# Postprint: Improved Collaborative Representation-Based Hyperspectral Image Anomaly Detection Algorithm

**Authors:** Liu Wanjun, Wu Xiaojie, Haicheng Qu, I have received the name "王凤". Please provide the full academic paper content with ···tags, LaTeX formulas, and citations that require translation. I am ready to process the complete manuscript according to the specified scientific translation protocols.

**Date:** 2018-05-20T00:00:00+00:00

## Abstract

To address the issue in collaborative representation-based hyperspectral image anomaly detection algorithms where the center of a dual-window is an anomalous pixel while the background dictionary contains the same type of anomalous pixels, resulting in a relatively small output for the center pixel that is difficult to distinguish from the background, an improved collaborative representation-based hyperspectral image anomaly detection algorithm is proposed. To reduce the weight of anomalous pixels in the background dictionary, the weights of dictionary atoms are adjusted using the distance between the atoms and the mean value, thereby increasing the output of the center pixel under the aforementioned scenario. Experimental results demonstrate that the proposed algorithm achieves favorable detection performance under different dual-window configurations, validating its effectiveness.

## Full Text

## Preamble

### Improved Collaborative Representation for Hyperspectral Imagery Anomaly Detection Algorithm

*Liu Wanjun, Wu Xiaojie, Qu Haicheng, Wang Feng*
(School of Software, Liaoning Technical University, Huludao, Liaoning 125105, China)

**Abstract:** In collaborative representation-based hyperspectral image anomaly detection algorithms, when the center pixel of a dual window is anomalous and the background dictionary contains the same type of anomalous pixels, the output of the center pixel becomes small and difficult to distinguish from the background. To address this problem, this paper proposes an improved collaborative representation algorithm for hyperspectral image anomaly detection. To reduce the weights of anomalous pixels in the background dictionary, the algorithm adjusts the weight of each atom based on its distance to the mean of the background dictionary, thereby increasing the output of the center pixel in the aforementioned scenario. Experimental results demonstrate that the proposed algorithm achieves superior detection performance across different dual-window configurations, validating its effectiveness.

**Keywords:** hyperspectral image; anomaly detection; anomalous pixel; collaborative representation; dual window

## 0 Introduction

Hyperspectral image target detection represents a crucial research direction in hyperspectral imaging with broad applications in military security, environmental pollution monitoring, geological exploration, and agricultural monitoring. Depending on whether prior information is required, hyperspectral target detection is typically categorized into spectral matching detection and anomaly detection. In practical scenarios, obtaining accurate ground object reflectance spectra and comprehensive spectral libraries is challenging, making prior information difficult to acquire. Consequently, anomaly detection algorithms that do not require prior information are particularly important and represent a current research focus.

The RX algorithm, proposed by Reed et al. [?] based on generalized likelihood ratio testing, stands as one of the most classical methods in hyperspectral anomaly detection. However, the RX algorithm neglects the rich nonlinear information present in hyperspectral images, resulting in relatively poor detection accuracy. Kernel-based methods have been developed to address this limitation. Commonly used kernel-based anomaly detection algorithms include the Kernel RX (KRX) algorithm and Support Vector Data Description (SVDD). Kwon et al. [?] proposed the KRX algorithm, which maps original data through nonlinear functions into high-dimensional feature spaces, rendering originally linearly inseparable data linearly separable and thereby better distinguishing background from target information. Banerjee et al. [?] introduced the SVDD algorithm, which constructs a nonlinear classifier that encloses samples with common characteristics within a minimal hypersphere while maximizing the exclusion of other class samples outside this sphere, achieving anomaly detection through the hypersphere boundary. Numerous researchers [?, ?, ?] have proposed improved algorithms based on these kernel methods, achieving favorable detection results.

Li et al. [?] proposed the Collaborative Representation-based Detector (CRD) algorithm, which first applied collaborative representation to hyperspectral image anomaly detection. The CRD algorithm is founded on the principle that background pixels can be approximately represented by surrounding pixels, whereas anomalous pixels cannot. It employs a sliding dual window to obtain a background dictionary and uses linear combinations of background dictionary atoms to generate approximations of the center pixel. The Euclidean distance between the center pixel and its approximation determines whether the center pixel is anomalous. However, the CRD algorithm considers only the spectral characteristics of hyperspectral images while ignoring their spatial characteristics. Zhang et al. [?] addressed this by proposing a joint kernel collaborative and sparse difference index representation model that employs sparse coding theory in kernel space while simultaneously considering spectral correlation and spatial correlation, thereby improving detection accuracy. Nevertheless, the kernel function parameters in this method are determined through extensive experimental optimization and cannot be selected adaptively. Tang et al. [?] subsequently proposed an adaptive kernel joint representation algorithm for hyperspectral image anomaly detection that sets kernel parameters based on local statistical properties of detection windows, enhancing the local adaptability of kernel parameters and improving detection performance.

A critical limitation of the CRD algorithm and its related improvements is their inability to effectively handle cases where the center pixel is anomalous while the background dictionary contains the same type of anomalous pixels. In such scenarios, the output of the center pixel becomes small and difficult to distinguish from background pixels. This paper proposes an improved collaborative representation algorithm for hyperspectral image anomaly detection that addresses this specific problem.

## 1 Collaborative Representation for Hyperspectral Image Anomaly Detection

The collaborative representation algorithm generates approximations of the center pixel through linear combinations of background dictionary atoms, where the background dictionary is obtained via a sliding dual window. Let $y \in \mathbb{R}^d$ denote the center pixel with $d$ spectral dimensions. The dual window consists of an outer window of size $w_{\text{out}} \times w_{\text{out}}$ and an inner window of size $w_{\text{in}} \times w_{\text{in}}$, where $w_{\text{out}} > w_{\text{in}}$. The background dictionary $X \in \mathbb{R}^{d\times s}$ comprises pixels from the area between the inner and outer windows, where $s = w_{\text{out}}^2 - w_{\text{in}}^2$ represents the number of background dictionary atoms.

The objective of collaborative representation is to find a weight vector $\alpha \in \mathbb{R}^s$ that minimizes both the reconstruction error $\|y - X\alpha\|_2^2$ and the weight vector magnitude $\|\alpha\|_2^2$. To ensure that atoms highly similar to the center pixel receive larger weights while dissimilar atoms receive smaller weights, a diagonal weighting matrix $\Gamma$ is introduced. The diagonal elements of $\Gamma$ represent the Euclidean distances between the center pixel and each background dictionary atom. To enhance solution stability and improve discriminative power, a sum-to-one constraint is imposed on $\alpha$ by introducing a row vector $y^T \in \mathbb{R}^{1\times s}$ with all elements equal to 1.

The optimization problem is formulated as:

$$\arg\min_{\alpha} \|y - X\alpha\|_2^2 + \lambda\|\Gamma\alpha\|_2^2 \quad \text{s.t.} \quad y^T\alpha = 1$$

Using the method of Lagrange multipliers, this becomes:

$$\arg\min_{\alpha} \left[\|y - X\alpha\|_2^2 + \lambda\|\Gamma\alpha\|_2^2 + 2\mu(y^T\alpha - 1)\right]$$

Taking the derivative with respect to $\alpha$ and setting it to zero yields:

$$\alpha = (X^T X + \lambda\Gamma^T\Gamma)^{-1}(X^T y - \mu y)$$

The algorithm' s output is defined as the residual error:

$$r = \|y - \hat{y}\|_2 = \|y - X\alpha\|_2$$

If $r$ exceeds a predefined threshold, $y$ is classified as anomalous; otherwise, it is classified as background.

## 2 Improved Collaborative Representation for Hyperspectral Image Anomaly Detection

In collaborative representation-based hyperspectral anomaly detection, a regularized diagonal matrix weighted by distance is used to adjust the weights of background dictionary atoms. Atoms with higher similarity to the center pixel receive larger weights, while less similar atoms receive smaller weights. However, when the center pixel is anomalous and the background dictionary contains the same type of anomalous pixels, these similar anomalous atoms receive large weights due to their small Euclidean distances to the center pixel. This causes the approximation $\hat{y}$ to be close to $y$, resulting in a small residual $r$. If $r$ falls below the threshold, the anomalous pixel is misclassified as background, leading to poor detection performance in such window configurations.

To address this issue, this paper proposes an improved collaborative representation algorithm. Let $\mu$ denote the mean of the background dictionary. The weight of each atom is adjusted based on its distance to this mean. Under relatively simple background conditions, atoms far from the mean are considered more anomalous and should receive smaller weights, while atoms near the mean are considered background pixels and should receive larger weights. The following diagonal matrix is used to adjust background dictionary atom weights:

$$\Gamma_\mu = \begin{bmatrix} \|x_1 - \mu\|_2 & 0 & \cdots & 0 \\ 0 & \|x_2 - \mu\|_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \|x_s - \mu\|_2 \end{bmatrix}$$

The new optimization problem becomes:

$$\arg\min_{\alpha} \|y - X\alpha\|_2^2 + \lambda\|\Gamma_\mu \alpha\|_2^2 \quad \text{s.t.} \quad y^T\alpha = 1$$

The solution is:

$$\alpha = (X^T X + \lambda\Gamma_\mu^T \Gamma_\mu)^{-1}(X^T y - \mu y)$$

The algorithm output remains $r = \|y - X\alpha\|_2$.

**Analysis of Algorithm Behavior Under Different Scenarios**

1. **Center pixel is background, background dictionary contains no anomalies**: All diagonal elements in $\Gamma_\mu$ are small. Since most dictionary atoms have small differences from the center pixel, the approximation $\hat{y}$ is close to $y$, resulting in a small residual $r$.
2. **Center pixel is background, background dictionary contains few anomalies**: Anomalous atoms correspond to larger diagonal elements in $\Gamma_\mu$, reducing their weights, while background atoms have smaller diagonal elements, increasing their weights. Due to the small proportion of anomalies, most weighted atoms remain similar to the center pixel, yielding a small residual $r$.
3. **Center pixel is anomalous, background dictionary contains no anomalies**: All diagonal elements in $\Gamma_\mu$ are small, but dictionary atoms differ significantly from the center pixel, making the approximation $\hat{y}$ substantially different from $y$ and producing a large residual $r$.
4. **Center pixel is anomalous, background dictionary contains few anomalies of different types**: Anomalous atoms have larger diagonal elements (reduced weights) while background atoms have smaller elements (increased weights). Since all atoms differ significantly from the center pixel, $\hat{y}$ differs markedly from $y$, yielding a large residual $r$.
5. **Center pixel is anomalous, background dictionary contains same-type anomalies**: Anomalous atoms have larger diagonal elements (reduced weights) while background atoms have smaller elements (increased weights. Anomalous atoms similar to the center pixel receive small weights, while dissimilar background atoms receive large weights, making $\hat{y}$ significantly different from $y$ and producing a large residual $r$.

This analysis demonstrates that the proposed algorithm adjusts background dictionary atom weights to produce small outputs for background center pixels and large outputs for anomalous center pixels, even when the background dictionary contains the same anomaly type.

**Algorithm Steps**

1. Input the 3D hyperspectral image, parameter $\lambda$, and dual-window dimensions.
2. For each pixel to be detected, treat it as the center pixel $y$ and obtain the background dictionary $X$ through the dual window.
3. Compute the background dictionary mean $\mu$ and diagonal matrix $\Gamma_\mu$.
4. Calculate the weight vector $\alpha$ using the solution equation.
5. Compute the distance $r$ between the center pixel and its estimated value.
6. Move to the next pixel and repeat steps 2-5 until all pixels are processed.

## 3 Experiments and Analysis

Experiments were conducted using MATLAB R2016a on a computer with an Intel(R) Core(TM) i7-6700 CPU and 16 GB RAM. The algorithm was validated using one simulated dataset and two real hyperspectral image datasets.

### 3.1 Data Description

**First Dataset (Simulated)**: This dataset uses AVIRIS hyperspectral imagery from the San Diego Naval Air Station. The original image has 400×400 pixels and 224 bands. After removing water absorption and low SNR bands (1-6, 33-35, 94-97, 107-113, 153-166, and 221-224), 186 bands remain for experimentation, with the first band shown in [Figure 1: see original paper]. A 100×100 pixel region was extracted as the background. Sixteen pixels from aircraft in the AVIRIS image were inserted into the background as anomalous targets, each containing 50% aircraft spectrum and 50% background spectrum. The simulated data is shown in [Figure 2: see original paper], where (a) displays the first band and (b) shows the target distribution.

**Second Dataset (Real AVIRIS)**: This dataset also uses AVIRIS imagery, extracting a 100×100 pixel region where aircraft serve as the anomalies to be detected. The real AVIRIS data is shown in [Figure 3: see original paper], containing 57 anomalous pixels.

**Third Dataset (Real HYDICE)**: This dataset uses HYDICE hyperspectral imagery from a suburban residential area in Texas. The original image has 307×307 pixels and 210 bands. After removing problematic bands (1-4, 76, 87, 101-111, 136-153, and 198-210), 162 bands remain. An 80×80 pixel region was extracted for experimentation, with vehicles as the anomalies. The HYDICE real data is shown in [Figure 4: see original paper], containing 14 anomalous pixels.

### 3.2 Simulated Data Experiments

Using the data shown in [Figure 2: see original paper], this experiment features relatively simple background with significant differences between anomalies and background, and high similarity among anomalous pixels. Under these conditions, the CRD algorithm performs poorly when the center pixel is anomalous and the background dictionary contains the same anomaly type. For comparison with CRD, the parameter $\lambda$ was set to the same value as in reference [15], and this value was used in all subsequent experiments.

When the outer window size is 11 and inner window size is 5, the pixel at coordinates (55,44) serves as the center pixel. In this case, the center pixel is anomalous and the background dictionary contains the same anomaly type. Comparing the weight vectors of both algorithms reveals significant differences. [Figure 5: see original paper] shows the weight vectors: (a) for the CRD algorithm and (b) for the improved algorithm. The anomalous atom in the background dictionary that matches the center pixel is atom 69. The CRD algorithm assigns a weight of 0.8879 to this atom, while the improved algorithm assigns only 0.0937. Using equations (7) and (11), the outputs are 0.0105 for the original algorithm and 0.0251 for the proposed algorithm, demonstrating that the improved algorithm reduces weights for anomalous atoms in the background dictionary and increases the center pixel output.

The Area Under the Curve (AUC) of the Receiver Operating Characteristic (ROC) curve evaluates detection performance, with larger AUC values indicating better classifier performance. shows the AUC values for the CRD algorithm across different window sizes. Notably, when the inner window size is 5 or the outer window size is 17, the AUC values are relatively small. Analysis of the target distribution reveals that these configurations exhibit the problematic scenario where the center pixel is anomalous while the background dictionary contains the same anomaly type. In such cases, the CRD algorithm produces small Euclidean distances between the center pixel and similar anomalies, resulting in large weights for these anomalous atoms. This makes the estimated value close to the center pixel, yielding small residuals that fall below the detection threshold and cause misclassification.

presents the AUC values for the improved algorithm across different window sizes. When the inner window size is 5 or outer window size is 17, the improved algorithm achieves higher AUC values than the original algorithm. By adjusting atom weights based on distance to the dictionary mean, the improved algorithm increases background pixel weights and decreases anomalous pixel weights. Consequently, anomalies similar to the center pixel receive smaller weights, creating larger differences between the center pixel and its estimate, which increases output values and improves detection accuracy. For other window sizes, the improved algorithm' s AUC values remain essentially identical to those of the original algorithm.

### 3.3 Real Data Experiments

**First Real Dataset (AVIRIS)**: Using the data shown in [Figure 3: see original paper], this dataset includes buildings, aprons, runways, and sparse vegetation as background. The target size is larger than in the simulated data, making small inner windows less effective for detection. shows the CRD algorithm' s AUC values across window sizes. The AUC decreases significantly when the inner window size is 7, 9, or 11, or when the outer window size is 27. Target distribution analysis confirms that these configurations exhibit the problematic scenario. shows that the improved algorithm achieves substantially higher AUC values under these conditions, while maintaining comparable performance for other window sizes.

**Second Real Dataset (HYDICE)**: Using the data shown in [Figure 4: see original paper], this dataset features seven vehicles of different sizes, some positioned close together. Large outer window sizes can cause the problematic scenario. shows the CRD algorithm' s AUC values, which are smaller when the outer window size is 15 or 17. demonstrates that the improved algorithm achieves higher AUC values under these conditions, again validating its effectiveness.

### 3.4 Algorithm Time Performance

During pixel-wise detection, both algorithms use different diagonal matrices to adjust background dictionary atom weights. The proposed algorithm requires computing the background dictionary mean and then the Euclidean distance between each atom and this mean, whereas the original algorithm directly computes distances between the center pixel and each atom. compares the runtime across the three datasets, with each value representing the average runtime across different window sizes. The results show that the proposed algorithm requires slightly more time, but the difference is negligible because the mean computation time is minimal relative to the total runtime.

## 4 Conclusion

This paper proposes an improved collaborative representation algorithm for hyperspectral image anomaly detection. By adjusting background dictionary atom weights based on distance to the dictionary mean, the algorithm reduces weights for anomalous pixels while increasing weights for background pixels. When the center pixel is anomalous and the background dictionary contains the same anomaly type, the proposed algorithm effectively reduces anomalous atom weights and increases the center pixel output, thereby improving detection accuracy. Experimental results demonstrate superior detection precision compared to the original algorithm under problematic window configurations, while maintaining comparable performance for other configurations.

## References

[1] Matteoli S, Diani M, Corsini G. A tutorial overview of anomaly detection in hyperspectral images [J]. IEEE Aerospace & Electronic Systems Magazine, 2010, 25(7): 5-28.

[2] Li M, Zhu G, Zhang X, et al. Hyperspectral anomaly detection algorithm based on multi-aperture mapping [J]. Acta Geodaetica et Cartographica Sinica, 2016, 45(10): 1222-1230.

[3] Zhao C, Deng W, Yao X. Real-time anomaly target detection in hyperspectral imagery based on line-by-line processing [J]. Acta Optica Sinica, 2017, 37(1): 0128002.

[4] He L, Pan Q, Di W, et al. Advances in hyperspectral image target detection [J]. Acta Electronica Sinica, 2009, 37(9): 2016-2024.

[5] Yuan Y, Wang Q, Zhu G. Fast hyperspectral anomaly detection via high-order 2-D crossing filter [J]. IEEE Trans on Geoscience & Remote Sensing, 2014, 53(2): 620-630.

[6] Zhao C, You W, Qi B, et al. Real-time hyperspectral remote sensing anomaly detection algorithm using polynomial recursive kernel [J]. Acta Optica Sinica, 2016, 36(2): 0228002.

[7] Zhang Y, Du B, Zhang L, et al. A low-rank and sparse matrix decomposition-based Mahalanobis distance method for hyperspectral anomaly detection [J]. IEEE Trans on Geoscience & Remote Sensing, 2016, 54(3): 1376-1389.

[8] Reed I S, Yu X. Adaptive multiple-band CFAR detection of an optical pattern with unknown spectral distribution [J]. IEEE Trans on Acoustics Speech & Signal Processing, 1990, 38(10): 1760-1770.

[9] Kwon H, Nasrabadi N M. Kernel RX-algorithm: a nonlinear anomaly detector for hyperspectral imagery [J]. IEEE Trans on Geoscience & Remote Sensing, 2005, 43(2): 388-397.

[10] Banerjee A, Burlina P, Diehl C. A support vector method for anomaly detection in hyperspectral imagery [J]. IEEE Trans on Geoscience & Remote Sensing, 2006, 44(8): 2282-2291.

[11] Zhao C, Li X, Wang Y. Research progress on hyperspectral image anomaly target detection [J]. Journal of Electronic Measurement and Instrumentation, 2014, 28(8): 803-811.

[12] Zhou J, Kwan C, Ayhan B, et al. A novel cluster kernel RX algorithm for anomaly and change detection using hyperspectral images [J]. IEEE Trans on Geoscience & Remote Sensing, 2016, 54(11): 6497-6504.

[13] Zhao C, Yao X F, Yan Y. Modified kernel RX algorithm based on background purification and inverse-of-matrix-free calculation [J]. IEEE Geoscience & Remote Sensing Letters, 2017, 14(4): 544-548.

[14] Mei F, Zhao C, Wang L, et al. Adaptive hyperspectral anomaly detection algorithm based on support vector data description [J]. Acta Photonica Sinica, 2009, 38(11): 2820-2825.

[15] Li W, Du Q. Collaborative representation for hyperspectral anomaly detection [J]. IEEE Trans on Geoscience & Remote Sensing, 2015, 53(3): 1463-1474.

[16] Zhang L, Zhao C, Cheng B. Joint kernel collaborative representation for hyperspectral image anomaly target detection [J]. Journal of Optoelectronics · Laser, 2015, 26(11): 2154-2161.

[17] Tang Y, Huang S, Ling Q, et al. Adaptive kernel joint representation anomaly detection for hyperspectral images [J]. High Power Laser and Particle Beams, 2015, 27(9): 49-55.

*Note: Figure translations are in progress. See original paper for figures.*