Postprint: Leader-Following Consensus for Second-Order Nonlinear Multi-Agent Systems

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Abstract

To reduce the impact of communication delays on system consensus, this research investigates the leader-following consensus for second-order nonlinear multi-agent systems with a leader, and proposes a novel concept of approximate stochastic impulsive delay applied to a new protocol. Compared with traditional protocols, when communication delays are small at impulsive moments, each agent in the new protocol predicts its current state based on the delayed state, and replaces the delayed state with its own future predicted state to send to neighboring agents while simultaneously compensating for delays in its own feedback channel, thereby enabling the system to achieve consensus more rapidly. Based on Lyapunov stability theory, two sufficient conditions guaranteeing system consensus are derived by utilizing the properties of a further generalized Halanay inequality. Finally, simulation examples demonstrate the superiority of the new protocol.

Full Text

Leader-Following Consensus of Second-Order Nonlinear Multi-Agent Systems

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Abstract

To mitigate the impact of communication delays on system consensus, this paper investigates the leader-following consensus problem for second-order nonlinear multi-agent systems with an active leader. We introduce the novel concept of approximate random pulse delay and apply it to a new control protocol. Compared with traditional protocols, when communication delays at pulse instants are small, each agent in the new protocol predicts its current state based on the delayed state, transmits its predicted future state to neighboring agents instead of the delayed state, and simultaneously compensates for the delay in its own feedback channel, thereby enabling the system to achieve consensus more rapidly. Based on Lyapunov stability theory and utilizing properties of a further generalized Halanay inequality, we derive two sufficient conditions that guarantee system consensus. Finally, simulation examples demonstrate the superiority of the proposed protocol.

Keywords: active leader; delay; multi-agent system; consensus

0 Introduction

Research on consensus problems in multi-agent systems emerged in the mid-to-late 20th century. In 1987, Reynolds built upon previous work to propose the Boid model, which modeled common biological swarm phenomena in nature. Vicsek et al. subsequently introduced the Vicsek model, simplifying the conditions required by the Boid model and demonstrating through simulation that system individuals would eventually converge to consensus. Jadbabaie et al. theoretically proved the consensus behavior of the Vicsek model. Building on the work of Fax et al., Olfati-Saber et al. established a theoretical framework for multi-agent system consensus problems and presented basic control protocols that solved consensus for first-order integrator systems. Ren et al. extended this work to study consensus under both fixed and switching topologies. In recent years, this research area has developed into a hot topic in control science, with applications in robot swarm control, formation control, and congestion control in communication and sensor networks.

Building upon these foundations, researchers have investigated consensus problems with leaders or various types of delays in second-order and higher-order multi-agent systems. Reference [10] studied average consensus for a class of second-order nonlinear multi-agent systems under topology switching. References [11-14] employed different methods to examine impulsive consensus in second-order multi-agent systems with input delays. Reference [15] investigated leader-following consensus for nonlinear multi-agent systems under distributed impulsive control. Reference [16] analyzed impulsive consensus in second-order multi-agent systems with active leaders and input delays. Reference [17] studied impulsive consensus for multi-agent systems with uncertain and randomly occurring nonlinear dynamics. Reference [21] utilized properties of a generalized Halanay inequality based on complex network models to derive sufficient conditions for system stability under different communication delays. To address adverse effects of communication delays on multi-agent system formation, reference [22] proposed a master-slave predictive formation control architecture where each agent could predict its current state based on delayed state.

In practical problems, time-varying communication delays exist within a certain range. Inspired by reference [22], when applying impulsive control to multi-agent systems, we consider having each agent use its predicted future state instead of the delayed state when pulse moment delays are small, while compensating for its own feedback channel delay. This paper presents preliminary research on this approach and introduces the concept of approximate random pulse delay, which can reduce control costs and improve control efficiency to some extent. Methodologically, by further generalizing Lemma 5 from reference [21], we obtain a Halanay inequality applicable to our conditions and use its unique properties to derive sufficient conditions for achieving leader-following consensus under time-varying communication delays. Finally, Matlab simulations verify the superiority of our theoretical results.

1.1 Graph Theory

Let $G = (V, \mathcal{E}, A)$ represent an N-order undirected communication topology graph, where $V = \{R_1, R_2, \dots, R_N\}$ are the nodes of graph G. A path between initial node i and terminal node j in graph G has the form $\{(i, i_1), (i_1, i_2), \dots, (i_l, j)\} \subset \mathcal{E}$. The weighted adjacency matrix is $A = [a_{ij}]$, where element $a_{ij} \geq 0$ represents the weight of the edge between nodes i and j. The weight is positive when nodes are connected and zero otherwise. The Laplacian matrix of graph G is $L = [l_{ij}]$ defined as:

$$l_{ij} = \begin{cases} \sum_{j=1, j\neq i}^{N} a_{ij}, & i = j \\ -a_{ij}, & i \neq j \end{cases}$$

If the connection between leader R_0 and follower nodes changes with topology switching, then this leader is called an active leader. Let $B = \text{diag}(b_1, b_2, \dots, b_N) \in \mathbb{R}^{N\times N}$ be a diagonal matrix where element b_i is the connection weight between leader R_0 and follower node i. If node i is connected to R_0, let $b_i > 0$; otherwise $b_i = 0$ for $i = 1, 2, \dots, N$.

2 Main Results

Consider the active leader dynamics model:

$$\begin{cases} \dot{x}_0(t) = v_0(t) \\ \dot{v}_0(t) = f(x_0(t), t) \end{cases}$$

where $x_0(t) \in \mathbb{R}^n$ and $v_0(t) \in \mathbb{R}^n$ represent the leader' s position and velocity states, respectively, and $f(x_0(t), t)$ is a nonlinear vector-valued function describing the leader' s dynamics.

Consider a multi-agent system with N agents. Assume each agent exchanges information with neighboring agents or receives information from the leader to update its state. The agent dynamics are given by:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = f(x_i(t), t) + u_i^1(t) + u_i^2(t) \end{cases}, \quad i = 1, 2, \dots, N$$

where $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ are the position and velocity states of agent i, respectively, $u_i^1(t)$ and $u_i^2(t)$ are control inputs, and $f(x_i(t), t)$ represents the nonlinear vector-valued function describing each agent' s intrinsic dynamics.

Definition 1. Under the assumptions and conditions of Theorem 1, if the solution of error system (8) satisfies $\lim_{t\to+\infty} x_i(t) = 0$ and $\lim_{t\to+\infty} v_i(t) = 0$, then the second-order nonlinear multi-agent system is said to achieve leader-following consensus under control protocols (3), (5), and (6).

Assumption 1. If the nonlinear vector-valued function f satisfies the Lipschitz condition: for any vectors $x, \bar{x} \in \mathbb{R}^n$, there exists a non-negative constant φ such that $\|f(x, t) - f(\bar{x}, t)\| \leq \varphi\|x - \bar{x}\|$.

Assumption 2. All agents have the same probability of experiencing effective delays at pulse instants.

In practical applications, due to physical limitations, information exchange between agents often involves time-varying communication delays. Inspired by reference [10], we design the following continuous control protocol for non-pulse moments, where agents update their states by exchanging information with neighbors:

$$u_i^1(t) = a \sum_{j\in\mathcal{N}_i} a_{ij}[v_j(t - \tau(t)) - v_i(t - \tau(t))]$$

where $a \in (0, 1)$ represents the coupling strength and $\tau(t)$ is the time-varying communication delay.

When the network topology of the multi-agent system switches, define a piecewise constant switching signal $s(t) : (0, +\infty) \to \mathcal{P} = \{1, 2, \dots, m\}$. Let $\{G_1, G_2, \dots, G_m\}$ denote the set of possible topology graphs. If $s(t)$ is constant, the topology is fixed; otherwise, the topology graph corresponding to the switching signal is denoted as $G_{s(t)}$.

From reference [10], for any pulse signal, we can deduce that as time increases, the agents' position and velocity states will eventually synchronize with the leader' s states. Therefore, to achieve leader-following consensus, we extend this result by applying the leader' s state information as a pulse signal to each follower node, i.e., $u_i^2(t) = \sum_{k=1}^{\infty} \delta(t - t_k) \cdot$ [pulse control term].

Lemma 1 [17]. If [mathematical condition].

Lemma 2 [18]. For any symmetric matrix $P \in \mathbb{R}^{n\times n}$ and any vectors $x, y \in \mathbb{R}^n$, we have $2x^T Py \leq x^T Px + y^T Py$.

Lemma 3 [19]. For any vectors $x, y \in \mathbb{R}^n$ and any Hermitian matrix $\Phi \in \mathbb{R}^{n\times n}$, there exists the inequality: $\lambda_{\min}(\Phi)\|x\|^2 \leq x^T\Phi x \leq \lambda_{\max}(\Phi)\|x\|^2$.

Lemma 4. Based on Lemma 5 in reference [20], we obtain the following impulsive differential inequality (a generalization of Halanay inequality). The derivation introduces mathematical expectation, and the specific proof steps are consistent with Appendix A in reference [20], so they are omitted here.

If there exist real numbers $a > b > 0$, $p_i > 0$, $q_i > 0$ satisfying:

$$\begin{cases} D^+V(t) \leq aV(t) + b\sup_{t-\tau\leq s\leq t} V(s), & t \neq t_k \\ V(t_k^+) \leq p_k V(t_k) + q_k \sup_{t_k-\tau\leq s\leq t_k} V(s), & k \in \mathbb{N} \\ \sum_{i=1}^{r} \ln(p_i + q_i) + a(t_{k+r} - t_k) + b\tau < 0 \end{cases}$$

then there exist constants $\xi > 0$ and $\lambda > 0$ such that $V(t) \leq \sup_{t_0-\tau\leq s\leq t_0} V(s)e^{-\lambda(t-t_0)}$ for $t \geq t_0$.

Theorem 1. Under Assumptions 1 and 2, if the system variables satisfy conditions and , then the impulsive dynamical system (7) achieves leader-following consensus under control protocols (3), (5), and (6), where all agent states converge to the leader state (1).

Proof. Construct the Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} \left(x_i^T(t)x_i(t) + v_i^T(t)v_i(t)\right)$$

For any time $t \in (t_{k-1}, t_k]$, the derivative of $V(t)$ along the solution of system (7) is:

$$\dot{V}(t) = \sum_{i=1}^{N} \left(x_i^T(t)\dot{x}_i(t) + v_i^T(t)\dot{v}_i(t)\right)$$

From Assumption 2, we know that Λ is a symmetric matrix, and R is a negative definite Hermitian matrix. By Lemma 3, we have:

$$\dot{V}(t) \leq \varphi V(t) + 3\lambda_1 V(t) + 2\lambda_1 V(t - \tau(t))$$

Taking expectation and applying Lemma 4, if conditions and hold, we obtain:

$$\mathbb{E}V(t) \leq \sup_{t_0-\tau\leq s\leq t_0} \mathbb{E}V(s)e^{-\lambda(t-t_0)}$$

As $t \to +\infty$, $\mathbb{E}V(t) \to 0$, which implies $\lim_{t\to+\infty} x_i(t) = 0$ and $\lim_{t\to+\infty} v_i(t) = 0$. Therefore, the second-order nonlinear multi-agent system (7) achieves leader-following consensus under control protocols (3), (5), and (6).

3 Simulation Examples

Assume the multi-agent system under study contains 5 agents, and each agent' s intrinsic nonlinear dynamics satisfy $f(x_i(t), t) = \sin(x_i(t))$, which satisfies the Lipschitz condition with $\varphi = 1$. The initial states of the leader and followers are set as $x_0(0) = (12, 3)$, $v_0(0) = (20, 9)$, $x_1(0) = (-2, 3)$, $v_1(0) = (-16, 9)$, $x_2(0) = (-26, 20)$, $v_2(0) = (3, 15)$. The coupling strengths are $\alpha = 0.1$, $\beta = 0.7$. The pulse interval is $t_{k+1} - t_k = 0.05$. The pulse gain matrices are $B_k = \text{diag}(0.7, 0.7, 0.7, 0.7, 0.7)$, with connection matrices $B_1 = \text{diag}(0, 0, 0, 1, 1)$, $B_2 = \text{diag}(1, 1, 0, 0, 0)$, and $B_3 = \text{diag}(0, 0, 1, 1, 0)$. For simplicity, define the non-pulse delay as:

$$\tau(t) = \begin{cases} 0.1, & 0 \le t < 1 \\ 0.05, & 1 \le t < 1.5 \\ 0.04, & 1.5 \le t < +\infty \end{cases}$$

At pulse moments, when $\phi(t_k) = 1$, the corresponding delay is $\tau(t_k) = 0.03$; when $\phi(t_k) = 0$, $\tau(t_k) = 0$. The random variable follows a Bernoulli distribution with $N = 10$ and $r = 0.6$.

The switching signal is defined as $s(t) = ((k-1) \mod 3)+1$ for $t \in [t_{k-1}, t_k)$, so the topology switching sequence is $\{G_1, G_2, G_3\}$. Calculations show that when the topology is G_1, $\lambda_1 = 0.1739$, $\lambda_2 = 0.9878$, $\lambda_3 = 0.0618$, $\lambda_4 = 0.2074$, and with $\varepsilon = 1$, condition is satisfied: $\lambda_2 + \lambda_3 + \lambda_4 - \varepsilon\lambda_1 = 0.6348 < 1$. Condition is also satisfied: $\ln(\lambda_2 + \lambda_3 + \lambda_4 - \varepsilon\lambda_1) + \lambda_1\varphi + \lambda_2 + \lambda_3 + \lambda_4 - \varepsilon\lambda_1 \approx -234.5410 < -\lambda_1\varphi - \lambda_2 - \lambda_3 - \lambda_4 + \varepsilon\lambda_1$. Similar results hold for topologies G_2 and G_3.

Matlab simulations yield Figures 2-5. Figure 2 shows the position states of followers and the leader under protocol (3,5), Figure 3 shows the velocity states, and Figures 4-5 show the corresponding errors. The results demonstrate that as time increases, the leader' s position and velocity exhibit nonlinear variations, while the followers' states gradually converge to the leader' s states.

For comparison, simulations under protocol (3,4) are also performed with small pulse delays of 0.01s in [0.2s, 0.3s] and 0.005s in [0.6s, 0.7s], while other pulse delays remain at 0.03s. The results are shown in Figures 6-7. Comparing the state evolution curves under protocols (4) and (5) reveals that within finite time, the system under protocol (4) shows convergence trends but with significantly slower and less effective convergence due to small delays. Moreover, the velocity states of followers under protocol (4) exhibit large error fluctuations when approaching the leader' s state, indicating poor stability. The new protocol (5), which eliminates the impact of small delays at pulse moments, resolves these issues with errors rapidly converging to zero stably. Thus, the simulation results validate the feasibility of our theoretical approach.

4 Conclusion

This paper investigated the leader-following consensus problem for second-order nonlinear multi-agent systems with an active leader and time-varying delays under switching topologies. A novel control protocol was proposed consisting of two parts: a discrete control protocol at pulse moments that introduces the approximate random pulse delay term, and a continuous control protocol at non-pulse moments that incorporates time-varying delays. Based on relevant theories and utilizing the unique properties of the further generalized Halanay inequality, sufficient conditions for achieving leader-following consensus were derived. The theoretical results were verified through Matlab simulation comparisons, demonstrating the effectiveness of the proposed approach.

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Note: Figure translations are in progress. See original paper for figures.