Postprint: Maximum Correntropy UKF-Based Joint Estimation Algorithm for Target State and System Bias in Sensor Networks

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Abstract

To address the problem of heavy-tailed or abrupt characteristics in observation noise when sensors observe targets in space-air-ground integrated sensor networks, as well as the impact of system bias on target state estimation, an algorithm for joint estimation of target state and system bias (ASMCUKF) based on Maximum Correntropy Unscented Kalman Filter (MCUKF) is proposed. The MCUKF algorithm first obtains the predicted state estimate and covariance matrix through Unscented Transform (UT), and then reconstructs the observation information using a nonlinear regression method based on the Maximum Correntropy Criterion (MCC), thereby enhancing the robustness of UKF against heavy-tailed noise. The ASMCUKF algorithm establishes the state equation and nonlinear observation equation with systematic errors through state vector augmentation, and performs bias registration according to the estimated system bias, thereby mitigating the impact of system bias on target state estimation. Simulation results demonstrate that ASMCUKF achieves better estimation performance for communication target state and system bias than conventional methods in environments with heavy-tailed non-Gaussian observation noise.

Full Text

Preamble

Augmented Target State Estimation Algorithm with Systematic Errors Based on Maximum Correntropy Unscented Kalman Filter in Sensor Networks

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Abstract: This paper proposes an augmented state estimation algorithm for target states and systematic errors based on the Maximum Correntropy Unscented Kalman Filter (MCUKF), referred to as ASMCUKF, to address the problems of heavy-tailed or abruptly changing observation noise in space-air-ground integrated sensor networks and the impact of systematic errors on target state estimation. The MCUKF algorithm first obtains the predicted state estimate and covariance matrix through Unscented Transformation (UT), then reconstructs the observation information using a nonlinear regression method based on the Maximum Correntropy Criterion (MCC), thereby enhancing the robustness of UKF against heavy-tailed noise. The ASMCUKF algorithm establishes state equations and nonlinear observation equations with systematic errors through state vector augmentation, and performs bias registration according to the estimated systematic errors, thereby mitigating the influence of systematic errors on target state estimation. Simulation results demonstrate that ASMCUKF achieves better estimation performance for communication target states and systematic errors compared to traditional methods in heavy-tailed non-Gaussian observation noise environments.

Keywords: space-air-ground integrated; sensor network; unscented Kalman filter; maximum correntropy criterion; systematic errors

0 Introduction

Space-air-ground integrated sensor networks, composed of remote sensing satellites, unmanned aerial vehicles, airships, balloons, ground sensors, and ground data processing centers, represent a new type of network architecture. These networks feature rapid deployment, strong robustness, and high fault tolerance, enabling data collection, processing, and transmission for communication targets. They have broad application prospects in national defense, military operations, satellite communications, aircraft communications, vehicular networks, industrial control, environmental monitoring, and health care. Target state estimation in sensor networks has become a prominent research focus, primarily involving real-time optimization algorithms to predict communication target location information. Common observation methods for position estimation include Angle of Arrival (AOA), Time of Arrival (TOA), Difference Time of Arrival (DTOA), Received Signal Strength Indicator (RSSI), and various hybrid approaches. Since target observations are typically nonlinear functions, traditional state estimation methods primarily employ Kalman filter variants such as Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF), along with improved filters like Maximum Likelihood-based Kalman filtering and Unscented Particle Filter (UPF). However, heavy-tailed non-Gaussian noise is common in practical sensor network applications, and both EKF and UKF, which are based on the minimum mean square error criterion, exhibit strong sensitivity to heavy-tailed noise and consequently poor filtering performance. To address this limitation, correntropy has been proposed as a robust method that captures not only second-order but also higher-order statistics. The Maximum Correntropy Criterion (MCC) has found extensive applications across various domains. For observation noise with heavy-tailed non-Gaussian characteristics, a novel MCUKF algorithm has been developed that combines MCC with UKF to enhance robustness. While multi-sensor fusion is typically employed to address uncertainty and limitations of single-sensor observations, most approaches neglect the impact of systematic errors, which can degrade estimation performance to levels worse than single-sensor systems. To overcome this challenge, an augmented state joint estimation algorithm (ASUKF) has been proposed to simultaneously estimate both systematic errors and target states.

1 Correntropy

Correntropy is an information-theoretic measure defined between random variables X and Y as:

$$V(X,Y) = E[\kappa_\sigma(X-Y)]$$

where E denotes the expectation operator, $\kappa_\sigma(\cdot)$ represents a kernel function, and σ is the kernel width. The Gaussian kernel function is commonly employed:

$$\kappa_\sigma(x-y) = G_\sigma(e) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{e^2}{2\sigma^2}\right)$$

where $e = x - y$. For random variables X and Y with joint distribution function F_{XY}, correntropy can be expressed as:

$$V(X,Y) = \int \kappa_\sigma(x-y)dF_{XY}(x,y)$$

In practical sensor network applications where data quantities are limited and the joint distribution function is unknown, correntropy can be estimated using finite sample averages:

$$\hat{V}(X,Y) = \frac{1}{M}\sum_{i=1}^{M}\kappa_\sigma(x_i - y_i)$$

where $\{(x_i, y_i)\}_{i=1}^{M}$ represents M samples from the joint distribution F_{XY}.

2 MCUKF Target State Estimation Algorithm

The MCUKF algorithm employs a nonlinear regression method based on MCC to reconstruct observation information, thereby enhancing UKF' s robustness against heavy-tailed noise. This algorithm is applied to target state estimation in sensor networks to address the impact of heavy-tailed non-Gaussian noise.

2.1 MCUKF Algorithm Description

Consider the nonlinear discrete-time system model:

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + \mathbf{q}_k \\ \mathbf{z}_{k+1} = \mathbf{h}(\mathbf{x}_{k+1}) + \mathbf{r}_{k+1} \end{cases}$$

where $\mathbf{x}_{k+1}$ represents the target state vector, $\mathbf{z}_{k+1}$ denotes the sensor network observation vector, $\mathbf{f}(\cdot)$ and $\mathbf{h}(\cdot)$ are continuous differentiable nonlinear system and observation functions respectively, $\mathbf{q}_k$ is process white noise with covariance matrix $\mathbf{Q}$, and $\mathbf{r}_{k+1}$ is observation white noise with covariance matrix $\mathbf{R}$.

The MCUKF filter derivation proceeds as follows. First, a set of sigma points is obtained through UT transformation:

$$\begin{cases} \chi_{0,k|k} = \hat{\mathbf{x}}_{k|k} \\ \chi_{i,k|k} = \hat{\mathbf{x}}_{k|k} + \left(\sqrt{(n+\lambda)\mathbf{P}_{k|k}}\right)_i, \quad i = 1, \dots, n \\ \chi_{i,k|k} = \hat{\mathbf{x}}_{k|k} - \left(\sqrt{(n+\lambda)\mathbf{P}_{k|k}}\right)_{i-n}, \quad i = n+1, \dots, 2n \end{cases}$$

where $\left(\sqrt{(n+\lambda)\mathbf{P}_{k|k}}\right)_i$ denotes the i-th column of the matrix square root, n is the state dimension, λ is a distribution control parameter typically set as $\lambda = \alpha^2(n+\kappa) - n$, α is a small positive parameter ($10^{-4} \le \alpha \le 1$), and κ is a tunable parameter usually set to $3 - n$.

The sigma points are propagated through the system equation to form $2n+1$ predicted sigma points:

$$\chi_{i,k+1|k} = \mathbf{f}(\chi_{i,k|k}), \quad i = 0, \dots, 2n$$

The predicted state estimate and covariance matrix are then computed as:

$$\begin{cases} \hat{\mathbf{x}}_{k+1|k} = \sum_{i=0}^{2n} \omega_i^{(m)} \chi_{i,k+1|k} \\ \mathbf{P}_{k+1|k} = \sum_{i=0}^{2n} \omega_i^{(c)} \left[\chi_{i,k+1|k} - \hat{\mathbf{x}}_{k+1|k}\right] \left[\chi_{i,k+1|k} - \hat{\mathbf{x}}_{k+1|k}\right]^T + \mathbf{Q} \end{cases}$$

where the weights are given by:

$$\begin{cases} \omega_0^{(m)} = \frac{\lambda}{n+\lambda} \\ \omega_0^{(c)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \\ \omega_i^{(m)} = \omega_i^{(c)} = \frac{1}{2(n+\lambda)}, \quad i = 1, \dots, 2n \end{cases}$$

with $\beta = 2$ being optimal for Gaussian distributions.

Based on the one-step prediction, a new set of sigma points is generated:

$$\begin{cases} \chi_{0,k+1|k} = \hat{\mathbf{x}}_{k+1|k} \\ \chi_{i,k+1|k} = \hat{\mathbf{x}}_{k+1|k} + \left(\sqrt{(n+\lambda)\mathbf{P}_{k+1|k}}\right)_i, \quad i = 1, \dots, n \\ \chi_{i,k+1|k} = \hat{\mathbf{x}}_{k+1|k} - \left(\sqrt{(n+\lambda)\mathbf{P}_{k+1|k}}\right)_{i-n}, \quad i = n+1, \dots, 2n \end{cases}$$

These predicted sigma points are substituted into the observation equation to obtain the corresponding observation sigma points:

$$\mathbf{z}_{i,k+1|k} = \mathbf{h}(\chi_{i,k+1|k}), \quad i = 0, \dots, 2n$$

The predicted observation mean and covariance are:

$$\begin{cases} \hat{\mathbf{z}}_{k+1|k} = \sum_{i=0}^{2n} \omega_i^{(m)} \mathbf{z}_{i,k+1|k} \\ \mathbf{P}_{zz,k+1|k} = \sum_{i=0}^{2n} \omega_i^{(c)} \left[\mathbf{z}_{i,k+1|k} - \hat{\mathbf{z}}_{k+1|k}\right] \left[\mathbf{z}_{i,k+1|k} - \hat{\mathbf{z}}_{k+1|k}\right]^T + \mathbf{R} \end{cases}$$

The cross-covariance matrix and Kalman gain are:

$$\begin{cases} \mathbf{P}_{xz,k+1|k} = \sum_{i=0}^{2n} \omega_i^{(c)} \left[\chi_{i,k+1|k} - \hat{\mathbf{x}}_{k+1|k}\right] \left[\mathbf{z}_{i,k+1|k} - \hat{\mathbf{z}}_{k+1|k}\right]^T \\ \mathbf{K}_{k+1} = \mathbf{P}_{xz,k+1|k} \mathbf{P}_{zz,k+1|k}^{-1} \end{cases}$$

Finally, the state update and covariance update are computed as:

$$\begin{cases} \hat{\mathbf{x}}_{k+1|k+1} = \hat{\mathbf{x}}_{k+1|k} + \mathbf{K}_{k+1}(\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k}) \\ \mathbf{P}_{k+1|k+1} = \mathbf{P}_{k+1|k} - \mathbf{K}_{k+1} \mathbf{P}_{zz,k+1|k} \mathbf{K}_{k+1}^T \end{cases}$$

3 ASMCUKF Target State Estimation Algorithm

The ASMCUKF algorithm incorporates systematic errors as components of the state vector, utilizing the MCUKF filter to achieve joint estimation of target states and systematic errors. This approach addresses the performance degradation caused by multi-sensor systematic errors on communication target state estimation under heavy-tailed non-Gaussian observation noise.

3.1 System Description

In a space-air-ground integrated sensor network, L sensors are deployed to estimate the target state, with sensor coordinates (x_i, y_i, z_i). Assuming the mobile communication target' s position and velocity at time k are $(x(k), y(k), z(k))$ and $(\dot{x}(k), \dot{y}(k), \dot{z}(k))$ respectively, the observation vector for sensor i is $\mathbf{z}_i(k) = [d_i(k), \theta_i(k), \varphi_i(k)]^T$, where $d_i(k)$, $\theta_i(k)$, and $\varphi_i(k)$ represent distance, azimuth angle, and elevation angle measurements. The distance bias, azimuth angle bias, and elevation angle bias for L sensors are defined as Δd_i, $\Delta\theta_i$, and $\Delta\varphi_i$ respectively, where $i = 1, 2, \ldots, L$.

The target state vector at time k is defined as:

$$\mathbf{x}(k) = [x(k), \dot{x}(k), y(k), \dot{y}(k), z(k), \dot{z}(k)]^T$$

Assuming the target undergoes constant velocity linear motion, the augmented state vector combining target states and systematic errors is:

$$\mathbf{x}_a(k) = [\mathbf{x}^T(k), \Delta d_1, \Delta\theta_1, \Delta\varphi_1, \ldots, \Delta d_L, \Delta\theta_L, \Delta\varphi_L]^T$$

The augmented state equation becomes:

$$\mathbf{x}_a(k+1) = \mathbf{F}_a \mathbf{x}_a(k) + \mathbf{G}_a \mathbf{w}_a(k)$$

where $\mathbf{F}_a = \mathrm{diag}(\mathbf{F}, \mathbf{I}_{3L})$, $\mathbf{G}_a = \mathrm{diag}(\mathbf{G}, \mathbf{0}_{3L})$, and $\mathbf{w}_a(k) = [\mathbf{w}^T(k), \mathbf{0}^T]^T$.

The system observation equation is defined as:

$$\mathbf{z}_i(k) = \mathbf{h}_i(\mathbf{x}(k)) + \Delta\mathbf{z}_i + \mathbf{e}_i(k)$$

where $\mathbf{h}_i(\mathbf{x}(k))$ represents the ideal observation without bias, $\Delta\mathbf{z}_i = [\Delta d_i, \Delta\theta_i, \Delta\varphi_i]^T$ denotes the systematic error vector, and $\mathbf{e}_i(k)$ is the observation noise.

3.2 ASMCUKF Algorithm Description

The ASMCUKF algorithm builds upon the MCUKF framework, using the state equation and observation equation to register systematic errors and mitigate the impact of heavy-tailed non-Gaussian noise on target state estimation. The algorithm proceeds as follows:

a) Select an appropriate kernel width σ, and initialize the state estimate $\hat{\mathbf{x}}_{a,0|0}$ and covariance matrix $\mathbf{P}_{0|0}$.

b) Calculate sigma points using the UT transformation and compute the one-step prediction of the sigma point set through the augmented state transition matrix $\mathbf{F}_a$.

c) Compute the predicted state estimate $\hat{\mathbf{x}}_{a,k+1|k}$ and covariance matrix $\mathbf{P}_{k+1|k}$.

d) Generate new sigma points and corresponding observations with systematic errors using the observation equation.

e) Obtain the modified observation covariance matrix through equations and compute the prior observation mean using the appropriate formulation.

f) Derive the bias-registered state estimate and covariance using the correntropy-based update equations.

g) Terminate when k reaches the predetermined time horizon; otherwise increment k and return to step b).

4 Simulation Results

4.1 Simulation Environment

The simulation employs the system described in Section 3.1 to estimate target states, using Root Mean Square Deviation (RMSD) and Time-Averaged RMSD (TARMSD) as performance metrics:

$$\mathrm{RMSD}(k) = \sqrt{\frac{1}{M}\sum_{m=1}^{M}[(\hat{x}_m(k|k) - x_m(k))^2 + (\hat{y}_m(k|k) - y_m(k))^2 + (\hat{z}_m(k|k) - z_m(k))^2]}$$

$$\mathrm{TARMSD} = \frac{1}{K}\sum_{k=1}^{K}\mathrm{RMSD}(k)$$

where $M = 100$ represents the total number of Monte Carlo runs and $K = 500$ is the total time steps per experiment.

The target moves with constant velocity in the sensor field according to:

$$\begin{cases} x(k) = 6000 + 50k \\ y(k) = 50000 - 10k \\ z(k) = 1000 + 8k \end{cases}$$

Four sensors are deployed with coordinates and systematic biases as follows (distance in meters, angles in radians):

- Sensor positions: $A(0, 40000, 0)$, $B(40000, 40000, 0)$, $C(0, 60000, 0)$, $D(40000, 60000, 0)$
- Distance biases: $\Delta d = [2000, 1500, 1500, 2000]$
- Azimuth biases: $\Delta\theta = [0.0087, 0.0025, 0.0070, 0.0078]$
- Elevation biases: $\Delta\varphi = [0.0087, 0.0025, 0.0070, 0.0078]$

The observation noise standard deviations are $\delta_d = 100$ m, $\delta_\theta = \delta_\varphi = 0.5^\circ$, and the process noise covariance is $\mathbf{Q} = \mathrm{diag}([10, 10, 10])$. The heavy-tailed non-Gaussian observation noise is modeled as a Gaussian mixture: $0.8\mathcal{N}(0, \sigma_1^2) + 0.2\mathcal{N}(0, \sigma_2^2)$, with $\sigma_1 = 10$ m for distance and $\sigma_1 = 0.5^\circ$ for angles, and $\sigma_2 = 10\sigma_1$.

The initial true state is $\mathbf{x}(0) = [6000, 50, 50000, -10, 1000, 80]^T$, while the initial state estimate is $\hat{\mathbf{x}}_{0|0} = [6250, 50, 50500, -10, 1250, 8, 500, 0.001, 0.001]^T$ with initial covariance $\mathbf{P}_{0|0} = \mathrm{diag}([100, 2, 100, 2, 100, 2, 10, 610, 610])$.

4.2 Simulation Results

The performance of ASMCUKF is compared against ASUKF and ASEKF under heavy-tailed non-Gaussian observation noise. [Figure 1: see original paper] illustrates the estimated trajectories from various filters as the communication target moves through the sensor network. When processing nonlinear observation equations, ASUKF and ASMCUKF demonstrate superior accuracy compared to ASEKF. In the presence of non-Gaussian observation noise, ASMCUKF exhibits stronger robustness than ASUKF. Due to initial estimation errors, all filters gradually converge toward the true trajectory.

[Figure 2: see original paper] depicts the RMSD performance across different filters. As iteration count increases, RMSD values show a decreasing trend, though fluctuations are evident due to heavy-tailed non-Gaussian noise. ASMCUKF with moderate kernel width ($\sigma = 2^2$) achieves the smallest RMSD among all filters. presents the TARMSD values, showing that ASMCUKF($\sigma = 2^2$) attains the minimum TARMSD of 316.1145 m, significantly outperforming ASUKF (324.9229 m) and ASEKF (339.7223 m). With large kernel width ($\sigma = 2^{20}$), ASMCUKF performance approaches that of ASUKF, while small kernel width ($\sigma = 2^2$) enables superior performance over both ASEKF and ASUKF.

[Figure 3: see original paper] examines the impact of sensor count on estimation performance. With other parameters fixed, RMSD decreases as the number of sensors increases. All joint algorithms effectively reduce systematic error effects, but ASMCUKF consistently achieves lower RMSD than ASEKF and ASUKF, particularly demonstrating the best estimation performance in multi-sensor scenarios when $\sigma = 2^2$.

5 Conclusion

This paper addresses the challenge of multi-sensor communication target state estimation under heavy-tailed non-Gaussian observation noise in sensor networks, where systematic errors can severely degrade estimation performance. The proposed ASMCUKF algorithm integrates MCUKF with state vector augmentation to jointly estimate target states and systematic errors. By leveraging the strong robustness of correntropy against non-Gaussian noise and employing bias registration through state augmentation, ASMCUKF significantly improves estimation accuracy. Simulation results validate that ASMCUKF outperforms

ASEKF and ASUKF in heavy-tailed noise environments, with performance advantages becoming more pronounced as sensor count increases. The kernel width parameter provides a trade-off between robustness and convergence, where moderate values yield optimal performance while extreme values approach conventional UKF behavior.

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Note: Figure translations are in progress. See original paper for figures.