Postprint: Cloud-PDR Method for Multi-Criteria Decision Making in Incomplete Systems

Authors: Yuan Xumei, Wang Yana, Zhang Xu

Date: 2018-05-20T00:00:00+00:00

Abstract

Addressing the significance of multi-criteria decision-making problems and the deficiencies of classical rough set methods in ranking alternatives within incomplete systems, this study proposes a multi-criteria decision-making method for incomplete systems based on cloud probability dominance relation (cloud PDR), building upon existing probability dominance relation (PDR) ranking methods and integrating cloud theory. The introduction of parameters such as expectation, entropy, and hyper-entropy into the PDR method fully accounts for the fuzziness, volatility, and randomness inherent in decision-making, rendering the evaluation process and results more objective and comprehensive. An applied research study on coal resource-based cities demonstrates the method' s capability to generate rankings of development levels and their associated volatilities, with the superiority and effectiveness of the proposed approach validated through comparative analysis with alternative methods.

Full Text

Cloud PDR Method for Multi-criteria Decision Making with Incomplete Systems

Yuan Xumei, Wang Yana, Zhang Xu
(School of Management & Economics, Yanshan University, Qinhuangdao, Hebei 066000, China)

Abstract: Multi-criteria decision making (MCDM) addresses the problem of selecting optimal alternatives or ranking them based on multiple criteria or attributes. While classical rough set methods have limitations in ranking alternatives within incomplete systems, this paper proposes a novel approach that integrates cloud theory with probabilistic dominance relation (PDR) sorting methods, termed the Cloud PDR method. By incorporating parameters such as expectation, entropy, and hyper-entropy into the PDR framework, the proposed method comprehensively accounts for fuzziness, volatility, and randomness in decision-making, yielding more objective and comprehensive evaluation processes and results. The method is applied to rank the development levels of coal resource-based cities, demonstrating its effectiveness through comparative analysis with existing methods.

Keywords: multi-criteria decision making; rough set; incomplete system; cloud PDR method; coal resource-based city; urban development level

0 Introduction

Multi-criteria decision making (MCDM) involves selecting optimal alternatives or ranking them according to multiple criteria or attributes. Numerous methods have been developed for such problems, including AHP, TOPSIS, PROMETHEE, factor analysis, principal component analysis, and rough set theory. Rough set theory, introduced by Polish scientist Professor Pawlak in 1982, provides a powerful tool for quantitatively analyzing and processing imprecise, inconsistent, and incomplete information. As research has progressed, rough set theory has matured and gained increasing attention, yielding substantial achievements. Notably, extensions of classical rough set models, particularly their integration with other methods, have proven effective for solving ranking problems in multi-attribute decision making and have found widespread application across various domains.

However, classical rough set methods exhibit limitations when applied to ranking problems in incomplete systems. While previous studies have combined rough sets with TOPSIS for third-party supplier evaluation and integrated rough sets with information entropy to assess coal resource-based city development levels, these approaches involve attribute reduction and weight determination, introducing subjectivity into the process. To address this, Greco et al. proposed a dominance relation-based rough set model, which spawned a series of methodological developments. Zhang et al. investigated dominance-based quantitative rough set models and approximate reasoning for set-valued decision tables with two categories. Lyu et al. developed a dominance relation-based ranking model for energy evaluation. Weng et al. analyzed new ranking models and reduction concepts through improvements to existing sorting models based on dominance relation rough set theory. Wei et al. introduced -dominance relations and designed attribute reduction algorithms for incomplete ordered information systems. Weng et al. explored probabilistic dominance relation rough set models and corresponding sorting methods, validating their rationality through logistics supply chain performance evaluation. Despite these advances, the probabilistic dominance relation method, while eliminating the need for weight determination, imposes strict requirements on dominance relations and demonstrates poor fault tolerance, limiting its practical effectiveness.

Consequently, Weng et al. proposed a probabilistic dominance relation rough set model that only requires a certain proportion of attributes to satisfy superiority relationships between alternatives, significantly simplifying the sorting and decision-making process. Li et al. further extended this approach to incomplete systems. Although existing methods have continuously addressed the shortcomings of classical rough set sorting approaches, they still inadequately consider decision-makers' subjective fuzziness and randomness, potentially leading to distorted or one-sided results. To overcome this limitation, this paper proposes a cloud model-based cloud probabilistic dominance relation method.

The cloud model, proposed by Chinese scholar Li Deyi based on probability theory and fuzzy set theory, provides a linguistic framework for describing the uncertainty transformation between qualitative concepts and quantitative values. As research on this model has matured, it has been applied across numerous fields. Wang et al. utilized cloud transformation to evaluate regional construction industry competitiveness in China. Yan et al. proposed a dynamic customer segmentation model based on cloud models to address uncertainty in customer behavior. Zhang et al. applied forward and backward cloud generators to comprehensively assess drought and flood conditions in Shandong Province. Zhou et al. developed an improved multi-level fuzzy comprehensive evaluation method based on cloud model theory. However, few studies have applied cloud model theory to analyze multi-criteria decision making problems in incomplete systems.

Building upon existing multi-criteria decision analysis methods, this paper integrates incomplete system PDR sorting with cloud theory to propose a cloud PDR-based rough set sorting method for incomplete systems. By introducing expectation (Ex), entropy (En), and hyper-entropy (He), the proposed method incorporates fuzziness and randomness into the decision process, providing a more realistic and practical analytical framework for alternative ranking under multiple criteria. The cloud PDR method is then applied to rank, compare, and analyze the development levels of coal resource-based cities.

1 Methodology

1.1 PDR Method

PDR (Probabilistic Dominance Relation) is a sorting method based on probabilistic dominance relations. In incomplete systems, its fundamental principle involves constructing a fault-tolerant probabilistic dominance relation, calculating dominance degrees, probabilistic dominance classes, probabilistic dominance matrices, and comprehensive dominance degrees based on conditional attribute values, and subsequently determining alternative rankings.

1.1.2 Related Concepts **Definition 1 (Incomplete System).** A quadruple $S = (U, A, V, f)$ defines a system, where $U = \{x_1, x_2, ..., x_m\}$ is a non-empty finite set of objects (the universe), $A = \{a_1, a_2, ..., a_n\}$ is a non-empty finite set of conditional attributes (criteria), $V = \bigcup_{k=1}^{n} V_k$ where V_k is the domain of attribute a_k, and $f : U \times A \to V$ is an information function. If there exists an attribute $a \in A$ such that $f(x, a)$ is unknown (denoted by *), and all domains are partially ordered sets (represented by $\succ$), then the system is called an incomplete system.

Definition 2 (Dominance Degree). Let $S = (U, A, V, f)$ be an incomplete system. For $\forall x_i, x_j \in U$ and $a \in A$, let $p^a_{ij}(x_i, x_j)$ denote the dominance degree of object x_j compared to x_i under attribute a. If $f(x_j, a) \succ f(x_i, a)$ under a, then the dominance degree of x_j over x_i is 1, i.e., $p^a_{ij}(x_i, x_j) = 1$; if $f(x_j, a) \prec f(x_i, a)$ under a, the dominance degree is 0, i.e., $p^a_{ij}(x_i, x_j) = 0$; if $f(x_j, a) = f(x_i, a)$ or one value is missing (*) under a, the dominance degree is 0.5, i.e., $p^a_{ij}(x_i, x_j) = 0.5$ (where "$\succ$" means "superior to", "$\approx$" means "equivalent to", and "$\prec$" means "inferior to"). Therefore, the dominance degree of object x_j not being inferior to x_i across the entire attribute set A is calculated as the ratio of the sum of attributes where $p^a_{ij}(x_i, x_j)$ equals 1 and those where it equals 0.5 to the total number of attributes.

Definition 3 (α-Probabilistic Dominance Class). In an incomplete system $S = (U, A, V, f)$, the α-probabilistic dominance relation is defined as $R^{\alpha\geq}_A = \{(x_i, x_j) \mid p^A_{ij}(x_i, x_j) \geq \alpha, 0.5 \leq \alpha \leq 1\}$. Given $x_i \in U$, the set $[x_i]^{\alpha\geq}_A = \{x_j \in U \mid (x_i, x_j) \in R^{\alpha\geq}_A\}$ is called the α-probabilistic dominance class of object x_i, representing the set of elements that are probabilistically superior to x_i.

Definition 4 (Dominance Matrix and Comprehensive Dominance Degree). Performing set operations on probabilistic dominance classes yields the dominance matrix, whose elements are denoted by $D^A_{ij}(x_i, x_j)$. The comprehensive dominance degree of each object, denoted as $d_A(x_i)$, is obtained by calculating the arithmetic mean of each row in the dominance matrix, enabling the ranking of all objects in U.

1.2 Cloud Model

1.2.1 Basic Concepts The cloud model, proposed by Professor Li Deyi, integrates traditional probability statistics and fuzzy mathematics to provide a linguistic framework for describing the uncertainty transformation between qualitative concepts and quantitative values. The cloud model characterizes the mapping between qualitative and quantitative representations using three parameters: expectation (Ex), entropy (En), and hyper-entropy (He). The expectation Ex represents the most typical sample point of the qualitative concept—the expected value of all cloud drops in the universe distribution. The entropy En measures the uncertainty of the qualitative concept, jointly determined by its randomness and fuzziness, representing the measurable granularity of the concept and reflecting the dispersion and ambiguity of its extension. Additionally, entropy captures the relationship between randomness and fuzziness. The hyper-entropy He measures the uncertainty degree of En—the entropy of entropy—jointly determined by the fuzziness and randomness of En, reflecting the randomness of samples representing the qualitative concept and the cohesion of cloud drops.

1.2.2 Cloud Model Comparison and Operation Rules The cloud operation and comparison rules between two objects x_i and x_j are presented in Table 1 . The dominance degrees between objects x_i, x_j, and x_k are shown in Table 2 .

1.3 Cloud PDR Method

Let $X = \{x_1, x_2, ..., x_m\}$ be m objects and $A = \{a_1, a_2, ..., a_n\}$ be n attributes. For object x_i under attribute a_j, let f_{ijk} denote the k-th value, where $i = 1, 2, ..., m; j = 1, 2, ..., n; k = 1, 2, ..., L$.

a) Attribute Value Cloud Conversion. According to cloud model concepts and characteristics, original data must first be converted into cloud drop form:

$$Ex_{ij} = \frac{1}{L}\sum_{k=1}^{L} f_{ijk}$$

$$En_{ij} = \sqrt{\frac{\pi}{2}} \times \frac{1}{L}\sum_{k=1}^{L} |f_{ijk} - Ex_{ij}|$$

$$He_{ij} = \sqrt{S^2 - En_{ij}^2}$$

where Ex_{ij}, En_{ij}, and He_{ij} represent the expectation, entropy, and hyper-entropy of object x_i under attribute a_j, respectively.

Let $X_i = (Ex_i, En_i, He_i)$ and $X_j = (Ex_j, En_j, He_j)$ denote the cloud attribute values of objects x_i and x_j under attribute a, respectively. Based on Definition 2, the dominance degrees for expectation, entropy, and hyper-entropy between two objects are shown in Table 2. Here, $p_{ij}^{a'}(x_i, x_j)$, $p_{ij}^{a''}(x_i, x_j)$, and $p_{ij}^{a'''}(x_i, x_j)$ represent the dominance degrees of x_j over x_i under attribute a with respect to expectation, entropy, and hyper-entropy, respectively. According to cloud drop comparison rules, $p_{ij}^{a'}(x_i, x_j) = 1$ indicates that x_j is superior to x_i in expectation Ex; $p_{ij}^{a'}(x_i, x_j) = 0.5$ indicates they are equivalent; conversely, $p_{ij}^{a''}(x_i, x_j) = 0$ and $p_{ij}^{a'''}(x_i, x_j) = 0$ indicate that x_j is inferior to x_i in entropy and hyper-entropy, meaning x_j exhibits greater volatility and uncertainty. The opposite relationships hold similarly.

In summary, the cloud dominance degree of object x_j compared to x_i under attribute a can be expressed as:

$$p_{ij}^{a}(x_i, x_j) = [p_{ij}^{a'}(x_i, x_j), p_{ij}^{a''}(x_i, x_j), p_{ij}^{a'''}(x_i, x_j)]$$

Similarly, the cloud dominance degree of object x_j not being inferior to x_i across the entire attribute set A is:

$$p_{ij}^{A}(x_i, x_j) = [p_{ij}^{A'}(x_i, x_j), p_{ij}^{A''}(x_i, x_j), p_{ij}^{A'''}(x_i, x_j)]$$

where the meanings of expectation, entropy, and hyper-entropy are as described in Section 1.2.1.

b) Cloud Dominance Degree Calculation. Let a' denote the number of attributes where x_j dominates x_i in expectation (value $= 1$), a'' denote the number of attributes where x_j dominates in entropy (value $= 1$), and a''' denote the number of attributes where x_j dominates in hyper-entropy (value $= 1$). Let b represent the number of attributes where x_j and x_i are equivalent (value $=$ 0.5). The total number of attributes is n.

c) Calculation of Probabilistic Cloud Dominance Classes, Cloud Dominance Matrix, and Comprehensive Cloud Dominance Degree.

(a) **Calculate Probabilistic Cloud Dominance Classes.** Based on Definition 3, the α-probabilistic cloud dominance relations for expectation, entropy, and hyper-entropy in incomplete system S under threshold α are:

$$R_A^{\alpha \geq'} = \{(x_i, x_j) \mid p_{ij}^{A'}(x_i, x_j) \geq \alpha, 0.5 \leq \alpha \leq 1\}$$

$$R_A^{\alpha \geq''} = \{(x_i, x_j) \mid p_{ij}^{A''}(x_i, x_j) \geq \alpha, 0.5 \leq \alpha \leq 1\}$$

$$R_A^{\alpha \geq'''} = \{(x_i, x_j) \mid p_{ij}^{A'''}(x_i, x_j) \geq \alpha, 0.5 \leq \alpha \leq 1\}$$

The corresponding probabilistic cloud dominance classes for object x_i are $[x_i]_A^{\alpha \geq'}$, $[x_i]_A^{\alpha \geq''}$, and $[x_i]_A^{\alpha \geq'''}$.

(b) **Calculate Cloud Dominance Matrix.** Based on Definition 4, the cloud dominance matrix elements $D_{ij}^{A}(x_i, x_j)$ for objects under the three characteristics in incomplete system S satisfy Equation (6).

(c) **Calculate Comprehensive Cloud Dominance Degree.** Finally, the comprehensive cloud dominance degrees $d'_A(x_i)$, $d''_A(x_i)$, and $d'''_A(x_i)$ for each object under expectation, entropy, and hyper-entropy are calculated using Equation (7).

The rankings of all objects x_i can be obtained according to their comprehensive cloud dominance degrees.

2 Application Study

2.1 Background and Data

Following the research problem of ranking coal resource-based city development levels from reference [1], this study applies four methods—principal component analysis [1], reduction-based rough set method [5], dominance relation-based rough set method [13], and the proposed cloud PDR sorting method—to rank six coal resource-based cities: Tangshan (x_1), Datong (x_2), Yangquan (x_3), Jincheng (x_4), Shuozhou (x_5), and Shizuishan (x_6). The evaluation employs 12 indicators: proportion of industrial added value in GDP (a_1), proportion of tertiary industry added value in GDP (a_2), natural population growth rate (a_3), proportion of non-agricultural population (a_4), proportion of tertiary industry employment (a_5), per capita GDP (a_6), average wage of employed staff (a_7), per capita retail sales of consumer goods (a_8), number of doctors per 100,000 people (a_9), per capita education expenditure (a_{10}), per capita park green space area (a_{11}), and built-up area green coverage rate (a_{12}).

For representativeness, the first three methods use 2009 data, while the cloud PDR method analyzes data from 2007-2009 (Table 3). Due to various reasons, some indicator data are missing in the original dataset, making this a ranking problem for incomplete systems.

2.2 Data Calculation

a) Attribute Value Cloud Conversion. According to Section 1.3, the cloud PDR method is applied to rank the development levels of cities in the incomplete system. First, the expectation Ex, entropy En, and hyper-entropy He of the original data are calculated using Equation (1), with results shown in Table 4 .

b) Cloud Dominance Degree Calculation. Taking Tangshan (x_1) and Datong (x_2) under indicator a_2 as an example, the cloud dominance degree is calculated. According to Table 4, Datong' s value for Ex (43.57) is larger than Tangshan' s (32.05), and its En and He values are also larger. Therefore, based on the calculation rules, $p_{21}^{a_2}(x_1, x_2) = [1, 0, 0]$. Similarly, all other cloud dominance degrees can be obtained. With 6 cities and 12 indicators, this generates 30 comparison pairs and 360 cloud dominance comparisons. Due to space limitations, individual results are not listed.

Based on cloud dominance degrees, the overall cloud dominance degree under the complete attribute set A can be calculated using Equation (4). For Tangshan (x_1) and Datong (x_2):

$$p_{12}^{A}(x_1, x_2) = [0.46, 0.29, 0.29]$$

$$p_{21}^{A}(x_2, x_1) = [0.54, 0.71, 0.71]$$

This indicates that for expectation, entropy, and hyper-entropy, Datong is superior to Tangshan with probabilities of 45.83%, 29.17%, and 29.17%, respectively. Conversely, Tangshan is superior to Datong with probabilities of 54.17%, 70.83%, and 70.83%. All $p_{ij}^{A}(x_i, x_j)$ values are calculated similarly using MATLAB, with results shown in Table 5 .

c) Probabilistic Cloud Dominance Classes Calculation. With threshold $\alpha = 0.5$ (majority principle), meaning city x_j must be superior to city x_i in at least 6 of the 12 indicators, the probabilistic cloud dominance classes for each city are obtained using Equation (5), as shown in Table 6 . For expectation Ex, $[x_1]_A^{0.5\geq'} = \{x_3, x_4, x_5, x_6\}$ indicates that Yangquan, Jincheng, Shuozhou, and Shizuishan are superior to Tangshan with majority probability, suggesting Tangshan has relatively low average development level. For entropy En and hyper-entropy He, $[x_1]_A^{0.5\geq''} = \{x_1\}$ and $[x_1]_A^{0.5\geq'''} = \{x_1\}$ indicate no other city is more stable than Tangshan, reflecting its relatively stable development level.

d) Cloud Dominance Matrix and Comprehensive Cloud Dominance Degree Calculation. The cloud dominance matrices for all pairwise comparisons under the three characteristics are constructed using Equation (6). The comprehensive cloud dominance degrees are then calculated by averaging each row of the matrices using Equation (7), with results presented in Table 7 .

2.3 Results Analysis and Comparison

2.3.1 Ranking of Coal Resource-Based City Development Levels

Based on comprehensive cloud dominance degrees, the six coal resource-based cities can be ranked. According to Ex, the ranking is: Shizuishan, Yangquan (Shuozhou), Jincheng, Tangshan (Datong). According to En: Tangshan, Yangquan (Jincheng, Shizuishan), Datong, Shuozhou. According to He: Tangshan, Yangquan, Datong, Jincheng (Shizuishan), Shuozhou.

Following cloud model ranking rules (priority to Ex, then En and He if Ex values are equal), the final development level ranking under the cloud PDR method is: Shizuishan, Yangquan, Shuozhou, Jincheng, Tangshan, Datong.

2.3.2 Comparison of Method Results

Table 8 compares the ranking results across four methods: principal component analysis [1], reduction-based rough set method [5], dominance degree-based rough set method [13], and the proposed cloud dominance degree-based rough set method. Due to space constraints, detailed calculations for other methods are omitted (see references [1,5,13]). Notably, references [1,5] require complete data, so missing values were supplemented for comparison.

The overall rankings show minor variations across methods, though some cities shift positions significantly. For example, Datong ranks relatively high in other methods but lower in the cloud PDR method. This is because the cloud PDR method comprehensively analyzes development level, volatility, and uncertainty. Using data from 2007–2009, the coal market experienced severe impacts from the 2008 global economic recession. As a typical coal city, Datong faced considerable difficulties with slow development, significant demand fluctuations (En), and high uncertainty risks (He), resulting in a lower ranking. This aligns with the cloud PDR method’ s results, demonstrating that the approach effectively captures both objective and subjective aspects of evaluation, yielding more comprehensive and realistic outcomes.

3 Conclusion

This paper proposes a cloud PDR-based rough set sorting method for incomplete systems by integrating cloud theory with probabilistic dominance relation rough set models. Through calculating cloud dominance degrees, probabilistic cloud dominance classes, and comprehensive cloud dominance degrees, the method ranks alternatives according to cloud comparison rules. A case study demonstrates the method’ s feasibility and effectiveness, while comparative analysis with existing literature validates its advantages. The digital characteristics of cloud models (expectation, entropy, hyper-entropy) are key to the method’ s superiority and ability to produce more objective and comprehensive results. The cloud PDR method can be applied to real-world multi-criteria decision making problems in incomplete systems that require consideration of uncertainty factors.

References

[1] Li Xinchun, Peng Hongjun, Zhao Jing. Research on development strategies for coal resource-based cities [J]. Soft Science, 2006(3): 81-85.
[2] Pawlak Z. Rough sets [J]. International Journal of Computer and Information Science, 1982, 11(5): 341-356.
[3] Pawlak Z, Skowron A. Rudiments of rough sets [J]. Information Sciences, 2006, 177(1): 3-27.
[4] Li Xiaoli. Third-party reverse logistics supplier evaluation based on grey TOPSIS with rough sets [J]. Science and Technology Management Research, 2013(14): 67-71.
[5] Yin Zongcheng, Ding Rijia, Zhao Zhenbao. Comprehensive evaluation of coal resource-based city development levels based on rough set theory [J]. Journal of China Coal Society, 2007, 32(10): 1112-1116.
[6] Greco S, Matarazzo B, Słowiński R. Rough sets theory for multicriteria decision analysis [J]. European Journal of Operational Research, 2001, 129(1): 1-47.
[7] Zhang H Y, Yang S Y. Feature selection and approximate reasoning of large-scale set-valued decision tables based on -dominance-based quantitative rough sets [J]. Information Sciences, 2016.
[8] Lyu Y J, Weng S Z, Chen Q M. Application of ranking model to energy evaluation based on dominance relation [J]. International Journal of Nonlinear Science, 2012, 14(3): 278-286.
[9] Weng S Z, Lyu Y J, Mo J L. Ranking model based on dominance relation

and its order-preserving reduction theory [J]. Journal of Guangxi Normal University: Natural Science Edition, 2013, 31(3): 37-44.
[10] Wei B P, Lyu Y J, Li J H. Attribute reduction of rough set model under -dominance relation [J]. CAAI Transactions on Intelligent Systems, 2014, 9(2): 251-258.
[11] Weng S Z, Lyu Y J. Ranking method based on probabilistic dominance relation and its application [J]. Journal of Shanxi University: Natural Science Edition, 2015(03): 439-446.
[12] Weng S Z, Lyu Y J. Ranking method for interval rough numbers and its application [J]. Journal of Nanjing University: Natural Science Edition, 2015(4): 818-825.
[13] Li Baimin, Weng S Z, Lyu Y J. Comprehensive evaluation of regional logistics competitiveness in Guangxi based on rough sets [J]. Journal of Jinan University: Natural Science Edition, 2016(05): 341-346.
[14] Wang Xueqing, Pan Hui, Liu Bingsheng. Evaluation of regional construction industry competitiveness in China based on cloud model [J]. Journal of Shanxi University of Finance and Economics, 2012(7): 55-66.
[15] Yan Changshun, Li Yijun. Research on dynamic customer segmentation model based on cloud model [J]. Journal of Harbin Institute of Technology, 2007(2): 299-302.
[16] Zhang Mingyuan, Gao Pan, Yuan Yongbo. Application of cloud model theory in regional drought and flood evaluation [J]. Fuzzy Systems and Mathematics, 2016, 30(1): 174-181.
[17] Zhou Yonglin, Wang Lei. Multi-level fuzzy comprehensive evaluation method based on cloud model theory [J]. Computer Simulation, 2016, 33(12): 390-395.

Note: Figure translations are in progress. See original paper for figures.