency point detection method from reference [13] to enhance signal sparsity;

c) Set a threshold to remove low-energy points, normalize the remaining observation signals column-wise, project them onto the $M$-dimensional upper hemisphere, and examine the clustering characteristics;

d) Apply the GASA-FCM clustering algorithm to the normalized observation signals to obtain cluster centers and preliminary mixing matrix estimates;

e) Perform Hough transform on each class of data obtained from GASA-FCM clustering, detect local maximum point coordinate parameters, calculate line vectors (i.e., mixing matrix column vectors), and obtain the refined mixing matrix estimate.

Hough transform maps data from graphical space to parameter space. After Hough transformation, points on the same line in graphical space correspond to curves in parameter space that intersect at a single point, creating an extremum point. The main idea of using Hough transform for mixing matrix estimation is: applying Hough transform to time-frequency domain observation signals, where all curves transformed from observation points on the same clustering line intersect to form extremum points. By accumulating these points to obtain local maximum positions, line vectors can be calculated, leading to mixing matrix estimation.

Assuming a line in graphical space passing through point $(x, y)$ is $y = ax + b$, its parameter equation in transform space is:

$$\rho = x \cos\theta + y \sin\theta$$

where $\rho$ represents the distance from the origin to the line in Cartesian coordinates, and $\theta$ represents the angle between the x-axis and the line normal. The Hough transform principle is illustrated in [Figure 4: see original paper], showing how points A, B, C, D on the same line in graphical space transform to curves A' , B' , C' , D' in parameter space that intersect at point P' . Detecting P' s coordinate parameters enables line direction estimation.

## 4 Experimental Simulation and Analysis

### 4.1 Mixing Matrix Evaluation Criteria

To evaluate the precision of underdetermined mixing matrix estimation, this paper adopts the normalized mean square error (NMSE) from reference [14] and the deviation angle metric from reference [15] to assess algorithm performance.

The normalized mean square error is expressed as:

$$\text{NMSE} = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{[a_{ij} - \hat{a}_{ij}]^2}{a_{ij}^2}$$

where $M$ and $N$ are the row and column dimensions of the mixing matrix, and $a_{ij}$ and $\hat{a}_{ij}$ are elements of the true mixing matrix $A$ and estimated matrix $\hat{A}$, respectively. A smaller NMSE indicates more accurate mixing matrix estimation.

The deviation angle is expressed as:

$$\mathrm{ang}(a_i, \hat{a}_i) = \arccos\left(\frac{a_i^T \hat{a}_i}{\|a_i\|\|\hat{a}_i\|}\right) \times \frac{180}{\pi}$$

which represents the angle between $a_i$ and $\hat{a}_i$, where $a_i$ is the $i$-th column of the true matrix $A$ and $\hat{a}_i$ is the $i$-th column of the estimated matrix $\hat{A}$ ($i = 1, 2, \dots, N$). A smaller deviation angle indicates better mixing matrix performance.

### 4.2 Experimental Simulation and Analysis

To verify the effectiveness of the proposed algorithm, a series of comparative simulation experiments were conducted in MATLAB R2010b using speech signals from the NOIZEUS database with an 8 kHz sampling frequency.

**Simulation 1:** Three source signals were mixed into two observation signals ($N = 3$, $M = 2$). The randomly generated $2 \times 3$ mixing matrix was:

$$A = \begin{bmatrix} 0.3633 & 0.6207 & 0.6848 \\ 0.9317 & 0.7841 & 0.7288 \end{bmatrix}$$

The three source signals were mixed using matrix $A$, producing two observation signals whose time-domain waveforms are shown in [Figure 4: see original paper]. Short-time Fourier transform was applied to the observation signals using a Hanning window with length 512 and overlap length 256. The resulting time-frequency domain scatter plot is shown in [Figure 5: see original paper].

As seen in [Figure 5: see original paper], the directionality of observation signals in the frequency domain is not clear. The proposed time-frequency single-source point detection method was applied to enhance signal sparsity, revealing good directionality. A threshold $\varepsilon = 0.15$ was set to remove low-energy points. The remaining observation signals were normalized column-wise and clustered to estimate the mixing matrix. The scatter plots after time-frequency single-source point detection and normalization are shown in [Figure 6: see original paper].

The mixing matrices estimated by different algorithms were:

- **FCM algorithm:**

$$\hat{A}_{\mathrm{FCM}} = \begin{bmatrix} 0.3761 & 0.6159 & 0.6735 \\ 0.9266 & 0.7872 & 0.7391 \end{bmatrix}$$

- **GAFCM algorithm:**

$$\hat{A}_{\mathrm{GAFCM}} = \begin{bmatrix} 0.3725 & 0.6220 & 0.6752 \\ 0.9280 & 0.7830 & 0.7376 \end{bmatrix}$$

- **GASAFCM algorithm:**

$$\hat{A}_{\text{GASAFCM}} = \begin{bmatrix} 0.3654 & 0.6189 & 0.6829 \\ 0.9308 & 0.7855 & 0.7305 \end{bmatrix}$$

- **Proposed algorithm:**

$$\hat{A}_{\text{proposed}} = \begin{bmatrix} 0.3652 & 0.6213 & 0.6836 \\ 0.9309 & 0.7836 & 0.7298 \end{bmatrix}$$

The NMSE and runtime were compared to evaluate mixing matrix estimation accuracy, as shown in . Deviation angles between the estimated and true mixing matrix columns were calculated to further analyze estimation precision, presented in .

**Simulation 2:** Four source signals were mixed into three observation signals ($N = 4$, $M = 3$). The randomly generated $3 \times 4$ mixing matrix was:

$$A = \begin{bmatrix} 0.9543 & 0.9748 & 0.2070 & 0.1936 \\ 0.2220 & 0.2149 & 0.6467 & 0.7505 \\ 0.2003 & 0.0606 & 0.7342 & 0.6318 \end{bmatrix}$$

The three observation signals are shown in [Figure 7: see original paper], with the time-frequency domain scatter plot in [Figure 8: see original paper]. After time-frequency single-source point detection and normalization, the scatter plots are shown in [Figure 9: see original paper].

The estimated mixing matrices were:

- **FCM algorithm:**

$$\hat{A}_{\text{FCM}} = \begin{bmatrix} 0.9330 & 0.9755 & 0.1056 & 0.1477 \\ 0.2171 & 0.2148 & 0.6782 & 0.7505 \\ 0.2447 & 0.0473 & 0.7269 & 0.6078 \end{bmatrix}$$

- **GAFCM algorithm:**

$$\hat{A}_{\text{GAFCM}} = \begin{bmatrix} 0.9520 & 0.9766 & 0.1967 & 0.1821 \\ 0.2163 & 0.2080 & 0.6583 & 0.7372 \\ 0.2166 & 0.0545 & 0.7267 & 0.6507 \end{bmatrix}$$

- **GASAFCM algorithm:**

$$\hat{A}_{\text{GASAFCM}} = \begin{bmatrix} 0.9570 & 0.9782 & 0.2094 & 0.1941 \\ 0.2160 & 0.1984 & 0.6444 & 0.7530 \\ 0.1938 & 0.0616 & 0.7355 & 0.6287 \end{bmatrix}$$

- **Proposed algorithm:**

$$\hat{A}_{\text{proposed}} = \begin{bmatrix} 0.9533 & 0.9757 & 0.2118 & 0.1967 \\ 0.2249 & 0.2108 & 0.6458 & 0.7500 \\ 0.2016 & 0.0640 & 0.7336 & 0.6315 \end{bmatrix}$$

The NMSE and runtime comparison is shown in , with deviation angles presented in .

As shown in and , although the proposed algorithm has longer runtime than other methods, it achieves smaller NMSE and more accurate mixing matrix estimation. and demonstrate that the proposed algorithm yields the smallest deviation angles for each column of the mixing matrix, confirming its superior estimation accuracy. These results consistently show that the proposed algorithm can effectively and accurately estimate the mixing matrix.

To verify the practicality of the proposed algorithm, extensive experimental data and simulations 1-2 were analyzed. The NMSE comparison is shown in [Figure 10: see original paper], revealing that NMSE increases as the number of source signals grows.

## 5 Conclusion

The accuracy of underdetermined mixing matrix estimation directly affects source signal separation performance. Traditional FCM algorithms exhibit excessive dependence on initial cluster centers and are prone to falling into local optima during the search process. This paper combines genetic algorithms with simulated annealing algorithms to leverage their complementary strengths, proposing an underdetermined mixing matrix estimation algorithm based on GASA-FCM hybrid clustering and Hough transform. This approach enhances both local and global search capabilities, effectively overcoming the local optima problem of traditional FCM algorithms. Additionally, to improve mixing matrix estimation accuracy, Hough transform is introduced to refine cluster centers obtained from clustering. Simulation results demonstrate that the proposed algorithm improves mixing matrix estimation accuracy and algorithm robustness, proving feasible and effective for mixing matrix estimation.

## References

[1] Dong T B, Yang J S. A robust underdetermined mixing matrix estimation algorithm [J]. Journal of Astronautics, 2013, 34 (3): 426-433.

[2] 毕晓君, 宫汝江. 基于混合聚类和网格密度的欠定盲矩阵估计 [J]. 系统工程与电子技术, 2012, 34 (3): 614-618.

[3] Alata M, Molhim M, Ramini A. Using GA for optimization of the fuzzy C-means clustering algorithm [J]. Research Journal of Applied Sciences Engineering & Technology, 2013, 5 (3): 695-701.

[4] 何嘉婧, 王晋东, 于智勇. 基于模糊 C-均值的改进人工蜂群聚类算法 [J]. 计算机应用研究,

2016, 33 (5): 1342-1345.
[5] 孙洁娣, 李玉霞, 温江涛, 等. 基于方向密度检测与霍夫变换的混合矩阵估计 [J]. 北京理工大学学报, 2016, 36 (11): 1188-1193.
[6] Llerena-Aguilar C, Gil-Pita R, Utrilla-Manso M, et al. A new mixing matrix estimation method based on the geometrical analysis of the sound separation problem [J]. Signal Processing, 2016, 134: 166–173.
[7] 李心杰, 贾进章, 李兵. 基于 SAGA-FCM 的煤与瓦斯突出预测方法 [J]. 煤田地质与勘探, 2016, 44 (2): 14-18.
[8] 熊拥军, 刘卫国, 欧鹏杰. 模糊 C-均值聚类算法的优化 [J]. 计算机工程与应用, 2015, 51 (11): 124-128.
[9] 殷旅江, 杨立君, 胡明茂, 等. 基于混合遗传模拟退火的模糊 C-均值聚类算法 [J]. 湖北汽车工业学院学报, 2015 (3): 62-65.
[10] 张焯, 徐玲, 杨丹. 基于改进模糊 C 均值的软件缺陷预测研究 [J]. 计算机工程与应用, 2015, 51 (7): 136-140.
[11] 孙洁娣, 李玉霞, 温江涛, 等. 基于密度的空间聚类与霍夫变换相结合的欠定盲源分离混合矩阵估计 [J]. 高技术通信, 2014, 24 (12): 1270-1275.
[12] Sun J, Li Y, Wen J, et al. Novel mixing matrix estimation approach in underdetermined blind source separation [J]. Neurocomputing, 2016, 173 (P3): 623-632.
[13] Guo Q, Ruan G, Nan P. Underdetermined mixing matrix estimation algorithm based on single source points [J]. Circuits Systems & Signal Processing, 2017: 1-15.
[14] Li Y, Nie W, Ye F, et al. A mixing matrix estimation algorithm for underdetermined blind source separation [J]. Circuits Systems & Signal Processing, 2016, 35 (9): 3367-3379.
[15] Zhi-Tao C, Ke J. A new algorithm to estimate mixing-matrix of underdetermined blind signal separation [C]// Proc of the 8th International Conference on Computational Intelligence and Security. [S.l.]: IEEE Computer Society, 2012: 374-377.
[16] Li Y, Nie W, Ye F, et al. A complex mixing matrix estimation algorithm in under-determined blind source separation problems [J]. Signal Image & Video Processing, 2016: 1-8.

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