Root Growth Swarm Optimization Algorithm Postprint

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Abstract

For global optimization problems, this paper proposes a novel intelligent optimization algorithm—the Root Growth Swarm Optimization (RGSO) algorithm—based on Support Vector Data Description (SVDD) and existing root growth algorithms, wherein the root system is partitioned into main root and lateral root populations. The growth behavior of the main root population is characterized using SVDD, with the location of highest nutrient concentration in soil serving as the global optimization objective, thereby constructing a root growth model. The mathematical model of RGSO is analyzed and its convergence is theoretically proven. Experimentally, comprehensive comparisons are conducted with three other state-of-the-art algorithms, and the impact of various parameters on optimization performance is investigated. The experimental results validate both the convergence and effectiveness of RGSO, establishing it as an effective algorithm for solving global optimization problems.

Full Text

Preamble

Root Growth Swarm Optimization Algorithm

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Abstract: Addressing global optimization problems, this paper proposes a novel intelligent optimization algorithm called Root Growth Swarm Optimization (RGSO), based on Support Vector Data Description (SVDD) and existing root growth algorithms. The root system is differentiated into taproot and lateral root groups. The growth behavior of the taproot group is described using

SVDD, with the position of highest nutrient concentration in soil serving as the global optimization target, thereby constructing a root growth model. The mathematical model of RGSO is analyzed, and its convergence is theoretically proven. In experiments, RGSO is comprehensively compared with three other state-of-the-art algorithms, and the impact of different parameters on optimization performance is observed. Experimental results verify the convergence and effectiveness of RGSO, demonstrating that it is an effective algorithm for solving global optimization problems.

Keywords: root growth algorithm; grouping mechanism; SVDD; optimization problem

0 Introduction

Optimization problems represent an ancient yet fundamental topic that has found successful applications in engineering technology, economic management, transportation, system control, artificial intelligence, pattern recognition, and numerous other fields. The mathematical formulation of an optimization problem is as follows:

$$\min f(x), \quad \text{s.t. } h(x) = 0; g(x) \geq 0$$

where x is an n-dimensional vector, f is the objective function, and h and g are constraint functions.

Bio-inspired intelligent algorithms constitute an emerging evolutionary computation technology. Natural biological phenomena follow certain intrinsic laws, and for centuries, scientists have sought to identify and document the physical principles underlying these phenomena. Inspired by various natural biological phenomena, computer scientists have developed computational techniques and methods for solving large-scale complex problems based on the behaviors of organisms in nature. The research objects of intelligent optimization algorithms include both the behaviors of organisms themselves and their natural environments, with the goal of discovering mathematical patterns within them. Traditional intelligent algorithms such as Genetic Algorithms, Immune Algorithms, and Particle Swarm Optimization all exhibit adaptive characteristics.

Particle Swarm Optimization (PSO), first proposed by Dr. Barnhart and Dr. Kennedy, originated from studies of bird flocking behavior. In 2014, Zhang et al. introduced the original root growth algorithm, which simulates plant root growth by targeting the location of highest nutrient concentration in soil as the optimization objective. Support Vector Data Description (SVDD) is a classification technique used for one-class classification problems. In 2013, Niazmardi et al. proposed an improved clustering framework using SVDD as a solution for scenarios lacking sufficiently high-quality training data during clustering.

Numerous improved algorithms based on traditional methods have been continuously proposed. For instance, research has applied backpropagation learning in feedforward neural networks to optimize FPGA chips, developed ant colony algorithms with interference cancellation for cooperative transmission, and created multi-objective simulated annealing and genetic algorithms for optimizing three-phase high-temperature superconducting transformers. Elite non-dominated sorting genetic algorithms have been used for multi-objective path planning, while artificial immune algorithms capable of solving time-varying nonlinear problems in dynamic environments have been developed. Other advances include a self-learning particle swarm optimization algorithm that enhances individual particle search capabilities and a comprehensive learning particle swarm optimization algorithm with improved neighbor structures. More recently, novel intelligent algorithms continue to emerge, such as improved social spider optimization algorithms for electromagnetic optimization, slime mold algorithms for solving Steiner tree problems, and mosquito tracking algorithms.

Building upon this related work, the root growth optimization model designed in this paper establishes an adaptive learning mechanism for the algorithm. Through this adaptive learning characteristic, the algorithm achieves global optimization. While most nature-inspired algorithms demonstrate a certain degree of effectiveness, they share common drawbacks: slow convergence speed and susceptibility to premature convergence and local optima. The RGSO algorithm proposed in this paper, based on SVDD, addresses these shortcomings by improving upon the original root growth algorithm. RGSO characterizes the growth patterns of root populations and the interactions between individuals, offering higher adaptability and self-learning capability compared to other algorithms, thus demonstrating greater effectiveness in solving large-scale complex problems.

1 Root Growth Model

1.1 Root Population

A plant root system consists of a series of root tips, which are the most active parts of the root and the primary sites for root growth, elongation, and water absorption. Consequently, root growth depends on the activity of root tips. The growth process of root tips is closely related to their environment, with soil nutrient concentration being the most influential factor. Empirical evidence shows that root tips always grow toward locations with the highest nutrient concentration in soil, as illustrated in [Figure 1: see original paper].

1.2 Initial State of Root System

The root system develops from a single seed, meaning the initial population consists of one root tip. This initial root tip is randomly generated within the feasible domain, and the entire root system develops from this starting point.

1.3 Taproot and Lateral Root Subgroups

In RGSO, the root population is divided into two subgroups: the taproot subgroup and the lateral root subgroup. The definitions of these subgroups are as follows:

Definition 1 (Taproot Subgroup). The taproot grows vertically downward through cell division and elongation of the radicle, representing the first root to emerge in a plant, also known as the primary root. In most gymnosperms and dicotyledons, the taproot continues to grow, becoming prominent and robust. The plant primarily relies on the continuous downward growth of the taproot to locate the position of highest nutrient concentration in soil.

Definition 2 (Lateral Root Subgroup). As the taproot grows through the soil and reaches a certain length, it produces lateral branches, forming an extensive root system. The primary motivation for forming lateral branches is to search a broader area for locations with higher nutrient concentrations. Once a location with higher nutrient concentration is discovered, this information is fed back to the entire root system, which then grows toward this better position.

Due to population diversity, each root tip occupies a different position with a corresponding fitness value. Based on the fitness values of root tips in the population, the growth strategies are divided into taproot tip growth strategy and lateral root tip growth strategy. The classification criterion depends on individual fitness values: root tips with fitness above the population average are classified as taproot tips, while those below the average are classified as lateral root tips.

1.4 Auxin Factor

Under normal growth conditions, plant root systems produce a growth hormone to promote root tip growth. The auxin level reflects the current fitness value of a root tip within the population. The better the fitness value (i.e., the smaller) of the current root tip’ s position compared to other individuals, the larger its auxin factor relative to others.

1.5 Soil Nutrient Distribution Model

In reality, soil nutrient distribution is non-uniform, generally exhibiting regional concentration patterns. Typically, if multiple locations in the soil have high nutrient concentrations, the spatial region determined by these locations is considered to have high nutrient concentration. To identify this region, Support Vector Data Description (SVDD) is introduced to construct a soil vector space model. This involves establishing the smallest possible hypersphere in vector space that encloses multiple vectors in the target region, with the sphere’ s center representing that region.

1.6 Growth Behaviors

Root growth behaviors primarily include root tip branching and elongation. The following assumptions constrain these behaviors:

Assumption 1. The growth direction of a root tip primarily depends on its perception of nutrient concentration in soil.

Assumption 2. During growth, if the total number of root tips has not reached the threshold, the taproot will branch.

Assumption 3. Different root tip individuals influence each other' s behavior, with superior individuals guiding the growth of less advanced ones.

Assumption 4. When the total number of root tips reaches a certain threshold, some individuals with poor fitness are eliminated.

1.6.1 Taproot Branching The root system develops from a single seed (one root tip), with growth manifested as continuous expansion of the root tip population. During root system development, superior individuals branch in their surrounding growth space, increasing the total number of root tips. In this paper, only taproot tip individuals are permitted to branch, with a constant number of branches produced each time.

1.6.2 Taproot Growth Taproot tip growth is relatively slow and influenced by two factors:

a) **Angle Factor.** Since taproots always grow toward regions with higher nutrient concentration, the growth direction is determined by the location of currently nutrient-rich regions. This region is defined by the spatial area determined by all root tips superior to the current individual. SVDD can be used to identify this space and calculate its center, toward which the current root tip should grow.

b) **Length Factor.** This length is determined by the distance from the mean position of all individuals in the population to the current individual' s position and by the current individual' s auxin factor.

1.6.3 Lateral Root Growth Lateral root tip growth is relatively fast and generally considered more scattered, with random growth angles and directions.

1.7 Elimination Mechanism

The root tip population has an upper limit, referred to as the equilibrium point of population size. When the population exceeds this equilibrium point, inferior individuals are eliminated to restore balance. The elimination criterion removes individuals with the worst fitness values until the total count returns to the equilibrium point.

2 Mathematical Model and Algorithm of RGSO

2.1 Mathematical Model of Root Population

Definition 3. The initial state of the root population is a single seed, meaning the algorithm begins with only one root tip. This root tip's position is randomly generated within the feasible domain, and the entire root system develops from it.

$$x_0 = l + \lambda \cdot (u - l)$$

where u and l represent the upper and lower bound vectors of the n-dimensional feasible domain, and λ is a random variable in $[0, 1]$.

Definition 4. For any root tip individual R_i, $R_i \in S$, where $S = \{M, L\}$. Here, S is the set of root tip populations, M is the set of taproot subgroups, and L is the set of lateral root subgroups.

Algorithm 1. Root Subgroup Classification

```
Input: Fitness function fit(x) and parameters
1: Calculate fitness fit(x_i) for all root tips
2: Calculate average fitness
3: for each root tip
4:   if fit(x_i) $\geq$ average
5:     The root tip belongs to taproot
6:   else
7:     The root tip belongs to lateral root
8:   end if
9: end for
Output: Classification of root tip individuals
```

2.2 Root Growth Model

Based on the root formation process, subgroup classification, and the branching and growth behaviors of root tips, the flowchart is shown in [Figure 2: see original paper].

2.2.1 New Root Tip Position Population growth includes the formation of new root tip positions, root tip branching, and root tip elongation.

Definition 5. The formation position of a new root tip is related to the original root tip' s position and is defined as:

$$x_{new} = x_{old} + \Delta x$$

where x_{old} represents the old root tip's position, and Δx is a normally distributed random variable following $N(0, 1)$.

2.2.2 Taproot Branching **Definition 6.** Only taproots branch; lateral roots do not branch.

Algorithm 2. Root Tip Branching Algorithm

```
Input: Positions of root tip individuals
1: for each root tip x_i
2:   if x_i is a taproot
3:     Generate new root tip position using Equation (2)
4:   end if
5: end for
Output: Positions of newly generated root tips
```

2.2.3 Soil Region Distribution Based on SVDD **Definition 7.** The region determined by all individuals superior to the current root tip is defined by a hypersphere enclosing these individual vectors. Let the sphere' s center be O and its radius be R.

$$\min_{R,O} R^2 + C\sum_{i=1}^{n}\xi_i$$

$$\text{s.t. } \|O - x_i\|^2 \leq R^2 + \xi_i, \quad \xi_i \geq 0$$

where $x_i \in A$, A is the set of all root tips superior to the current individual, O is the hypersphere center, R is the hypersphere radius, ξ_i are slack variables, and C is a penalty factor. Based on SVDD, O and R can be calculated from set A.

2.2.4 Taproot and Lateral Root Growth Strategies In RGSO, the root population is divided into taproot and lateral root subgroups, each with distinct growth patterns.

Definition 8. The growth angle of each taproot individual is determined by other root tips superior to the current individual.

$$\theta_i = O_i - x_i$$

where O_i is the center of the hypersphere determined by all individuals superior to x_i.

Definition 9. Auxin reflects the current fitness level of a root tip individual. The smaller the fitness value, the larger the auxin value.

$$E_i = \exp\left(-\frac{\text{fit}(x_i)}{\sum_{j\in S}\text{fit}(x_j)}\right)$$

where S represents the set of all root tip individuals in the current population.

Definition 10. The growth length γ of a taproot tip is related to its current auxin value: larger auxin values result in smaller growth lengths, and vice versa.

$$\gamma = \frac{1}{n} \cdot \sum_{j \in S} \text{rand} \cdot \|x_j - x_i\| \cdot E$$

where n is the current total number of individuals, E is the auxin factor, and rand is a random number in $[0, 1]$.

Definition 11. The scaling operator ω is related to the current root tip' s fitness: smaller fitness values indicate superior root tips with higher probability of retaining their original position.

$$\omega_i = \frac{1}{\sum_{j \in S} \text{fit}(x_j)}$$

Definition 12. Taproot tip position update is defined as:

$$x_i(t+1) = \omega \cdot x_i(t) + \alpha \cdot \theta_i \cdot \gamma$$

where $x_i(t)$ represents the position of the i-th taproot tip at time t, α is a control parameter in $[0, 1]$, θ_i represents the growth angle, and γ represents the growth length.

Definition 13. Lateral root tip position update is defined as:

$$x_i(t+1) = \beta \cdot \text{rand} \cdot x_i(t)$$

where $x_i(t)$ represents the position of the i-th lateral root tip at time t, employing a random walk growth strategy, β is a control parameter, and rand is a random number in $[0, 1]$.

Algorithm 3. Root Tip Growth Algorithm

```
Input: Positions, categories of root tip individuals, and parameters
1: for each root tip
2:   if root tip is taproot
3:     Calculate taproot position using Equation (8)
4:   else
5:     Calculate lateral root position using Equation (9)
6:   end if
7: end for
Output: Updated root tip positions
```

Each branching process increases the population size. Therefore, an upper limit Num is set for the total number of root tips. When the population exceeds Num, individuals with poor fitness are eliminated based on their fitness values. The elimination criterion removes the worst-performing individuals until the total count falls below the threshold Num.

3 Theoretical Analysis of the Root Growth Optimization Model

3.1 Convergence Proof

Lemma 1. In a geometric sequence $\{a_n\}$ with general term $a_n = a_1 \cdot q^{n-1}$ and $a_1 \neq 0$, the sequence $\{a_n\}$ converges if and only if $|q| < 1$.

Lemma 2. For the first-order linear non-homogeneous difference equation:

$$x_{k+1} = a \cdot x_k + f(k)$$

the solution converges if $|a| < 1$. The characteristic equation is $\lambda - a = 0$, yielding the homogeneous solution $x(k) = C \cdot a^k$. To ensure convergence, we must have $|a| < 1$, which implies $\lim_{k\to\infty} a^k = 0$.

Theorem 1. For any vector set A where each vector is n-dimensional, there exists an n-dimensional hypersphere that encloses all vectors in set A.

Theorem 2. If $0 < a < 1$ and $0 < b < 1$, then $0 < |a \cdot b| < 1$.

Theorem 3. The taproot growth behavior of the Root Growth Swarm Optimization algorithm is convergent.

To simplify the proof, we consider the n-dimensional vector in one dimension $(n = 1)$. The taproot growth expression is:

$$x_i(t+1) = \omega \cdot x_i(t) + \alpha \cdot \theta_i \cdot \gamma$$

Expanding θ_i:

$$\theta_i = O_i - x_i(t)$$

Converting to difference equation form:

$$x_i(t+1) = \omega \cdot x_i(t) + \alpha \cdot \gamma \cdot (O_i - x_i(t))$$

Rearranging:

$$x_i(t+1) = (1 - \alpha \cdot \gamma) \cdot x_i(t) + \alpha \cdot \gamma \cdot O_i$$

Let $k = 1 - \alpha \cdot \gamma$. Expanding γ:

$$\gamma = \frac{1}{n} \cdot \sum_{j \in S} \text{rand} \cdot \|x_j - x_i\| \cdot E$$

As $t \to \infty$, we have:

$$\lim_{t \to \infty} \frac{1}{n} \cdot \sum_{j \in S} \|x_j - x_i\| = p$$

where p is a constant. Therefore:

$$k = 1 - \alpha \cdot \text{rand} \cdot p \cdot E$$

Given $0 < \alpha < 1$, $0 < 1 - E < 1$, and $0 < \text{rand} < 1$, we have $0 < \alpha \cdot \text{rand} \cdot p \cdot E < 1$. By Theorem 2, $0 < |k| < 1$. According to Lemma 2, the necessary and sufficient condition for the difference equation solution to converge is $0 < |k| < 1$.

3.2 Effectiveness Analysis

Due to the nutrient-seeking nature of plant roots, root tips always grow toward regions of higher nutrient concentration in soil, a characteristic well-simulated by RGSO. To maintain population diversity, RGSO divides growth points into taproot and lateral root growth points. Based on the convergence property of the difference equation, the convergence of optimization results can be derived. Experiments demonstrate that RGSO can indeed quickly and effectively find extremum points.

4 Experiments

Experimental Environment: MATLAB R2014b on the HPCC of Lenovo Shenteng 6800, a cluster with 8 compute nodes and one master node. Each compute node is a high-performance server with 24 GB memory and a quad-core 2.4 GHz CPU. All servers run Red Hat Enterprise Linux 7.

4.1 Benchmark Functions

The benchmark functions used in experiments include 10 commonly used functions in intelligent evolutionary algorithms, listed in .

4.2 Growth Strategy and Parameter Analysis

The total number of iterations is set to 1000. Parameter settings for RGSO are shown in , with each parameter determined through extensive testing to ensure optimal performance.

Taproot growth references superior individuals in the population by constructing a hypersphere containing all currently superior individuals, using the sphere' s center as the target for the current individual. This maximizes utilization of all useful information and helps escape local optima. Lateral root growth follows random walk principles, providing a mechanism to escape local optima when taproot growth becomes trapped.

Parameter α regulates taproot growth step size. The algorithm's numerical optimization results varying with α are shown in [Figure 3: see original paper], where the vertical axis uses natural logarithm base e. Algorithm performance peaks when α is in $[0.5, 0.6]$. Parameter b affects convergence speed and precision: too small a value reduces precision, while too large a value slows convergence. Parameter β regulates lateral root growth step size. Through multiple experiments, optimal performance is achieved when β is in $[5, 10]$ in combination with the above parameters.

4.3 Comparison with Other Algorithms and Analysis

This section compares RGSO with RGA, SSO, and PSO to analyze RGSO' s performance. The total iterations are set to 1000, with each algorithm independently run 30 times to calculate mean and variance. Experiments terminate when reaching maximum iterations or when the result error falls below 10^{-10}.

Comparison results for $D = 60$ are shown in , and for $D = 100$ in . Algorithm rankings for $D = 60$ and $D = 100$ are presented in [Figure 4: see original paper] and [Figure 5: see original paper], respectively.

At $D = 60$, RGSO demonstrates highly effective results on functions f_1, f_3, f_5, and f_7, with good performance on remaining benchmark functions. At $D = 100$, RGSO similarly shows excellent results on f_1, f_3, f_5, and f_7, maintaining good performance on other functions. Compared to other algorithms, RGSO achieves more precise results on f_1, f_2, f_3, f_5, f_6, and f_7.

RGSO exhibits good performance and stability on most benchmark functions. However, its performance on certain multimodal functions is not outstanding. Analysis reveals two primary reasons: (a) RGSO' s theoretical foundation of developing from a single seed means the initial seed position selection influences final iteration precision and speed; (b) The selection of branching numbers also affects results.

5 Conclusion

Inspired by plant root growth processes, this paper proposes the Root Growth Swarm Optimization (RGSO) algorithm as an improvement upon existing root growth algorithms for solving optimization problems. By constructing a root population model, the algorithm describes overall root system development and root tip classification. Through a population growth model and SVDD-based soil nutrient distribution model, the growth patterns of differently classified root tips are elaborated, enabling more reasonable intra-population classification and accelerating convergence speed and precision. Global convergence is theoretically proven. Parameter tuning experiments identify optimal parameter values. Comparisons with state-of-the-art algorithms including PSO, SSO, and RGA demonstrate RGSO' s high effectiveness and global convergence.

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Note: Figure translations are in progress. See original paper for figures.