Postprint: K-means Clustering Demodulation Method for BPSK Signals Based on Chaotic Duffing Oscillator

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Date: 2018-05-20T00:00:00+00:00

Abstract

To address the issue of high bit error rate in the demodulation of binary phase shift keying (BPSK) signals under low signal-to-noise ratio (SNR) conditions, a K-means demodulation method based on chaotic Duffing oscillator is proposed. The methodology leverages the sensitivity to initial conditions and noise immunity characteristics of chaotic Duffing oscillator systems. When a BPSK signal is input to the Duffing oscillator system at low SNR, the phase jumps of the BPSK signal between 0° and induce changes in the output phase trajectory state of the Duffing oscillator. To capture these changes in phase trajectory state, the K-means clustering algorithm is employed to iteratively compute centroids of the phase trajectories, and decision-based demodulation of the BPSK signal is performed according to the inter-centroid distance after convergence. Simulation results demonstrate that, compared with several existing demodulation methods, the K-means clustering demodulation method for BPSK signals based on chaotic Duffing oscillator achieves substantial improvements in demodulation speed and demodulation accuracy under low SNR conditions.

Full Text

Preamble

K-means Clustering Demodulation Method for BPSK Signal Based on Chaotic Duffing Oscillator

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Abstract: Binary phase shift keying (BPSK) signals typically exhibit high bit error rates (BER) when demodulated in low signal-to-noise ratio (SNR) environments. To address this issue, this paper proposes a K-means clustering demodulation method based on chaotic Duffing oscillators. The method leverages two key characteristics of chaotic Duffing oscillators: extreme sensitivity to initial conditions and inherent immunity to noise. When a BPSK signal is input to a Duffing oscillator system at low SNR, the phase transitions of the BPSK signal between 0° and cause corresponding changes in the output phase trajectory state of the oscillator. To detect these phase trajectory variations, the K-means clustering algorithm is employed to iteratively compute the centroids of the phase trajectories. Demodulation is then performed by comparing the distance between the converged centroids. Simulation results demonstrate that compared with existing demodulation methods, the proposed K-means clustering demodulation method based on chaotic Duffing oscillators achieves significant improvements in demodulation speed and accuracy under low SNR conditions.

Keywords: Duffing oscillator; phase trajectory; K-means clustering; bit error rate

0 Introduction

In digital communication technology, BPSK modulation signals are widely employed in satellite communications, digital television, and other communication systems due to their excellent spectral efficiency, strong anti-interference capabilities, and straightforward hardware implementation. However, modern communication channels exhibit increasingly complex characteristics, with factors such as background noise and inter-symbol interference contributing to high demodulation error rates at the receiver. Consequently, reducing signal demodulation BER in low-SNR environments is of paramount importance for enhancing reception performance and ensuring communication quality.

Numerous scholars have introduced nonlinear processing methods to address BPSK demodulation challenges in low-SNR scenarios. References [1,2] utilized chaotic Duffing oscillators for BPSK signal processing and proposed mean-square deviation algorithms for demodulation decisions, though these approaches required excessively high sampling rates and neglected inter-symbol interference in the BPSK baseband signal. References [3,4] employed stochastic resonance methods for 2FSK and 4FSK signal demodulation, but similarly overlooked baseband signal inter-symbol interference while also being constrained by signal frequency magnitude. References [5,6] investigated a parameter-tuned stochastic resonance approach for BPSK signal demodulation, which achieved some success in low-SNR environments. However, this method proved highly sensitive to algorithm parameters, signal frequency, and amplitude, making parameter matching between the stochastic resonance system and signal parameters extremely difficult.

A comprehensive review of existing literature reveals several critical limitations in current BPSK demodulation techniques: (a) many advanced demodulation methods face significant difficulties in parameter adjustment, and (b) most studies on BPSK demodulation ignore inter-symbol interference phenomena in baseband signals. To address these issues, this paper considers inter-symbol interference in baseband signals under Gaussian channel transmission conditions and designs an interference-resistant shaping filter suitable for channel transmission. By integrating chaos theory, the Duffing oscillator is applied to BPSK signal processing, and a K-means clustering demodulation algorithm is employed for the processed signal, thereby reducing the BER of BPSK signal demodulation in low-SNR environments.

1.1 BPSK Signal Modulation Principle

BPSK signals typically represent binary symbols "1" and "-1" using phases of 0° and , respectively. The time-domain expression is given by:

$$S_{\text{bpsk}}(t) = A\cos(2\pi f_c t + \phi)$$

where A is the signal amplitude, f_c is the carrier frequency, and ϕ is the phase (for symbol "-1" and 0° for symbol "1"). As shown in Figure 1a [Figure 1: see original paper], this represents the time-domain waveform of an unshaped BPSK signal.

Most existing literature analyzes BPSK signals using the waveform shown in Figure 1(a). However, in practical communications, adjacent symbols interfere with each other due to pulse tailing phenomena (as illustrated in Figure 1b). Therefore, the unshaped BPSK signal in Figure 1(a) cannot be transmitted directly through communication channels. To eliminate inter-symbol interference, the signal must undergo shaping filter processing.

The common approach for eliminating inter-symbol interference involves designing shaping filters using low-pass impulse responses [7]. The filter expression is:

$$h(t) = \mathrm{Sa}\left(\frac{\pi t}{T_s}\right) \cdot \frac{\cos(\alpha\pi t/T_s)}{1 - 4\alpha^2 t^2/T_s^2}$$

where T_s is the sampling interval and α is the raised cosine roll-off coefficient satisfying $0 \leq \alpha \leq 1$. With α values of 0, 0.5, 0.8, and 1, the corresponding impulse responses of the shaping filters are shown in Figure 1c. Larger roll-off coefficients result in faster tail decay of the impulse response function $h(t)$, reducing timing precision requirements. However, increased roll-off coefficients expand the transmission bandwidth to $B = (1+\alpha)f_s$, which decreases bandwidth utilization to $\eta = R_B/B = 2/(1+\alpha)$ [8].

Combining equations (1) and (2), the time-domain expression for the shaped BPSK modulation becomes:

$$S_{\text{bpsk}}(t) = A(t)\cos(2\pi f_c t + \phi)$$

where $A(t) = a(t) * h(t)$ and $a(t)$ represents the symbol sequence. Using a random sequence of length $L_{en} = 20$ symbols with baseband amplitude 0.1 V, symbol rate $R_b = 1$ kbaud, carrier frequency $f_c = 8$ kHz, sampling frequency $f_s = 40 f_c = 320$ kHz, and roll-off coefficient $\alpha = 0.7$, the BPSK signal shaping modulation process is shown in Figure 2a [Figure 2: see original paper], with BPSK signals at different SNRs displayed in Figure 2b.

1.2 Chaotic Duffing Oscillator Overview

Chaotic Duffing oscillators exhibit sensitivity to initial conditions and immunity to noise. Minute variations in initial conditions cause substantial changes in system output [9], while the oscillator maintains its rich dynamic characteristics despite noise perturbations [10]. These properties offer significant advantages for weak signal detection and demodulation in low-SNR environments. This paper exploits these characteristics to achieve BPSK signal demodulation at low SNR.

The standard dynamic equation describing Duffing oscillator behavior is:

$$f(x) + k\dot{x} + x = \gamma\cos(2\pi f_c t)$$

where x is the system output, k is the damping coefficient, $\gamma\cos(2\pi f_c t)$ is the periodic driving force with amplitude γ, and $f(x)$ is a nonlinear restoring force function expressed as $f(x) = -ax + bx^3$. Parameters a and b are nonlinear restoring force coefficients [11][12] satisfying $a > 0, b > 0$. The Duffing oscillator equation can therefore be written as:

$$\ddot{x} + k\dot{x} - ax + bx^3 = \gamma\cos(2\pi f_c t)$$

This can also be expressed in state-space form as:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -ky + ax - bx^3 + \gamma\cos(2\pi f_c t) \end{cases}$$

For the Holmes-type Duffing oscillator with $a = 1, b = 1$ and damping coefficient $k = 0.5$, the Melnikov method [13] yields driving force amplitudes γ of 0.8, 0.826, and 0.83. Using fourth-order Runge-Kutta integration of equation (6), the resulting phase trajectories are shown in Figures 3(a)-(c) [Figure 3: see original paper]. As the periodic driving force amplitude increases, the Duffing oscillator transitions from chaotic to critical chaotic and finally to periodic states. By setting an appropriate driving force amplitude ($\gamma = 0.826$ in this paper), the phase trajectory state can be altered when the signal under test is applied.

In this work, the BPSK signal to be demodulated is injected into the Duffing oscillator system, modifying equation (5) to:

$$\ddot{x} + k\dot{x} - ax + bx^3 = \gamma \cos(2\pi f_c t) + s_n(t)$$

where $s_n(t)$ is the BPSK signal with channel noise, i.e., $s_n(t) = S_{\text{bpsk}}(t) + n(t)$, and $n(t)$ is additive white Gaussian noise with zero mean and variance δ^2. The input SNR is defined as:

$$\text{SNR}_{\text{in}} = 10 \log_{10} \left(\frac{P_{\text{bpsk}}}{P_n} \right) = 10 \log_{10} \left(\frac{A^2(t)}{2\delta^2} \right)$$

where P_{bpsk} and P_n are the BPSK signal power and Gaussian noise power, respectively. When the BPSK signal is applied, the Duffing system equation becomes:

$$\ddot{x} + k\dot{x} - ax + bx^3 = \gamma \cos(2\pi f_c t) + A(t) \cos(2\pi f_c t + \phi) + n(t)$$

Applying trigonometric identities to the right side yields:

$$\gamma \cos(2\pi f_c t) + A(t) \cos(2\pi f_c t + \phi) = \beta(t) \cos(2\pi f_c t + \theta(t))$$

where

$$\begin{cases} \beta(t) = \sqrt{\gamma^2 + 2\gamma A(t) \cos\phi + A^2(t)} \\ \theta(t) = \arctan\left(\frac{A(t)\sin\phi}{\gamma + A(t)\cos\phi}\right) \end{cases}$$

For BPSK signals, ϕ assumes only values of 0 and , making $\theta(t) = 0$, with:

$$\begin{cases} \beta(t) = |\gamma - A(t)|, & \phi = \pi \\ \beta(t) = \gamma + A(t), & \phi = 0 \end{cases}$$

Thus, based on the amplitude variation $\beta(t)$ after superposition of the periodic driving force and BPSK signal, the Duffing oscillator' s phase trajectory alternates between periodic and chaotic states. This alternation corresponds directly to the BPSK symbol transitions between "-1" and "1" , as modeled in Figure 4 [Figure 4: see original paper]. Following system output of the phase trajectory, a decision mechanism can be designed for demodulation. Based on the characteristics of the chaotic Duffing oscillator' s output phase trajectory, this paper proposes a K-means clustering demodulation method.

2 K-means Clustering Demodulation Algorithm for BPSK Signals

The K-means clustering algorithm is a simple unsupervised learning method. The process involves: (1) clustering samples into K clusters with randomly selected initial centroids, (2) computing new centroids for each cluster and reassigning points, and (3) iteratively recalculating centroids until convergence is achieved [15,16]. The K-means centroid iteration flowchart is shown in Figure 5 [Figure 5: see original paper].

When noise-free BPSK signals are input to the Duffing oscillator, the phase trajectories generated by symbol "1" (phase 0) and symbol "-1" (phase) are shown in Figure 6a [Figure 6: see original paper]. At SNR = -20 dB, the phase trajectories for both symbols are displayed in Figure 6b. Notably, noise only roughens the trajectories without altering their fundamental states.

The Duffing oscillator' s phase portrait exhibits two saddle points at (±1,0) according to Melnikov method analysis [14]. Thus, the initial centroids are (±1,0) for noise-free conditions. For noisy signals, after K-means clustering iteration, the converged centroids for symbols "1" and "-1" are shown in Figure 6c. The centroid for symbol "1" exhibits minimal offset from its initial position, while the symbol "-1" centroid shows substantially greater displacement.

Using the horizontal distance between converged centroids $d = |x_l - x_r|$ as a metric (where x_l and x_r are the x-coordinates of left and right centroids), Figure 7 [Figure 7: see original paper] shows the distribution of centroid distances d for 40 samples each of symbols "1" and "-1" at input SNRs of -15 dB, -20 dB, -25 dB, and -30 dB. The results clearly show that after K-means convergence, symbol "1" trajectories yield centroid x-coordinate distances in the range of approximately 1.76~2.0, while symbol "-1" trajectories produce distances of approximately 1.48~1.74. By establishing an appropriate distance threshold and applying K-means clustering to phase trajectory data, the converged centroid distance can be compared against this threshold to determine the transmitted symbol.

The demodulation procedure is summarized as follows:

a) For n points on the Duffing oscillator output phase trajectory, initialize centroids at (±1,0). Compute distances from each trajectory point to centroids (-1,0) and (1,0) as $d_{i1} = \sqrt{(x_{i1}+1)^2 + (y_{i1}-0)^2}$ and $d_{i2} = \sqrt{(x_{i2}-1)^2 + (y_{i2}-0)^2}$, respectively.

b) Classify each point to the nearest centroid, dividing the trajectory into two clusters with n_1 and n_2 points ($n_1 + n_2 = n$).

c) Recalculate new centroids (X_1, Y_1) and (X_2, Y_2) for each cluster, where $X_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$ and $Y_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} y_i$ (similarly for X_2, Y_2).

d) Repeat steps (a)-(c) until centroids cease moving (or satisfy a minimal displacement criterion).

e) Compute the distance between converged centroids $d = |x_l - x_r|$ and compare with threshold V_{thd}. Symbols are decoded as "1" when $d > V_{thd}$ and "-1" otherwise.

This K-means clustering decision algorithm exhibits low computational complexity. Compared with coherent demodulation, it eliminates the need for carrier recovery and low-pass filtering, reducing receiver computational load. Relative to the stochastic resonance method in [5], which involves numerous parameters requiring complex tuning and precise matching between signal and system parameters, the proposed method avoids such computational complexity and parameter matching requirements.

Matlab simulations were conducted at SNRs of -25 dB, -20 dB, -15 dB, and -10 dB, comparing the proposed Duffing-K-means method with the stochastic resonance approach from [5] and conventional coherent demodulation. Performance metrics included simulation time (computational complexity) and demodulation BER, with results summarized in Table 1 .

Table 1 Simulation Results of K-means Clustering Demodulation at Different SNRs

Simulation parameters: Symbol length $L_{en} = 500$, symbol rate $R_b = 1$ kbaud, carrier frequency $f_c = 8$ kHz, sampling frequency $f_s = 40f_c = 320$ kHz, roll-off coefficient $\alpha = 0.7$, centroid distance threshold $V_{thd} = 1.75$.

SNR (dB)	Avg. Simulation Time (s)	BER - Proposed Method (%)	BER - Stochastic Resonance (%)	BER - Coherent Demodulation (%)
-25				
-20				
-15				
-10				

The results demonstrate that at equivalent SNRs, the proposed method achieves significantly lower BER and faster computation than both alternative approaches. Extending the SNR range with 20 simulations per point (other parameters unchanged) yields the SNR-BER curves shown in Figure 8 [Figure 8: see original paper]. At -20 dB input SNR, the proposed method achieves approximately 10% BER, compared to 16% and 22% for stochastic resonance and coherent demodulation, respectively. At -10 dB SNR, the proposed method' s BER is only ~1%, versus 4% and 6% for the other methods, confirming its superiority in low-SNR demodulation.

3 Conclusion

Low-SNR BPSK signal demodulation presents significant challenges in digital communications. To enhance communication quality, this paper integrates chaotic oscillator theory with a straightforward K-means clustering demodulation algorithm. By injecting low-SNR BPSK signals into a chaotic Duffing oscillator and performing nonlinear iterative solving, the output phase trajectory is obtained. K-means clustering then iteratively computes trajectory centroids, using the converged inter-centroid distance as a decision metric for baseband symbol detection. Simulations demonstrate that at -10 dB SNR, the proposed method achieves a BER of merely 1%, offering substantial improvements over alternative approaches.

References

[1] Xu Lizhen. Research on weak BPSK signal detection technology based on chaos theory [D]. Harbin: Harbin Institute of Technology, 2012.

[2] Ji Jinjie. BPSK signal reception technology based on chaotic synchronization under low SNR [D]. Harbin: Harbin Institute of Technology, 2014.

[3] Liu Jin, Li Zan, Gao Rui, et al. A novel detector based on parameter-induced bistable stochastic resonator for binary PAM signal processing at low SNR [J]. IEEE Communications Letters, 2014: 427-430.

[4] Wang Ruifeng, Zhang Hongyan. Research on 2FSK signal detection method based on Duffing oscillator [J]. Journal of the China Railway Society, 2013, 7(35): 63-67.

[5] Shang Jinhong, Wang Fuzhong, Zhang Guanglu. Research on coherent reception BER of 2PSK signals based on stochastic resonance [J]. Applied Acoustics, 2015, 34(6): 495-500.

[6] Zhang Fujia, Jiang Hong, Zhang Qiuyun. Research on coherent reception BER of large-frequency signals based on stochastic resonance [J]. Process Automation Instrumentation, 2017, 38(7): 50-53.

[7] Zhan Yafeng, Cao Zhigang, Ma Zhengxin. Influence of roll-off coefficient error on BER performance of MPSK signals [J]. Journal on Communications, 2003, 10(24): 125-130.

[8] Zuo Jinzhong, Ma Yimin, Xi Qingling. Performance impact of roll-off coefficient mismatch on baseband transmission systems [J]. Foreign Electronic Measurement Technology, 2011, 30(8): 21-28.

[9] Xie Tao, Wei Xueye. Reliability research of chaotic oscillator in weak signal detection [J]. Chinese Journal of Scientific Instrument, 2008, 29(6): 1265-1270.

[10] Fan Jian. Research on application of stochastic resonance and chaos theory in weak signal detection [D]. Tianjin: Hebei University of Technology, 2014.

[11] Leng Yonggang, Lai Zhihui. Generalized parameter-tuned stochastic resonance of Duffing oscillator based on Kramers escape rate [J]. Acta Physica Sinica, 2014, 63(2): 25021-25029.

[12] Lai Zhihui. Research on weak signal detection methods based on chaotic and stochastic resonance characteristics of Duffing oscillator [D]. Tianjin: Tianjin University, 2014.

[13] Wang Guanyu, He Sailin. A quantitative study on detection and estimation of weak signals by using chaos Duffing oscillators [J]. IEEE Trans on Circuits and Systems, 2003: 945-953.

[14] Nie Chunyan. Chaotic Systems and Weak Signal Detection [M]. Beijing: Tsinghua University Press, 2009.

[15] Wan Jing, Zhang Yi, He Yunbin, et al. Research on KD-tree and K-means dynamic clustering method [J]. Application Research of Computers, 2015, 32(12): 3590-3595.

[16] Zhai Donghai, Yu Jiang, Gao Fei, et al. Research on K-means text clustering algorithm with maximum distance selection of initial cluster centers [J]. Application Research of Computers, 2014, 31(3): 713-716.

Note: Figure translations are in progress. See original paper for figures.