Postprint of an Evaluation Model for Urban Road Traffic Network Failure from a Dynamic Perspective

Authors: Yuan Pengcheng, Lin Xuxun

Date: 2018-05-20T00:00:00+00:00

Abstract

Based on reliability-related theories and an analysis of traffic network state variation patterns, this paper presents an expression method for the 'failure-non-failure' states of urban road traffic units and traffic networks. The time distribution law governing the evolution of road section units and traffic network states from the observation starting point t0 to the first occurrence of the 'failure state' is analyzed. Building upon this foundation, failure evaluation models for urban road units and traffic networks are derived, encompassing failure probability, mean time to failure, failure rate, and conditional failure time, along with a corresponding model calibration methodology. Employing a road network composed of five road units in the core area of a domestic city as a case study, distribution fitting and hypothesis testing are performed on the time to first failure, the model is calibrated, and the calculated results are compared with actual observational data. The results demonstrate that the lognormal distribution provides an ideal fit for the time to first failure, with model calculations closely matching the observed data, thereby indicating that the proposed model exhibits good practical applicability.

Full Text

Preamble

Urban Traffic Network Failure Evaluation Model from a Dynamic Perspective

Yuan Pengcheng[1]†, Lin Xuxun[2]
(1. School of Management, University of Shanghai for Science and Technology, Shanghai 200093, China; 2. School of Business, Changzhou University, Changzhou, Jiangsu 213164, China)

Abstract: Based on reliability theory, this paper proposes a method for representing the 'failure-non-failure' states of urban road traffic units and networks through analysis of traffic network state transition patterns. We analyze the time distribution of state transitions from the observation starting point t_0 to the first occurrence of the 'failure state' for both link units and the traffic network. Building upon this analysis, we derive failure evaluation models for urban road units and traffic networks, including failure probability, average failure duration, failure rate, and conditional failure duration, along with model calibration methods. Using a road network composed of five road units in the core urban area of a Chinese city as a case study, we perform distribution fitting and hypothesis testing for the first failure time, calibrate the model, and compare the model calculations with actual observations. The results demonstrate that the lognormal distribution provides an excellent fit for the first failure time, with model outputs closely matching observed data, confirming the model' s strong practical applicability.

Keywords: road unit; traffic network; failure state; failure probability

0 Introduction

The study of traffic system reliability has evolved from initially static characteristics. To date, substantial research has accumulated on static reliability evaluation of traffic networks, with researchers proposing various models and methods that reflect static network features from different perspectives. These primarily include connectivity reliability evaluation [4-6], travel time reliability evaluation [7-11], capacity reliability evaluation [12-14], and other reliability measures such as behavioral reliability [15-17] and potential reliability [18]. Representative contributions include Mine and Kawai' s pioneering concept of traffic network connectivity reliability [19], Chen et al.' s extension to capacity reliability [20], and Asakura et al.' s concept of travel time reliability. Following the establishment of these reliability metrics, subsequent research has focused on applying them to evaluate traffic network reliability, primarily through two approaches: analytical methods [21] and simulation [13, 22].

Analytical studies attribute traffic network stochasticity to randomness in supply and demand. These methods first calibrate the probability distributions of network supply and demand, then calculate the overall network travel time distribution based on link connectivity relationships, flow-impedance relationships, and network equilibrium characteristics, ultimately deriving network reliability. Simulation approaches, conversely, primarily use traffic flow simulators to generate network flows and employ Monte Carlo methods to simulate and calibrate the random probability distributions of reliability evaluation parameters.

Notably, recent advances in complexity science have spurred increasing research on simulating the evolution of urban traffic network reliability (referred to as 'cascading failure' in most literature) using complex network theory. For instance, Wu Jianjun et al. [23–26] developed early cascading failure models for urban traffic systems incorporating traffic flow assignment theory. Sun et al. [25] proposed a robust allocation model to defend against cascading failures. Wang Zhengwu et al. [27] developed a cascading failure model for traffic networks based on a bi-level network concept, applied it to evaluate node importance, and constructed a disaster spread dynamics model for urban road traffic network cascading failures [28]. Wang Fang et al. [29] simulated cascading failure processes using flow-capacity models. Chen Xiaolan [30] identified factor combinations that minimize and maximize cascading failure damage through simulation. Yin Hongying et al. [31] analyzed how network topology, link removal methods, and coupling strength affect cascading failures. Xu Xiaoyang [32] represented urban road traffic networks as complex networks with special statistical characteristics, adapted the SIS (susceptible-infected-susceptible) mechanism from epidemic spreading theory, and proposed a traffic congestion propagation simulation model, using it to evaluate node importance by comparing network parameters before and after random/deliberate attacks on Beijing' s road network.

While complex network theory offers value for large-scale networks (with at least tens of thousands of nodes) exhibiting significant statistical features, it becomes unsuitable for smaller networks with clear topological structures (e.g., networks comprising only dozens of roads), as network parameters lose their statistical significance. In practical traffic engineering, we often need to analyze and apply failure patterns and characteristics of local, limited-scale traffic networks. Moreover, although 'cascading failure' is inherently a dynamic concept, current research focuses on the propagation of failures triggered by initial node or link failures, rather than being truly 'dynamic' in the sense of capturing the specific failure state of a network at a given moment.

Therefore, unlike Xu Xiaoyang's complex network approach and simulation analysis, this paper examines the dynamic characteristics of traffic network failures from the perspective of general traffic networks, using stochastic process theory to derive time-varying patterns of road unit failures and establish analytical models for urban traffic network unit failures.

---

1 Model Construction

1.1 Traffic Network Failure State

Road segments constitute the basic units of traffic networks. In a traffic network system, each road performs different functions, and all road units work collectively and coordinate to ensure traffic flow distribution and fulfill the network' s 'collection and distribution' function. When certain road units in the system cannot perform their intended functions due to internal or external factors, these units are considered to have failed.

Consider a traffic network system N composed of n road units. Let the set of traffic flow states for each unit i be J_i. At time t, if the state of unit i belongs to a subset $K_i \subseteq J_i$ that represents loss of functionality, then unit i is defined as being in a failure state at time t. Denote the state of unit i at time t as $S_i(t)$, where:

$$S_i(t) = \begin{cases} 1, & \text{if the state at time } t \text{ belongs to } K_i \\ 0, & \text{if the state at time } t \text{ belongs to } J_i \setminus K_i \end{cases}$$

$S_i(t)$ is a random variable: $S_i(t) = 1$ indicates unit i is in a non-failure state, while $S_i(t) = 0$ indicates a failure state. Similarly, the state of the traffic network at time t can be denoted as $S(t) = \{S_1(t), S_2(t), \dots, S_n(t)\}$, which indicates which roads are operating normally and which have failed at time t.

The concept of 'road unit traffic flow state' proposed above is a fundamental concept for describing road traffic flow operational characteristics and forms the basis for defining road failure. In traffic engineering, traffic flow states are often categorized as 'smooth, congested, blocked,' etc. In reality, traffic flow state is a continuous concept, and thus the classification method and number of state categories are not unique. Regardless of the classification approach, each has its rationality. This paper does not debate the merits of various traffic state classification methods but uses the multi-state characteristic of traffic unit systems to illustrate the definition of unit failure states. Nor does it dwell on which specific state should be defined as 'failure'; this definition depends entirely on practical needs and managerial expectations. For example, among six possible states—free, smooth, busy, congested, blocked, and severed—one might consider only the 'severed' state as failure, or alternatively, define 'congested, blocked, and severed' collectively as the 'failure' state.

1.2 Road Unit Failure Function

In reality, the traffic states of network units constantly change. A road unit' s state transitions among 'smooth-congested-blocked' over time. If 'blocked' is defined as the unit' s 'failure state,' the unit essentially alternates between 'non-failure' and 'failure' states.

Let the initial state of unit i at time t_0 be $S_i(t_0) = 1$. After a period T_i, when the unit' s state first becomes 'failed,' T_i is called the first failure duration of unit i. Clearly, T_i is a continuous random variable. Let its probability density function be $f_i(t)$ and distribution function be $F_i(t) = P(T_i \leq t)$, which represents the probability that unit i will be in a failure state within the time interval $(t_0, t_0+t]$. This is called the failure function of unit i. If we define $R_i(t) = 1 - F_i(t)$, it represents the probability that unit i remains in a non-failure state during the interval $(t_0, t_0 + t]$.

Based on the probability density function of unit i's failure time, we can calculate the expected failure time:

$$E[T_i] = \int_0^\infty t f_i(t)\, dt = \int_0^\infty R_i(t)\, dt$$

If unit i is still in a non-failure state at time $t_0 + t$, the probability that it will reach the failure state within $(t, t + \Delta t]$ can be calculated as:

$$P(t < T_i \leq t + \Delta t \mid T_i > t) = \frac{F_i(t + \Delta t) - F_i(t)}{P(T_i > t)} = \frac{F_i(t + \Delta t) - F_i(t)}{R_i(t)}$$

Dividing both sides by Δt and taking the limit as $\Delta t \to 0$ yields the failure rate function for unit i:

$$z_i(t) = \lim_{\Delta t \to 0} \frac{F_i(t + \Delta t) - F_i(t)}{\Delta t} \cdot \frac{1}{R_i(t)} = \frac{f_i(t)}{R_i(t)}$$

This function characterizes the transition from the observation moment t_0 until the first occurrence of the failure state. In traffic engineering, particularly in intelligent transportation systems, historical traffic flow data can be used to predict impending network failure states. When the network is already in a failure state, these models can predict the dissipation duration, providing decision-making support for route selection.

If unit i is in a non-failure state at time $t_0 + t$ and has not yet reached the failure state after time x, the probability that it remains in a non-failure state is denoted as the conditional reliability function $R_i(x \mid t)$. From this, we can calculate the 'mean residual failure time' (the expected remaining time until failure for unit i), also called the average residual failure time:

$$E[T_{ri}] = \int_0^\infty R_i(x \mid t)\, dx$$

1.3 Traffic Network Failure Function

Consider a traffic network N composed of n road units, each having two states: 'failure' and 'non-failure.' According to combinatorial principles, the network has 2^n possible states. If m of these states result in the network losing its road 'evacuation' function (i.e., becoming 'failed'), then the network is considered failed when any of these m states occur.

Let the initial state of network N at time t_0 be $S(t_0)$. After a period T, when the network state first becomes 'failed,' T is called the first failure duration of network N. Similarly, we can derive the network failure function $F(t)$, reliability function $R(t) = 1 - F(t)$, expected failure time $E[T]$, failure rate function $z(t)$, and mean residual failure time $E[T_r]$.

2 Parameter Calibration Process

The core variables in the above traffic network failure evaluation models are the probability distributions of the first failure times T_i for road units and T for the traffic network. Therefore, the primary task is parameter estimation and distribution hypothesis testing for these random variables. Here, we apply the χ^2 goodness-of-fit test.

Assume random variable T follows a distribution function $F(t; \theta_1, \theta_2, \dots, \theta_r)$ with unknown parameters $\theta_1, \theta_2, \dots, \theta_r$. Let $T_1, T_2, \dots, T_n$ be an observed sample.

a) Divide the range of T into k continuous, non-overlapping intervals $A_1, A_2, \dots, A_k$ (e.g., $(a_0, a_1], (a_1, a_2], \dots, (a_{k-1}, a_k]$).

b) Let f_i denote the number of samples falling into interval A_i (the group frequency), with $\sum_{i=1}^{k} f_i = n$.

c) Estimate the unknown parameters $\theta_1, \theta_2, \dots, \theta_r$ using maximum likelihood estimation based on the sample $T_1, T_2, \dots, T_n$, obtaining estimates $\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_r$.

d) Substitute the estimates into the distribution function to obtain $F(t; \hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_r)$, yielding a distribution function without unknown parameters.

e) Calculate the theoretical probability $p_i = F(a_i; \hat{\theta}_1, \dots, \hat{\theta}_r) - F(a_{i-1}; \hat{\theta}_1, \dots, \hat{\theta}_r)$ for each interval A_i.

f) Compute the χ^2 statistic:

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - np_i)^2}{np_i}$$

g) For a given significance level α, if $\chi^2 < \chi^2_{1-\alpha}(k - r - 1)$, accept the null hypothesis that T follows the specified distribution; otherwise, reject it and consider alternative distributions.

---

3 Empirical Analysis

3.1 Data Source

We analyze a road network consisting of five one-way road units in the core urban area of a Chinese city [Figure 1: see original paper]. Data were collected by loop detectors embedded in the roads, which measure vehicle speed, flow, and time occupancy. The statistical interval is 3 minutes, meaning the detectors aggregate traffic flow parameters (including volume, average speed, average time occupancy, and average headway) every three minutes and upload them to a database. We obtained continuous traffic flow data for two months (60 days). Descriptive statistics for the five road units are shown in .

3.2 Failure States of Road Units and Traffic Network

Based on the fundamental definition of failure, a traffic network is considered 'failed' when it can no longer perform its original 'evacuation' function. Accordingly, when the network shown in [Figure 1: see original paper] exhibits the link state combinations shown in (where 0 represents 'failure state' and 1 represents 'non-failure state'), the network becomes disconnected and is thus considered failed.

Kerner' s three-phase traffic flow theory [33] is widely recognized in the theoretical community. Subsequent research extended this to four-phase (free flow, synchronized flow, wide moving jam) and five-phase (free flow, smooth flow, capacity state, synchronized flow, jam state) classifications. Considering the characteristics of the data, we adopt Cai Yanfei' s five-phase traffic state model based on flow-speed-occupancy data [34] for traffic state identification and classification (see [35, 37] for details). Here, we define a road unit as 'failed' when it is in synchronized flow or jam flow states, yielding the 'failure-non-failure' time series for each road unit shown in [Figure 2: see original paper].

3.3 Parameter Calibration

Using 0:00 as the initial observation time t_0, we observe the first failure durations for each road unit and the traffic network [Figure 4: see original paper]. Note that if a unit or network never reaches a failure state on a given day, we record $T = 1440$ minutes (the number of minutes in a day). Based on the failure state definitions for road units and the network, we obtain the network' s failure state time evolution process shown in [Figure 3: see original paper].

Applying the parameter calibration method from Section 2, we test the null hypothesis that the first failure times follow a lognormal distribution with parameters μ and σ^2. The goodness-of-fit test results are shown in .

3.4 Results Comparison

provides the probability density functions for the first failure times of each road unit and the network at observation start time t_0. Given a time duration t, we can calculate failure/non-failure probabilities, expected failure times, conditional reliability, mean residual failure times, and failure rate functions using equations (14)-(17). For $t_0 = 0:00$ and $t = 450$ minutes, the failure probabilities over time are shown in [Figure 5: see original paper].

Note that Road Unit 4 is excluded from distribution fitting as it only experienced failure on 2 days.

The results show that, except for Road Unit 4, all other units and the network' s failure durations can be fitted with lognormal distributions. The goodness-of-fit is excellent for Road Unit 2 and the network (p-values of 54% and 63%, respectively), and acceptable for Road Units 1 and 5 (p-values > 0.05 at the 5% significance level). For a lognormal distribution with parameters $\hat{\mu}$ and $\hat{\tau}$, the probability density function is:

$$f(t) = \begin{cases} \frac{1}{\sqrt{2\pi}\hat{\tau}t} e^{-\frac{(\ln t-\hat{\mu})^2}{2\hat{\tau}^2}}, & t > 0 \\ 0, & \text{otherwise} \end{cases}$$

The distribution function is:

$$F(t) = \Phi\left(\frac{\ln t - \hat{\mu}}{\hat{\tau}}\right)$$

From this, we derive:

$$E[T] = e^{\hat{\mu}+\frac{\hat{\tau}^2}{2}}$$

$$R(x \mid t) = \frac{1 - \Phi\left(\frac{\ln(t+x)-\hat{\mu}}{\hat{\tau}}\right)}{1 - \Phi\left(\frac{\ln t-\hat{\mu}}{\hat{\tau}}\right)}$$

To validate model accuracy, we compare model estimates with observed data by calculating: failure frequency = observed failures/observation periods; average failure duration = total failure duration/observation periods; conditional reliability frequency = failures occurring after t_0 given non-failure at t_0/observation periods; average residual failure time = failure durations in $(t_0 + t, t_0 + t + x]$/observation periods. The comparison results are shown in .

The close match between model-estimated failure probabilities and observed failure frequencies demonstrates high model accuracy and practical utility.

---

4 Conclusion

This paper proposes failure evaluation models for road units and traffic networks that capture the time-varying characteristics of urban traffic flow, using reliability and probability statistics theory from a dynamic perspective. We first present a state representation method for road units, then transform unit states into a 'failure–non-failure' binary representation based on the relationship between unit states and functionality. Building on this, we define an identification method for network 'failure-non-failure' states according to inter-unit relationships. We analyze the probability distribution of first failure times from observation point t_0 for both units and networks, provide methods for parameter estimation and hypothesis testing, and propose evaluation metrics including failure probability, expected failure time $E[T]$, conditional reliability $R(x \mid t)$, and failure rate.

Empirical validation using a real road network shows: (a) both road units and the network' s failure times pass lognormal distribution tests at the $\alpha = 0.05$ significance level, confirming lognormal distribution as a reliable choice; (b) model results closely match observations, indicating high precision.

The proposed models offer a novel perspective for evaluating road operational conditions and provide theoretical support for traffic planners and engineers. Current engineering practice relies heavily on static deterministic metrics like operating speed and congestion duration. While connectivity reliability and travel time reliability represent advances as static non-deterministic parameters, they fail to capture the time-varying nature of traffic network states. Our dynamic failure evaluation models reveal these temporal patterns—for example, [Figure 5: see original paper] shows how network failure probability evolves over time, enabling proactive traffic management and post-implementation effectiveness evaluation. The same principles can also evaluate failure state dissipation processes, which is highly significant.

Future research should address current limitations: the models are based on single failure events, whereas real networks alternate continuously between failure and non-failure states. Developing models that capture this dynamic alternation would be valuable. Additionally, limited data collection (only 2-3 months of continuous data, yielding 60 samples for first failure time distribution fitting) may introduce parameter estimation bias. Future studies should collect more extensive regional network data to further validate the models.

References

[1] Zou Zhiyun, Mao Baohua, Gong Quanzhou. Correlation analysis of urban road network failure (congestion) [J]. Transportation Systems Engineering and Information, 2009, 9(2): 110-114.

[2] Marvin Rausand. System Reliability Theory: Models, Statistical Methods, and Applications [M]. 2nd ed. Beijing: National Defense Industry Press, 2010.

[3] Kamm L. Reliability and maintenance [J]. Social Science Electronic Publishing, 2017, 81(4): 147-147.

[4] Xiao Xuemei, Jia Limin. Evaluation model for connectivity reliability of urban rail transit networks [J]. China Railway Science, 2016, 37(1): 132-137.

[5] Ma Hongwei, Zhou Xizhao. Asymptotic connectivity reliability analysis of stochastic traffic networks [J]. Operations Research and Management Science, 2015, (3): 45-50.

[6] Sun H, Wu J, Wang W, et al. Reliability-based traffic network design with

advanced traveler information systems [J]. Information Sciences, 2014, 287: 262-272.

[7] Sumalee A, Watling D, Nakayama S. Reliable network design problem: case with uncertain demand and total travel time reliability [J]. Transportation Research Record: Journal of the Transportation Research Board, 2006, 1964(1): 81-90.

[8] Bhat C R, Sardesai R. The impact of stop-making and travel time reliability on commute mode choice [J]. Transportation Research Part B: Methodological, 2006, 40(9): 709-730.

[9] Clark S, Watling D. Modelling network travel time reliability under stochastic demand [J]. Transportation Research Part B: Methodological, 2005, 39(2): 119-140.

[10] Liu H X, Recker W, Chen A. Travel Time Reliability [M]. Springer, 2012.

[11] Lin Xuxun, Yuan Pengcheng, Huo Liang' an. Dynamic travel time reliability calculation model based on probability density evolution theory [J]. Journal of Industrial Engineering and Engineering Management, 2017, 31(3): 141-148.

[12] Liu H X, Yun P U. Road network capacity reliability based on link travel time reliability [J]. Journal of Southwest Jiaotong University, 2004, 39(5): 573-577.

[13] Chen A, Yang H, Hong K L, et al. Capacity reliability of a road network: an assessment methodology and numerical results [J]. Transportation Research Part B: Methodological, 2002, 36(3): 225-252.

[14] Guo Changbo, Zhang Ning. Reliability simulation study on expressway exit capacity [J]. Journal of System Simulation, 2003, 15(11): 1595-1599.

[15] Guo Hongyang, Han Xuesong, Liu Lan, et al. Research on reliability risk measurement of driver traffic safety behavior [J]. China Safety Science Journal, 2013, 23(6): 103-109.

[16] Guo Zizheng, Zhang Dianye, Jin Jian, et al. Research on locomotive double-driver driving behavior reliability [J]. China Safety Science Journal, 2007, 17(2): 162-168.

[17] Tan Z, Yang H, Guo R. Pareto efficiency of reliability-based traffic equilibria and risk-taking behavior of travelers [J]. Transportation Research Part B: Methodological, 2014, 66(8): 16-31.

[18] Uno N, Iida Y, Kawaratani S, et al. An analysis of potential of providing information on traffic accident to enhance travel time reliability [C]// Proc of the 1st International Symposium on Transportation Network Reliability. 2001.

[19] Yuan Pengcheng, Han Yin, Ma Wanda. Stochastic traffic network equilibrium model based on travel time reliability [J]. Journal of University of Shanghai for Science and Technology, 2008, 30(4): 352-356.

[20] Chen A, Kasikitwiwat P, Yang C. Alternate capacity reliability measures for transportation networks [J]. Journal of Advanced Transportation, 2013, 47(1): 79-104.

[21] Palma A D, Picard N. Route choice decision under travel time uncertainty [J]. Transportation Research Part A: Policy and Practice, 2005, 39(4): 295-324.

[22] Hong K L, Luo X W, Siu B W Y. Degradable transport network: travel time budget of travelers with heterogeneous risk aversion [J]. Transportation Research Part B: Methodological, 2006, 40(9): 792-806.

[23] Wu J J, Sun H J, Gao Z Y. Cascading failures on weighted urban traffic equilibrium networks [J]. Physica A: Statistical Mechanics and Its Applications, 2007, 386(1): 407-413.

[24] Zheng J F, Gao Z Y, Zhao X M. Modeling cascading failures in congested complex networks [J]. Physica A: Statistical Mechanics and Its Applications, 2007, 385(2): 700-706.

[25] Sun H J, Zhao H, Wu J J. A robust matching model of capacity to defense cascading failure on complex networks [J]. Physica A: Statistical Mechanics and Its Applications, 2008, 387(25): 6431-6435.

[26] Wu Jianjun. Study on topological complexity of urban traffic networks [D]. Beijing: Beijing Jiaotong University, 2008.

[27] Wang Zhengwu, Kuang Aiwu, Wang Hejie. Node importance measurement in traffic networks considering cascading failure [J]. Journal of Highway and Transportation Research and Development, 2012, 29(5): 96-101.

[28] Wang Zhengwu, Peng Shuo, Huang Zhongxiang. Disaster spread dynamics model for cascading failure in urban road traffic networks [J]. Journal of Safety and Environment, 2014, 14(3): 1-4.

[29] Wang Fang. Research on vulnerability of equilibrium traffic networks based on cascading failure [D]. Chengdu: Southwest Jiaotong University, 2012.

[30] Chen Xiaolan. Identification of impact sources for cascading failure in urban traffic complex networks [D]. Changsha: Changsha University of Science and Technology, 2013.

[31] Yin Hongying, Quan Xiaofeng. Influence law and scope of cascading failure in transportation networks [J]. Journal of Systems & Management, 2013, 22(6): 869-875.

[32] Xu Xiaoyang. Research on urban road traffic network failure model for node importance evaluation [D]. Beijing: North China University of Technology, 2017.

[33] Kerner B S, Rehborn H. Recognition and tracking of spatial-temporal congested traffic patterns on freeways [J]. Transportation Research Part C: Emerging Technologies, 2004, 12(5): 369-400.

[34] Cai Yanfei. Traffic system state identification based on traffic phase characteristics [D]. Guangzhou: South China University of Technology, 2011.

[35] Yuan Pengcheng, Juan Zhicai. Semi-Markov chain model for regional traffic network travel time estimation [J]. Journal of Systems & Management, 2014, 23(5): 690–697.

[36] Yuan Pengcheng, Lin Xuxun. Dynamic travel time reliability calculation model based on probability density evolution theory [J]. Journal of Industrial Engineering and Engineering Management, 2017, 31(3): 141–148.

[37] Yuan Pengcheng, Lin Xuxun. Dynamic travel time reliability calculation model based on probability density evolution theory [J]. Journal of Industrial Engineering and Engineering Management, 2017, 31(3): 141–148.

Note: Figure translations are in progress. See original paper for figures.